Realisability of systems by discrete groups

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Let p be a fixed prime. A discrete p-toral group is a group that contains a normal subgroup $T\cong Z/p^\infty^{\times r}$ of p-power index, where $r\geq 0$. A fusion system over a discrete p-toral group S is a category whose objects are the subgroup of S and whose morphisms are homomorphisms between them that satisfy a set of axioms. Fusion systems over discrete p-toral groups have been shown to admit a ``classifying space” that is unique up to equivalence. Particular examples arise from finite groups (the finite case), compact Lie groups, p-compact groups and linear torsion groups. In all these cases the classifying space of the corresponding fusion system is homotopy equivalent to the p-completed classifying space of the object that gives rise to it. A fusion systems over a finite $p-group is said to be exotic if it does not arise from a genuine finite group. We found many such examples. Until recently it was not clear what should be meant by an exotic fusion system over an infinite discrete p-toral group. The project I will report on consists of two parts. In the first part we investigated four different ways of realising fusion systems over discrete p-toral groups by discrete groups, the most general of which we call ``sequential realisability”. This means that the fusion system in question is in the appropriate sense a colimit of a sequence of p-local finite groups, each of which is realisable by a genuine finite group (but there need not be any nontrivial homomorphism between any of these groups). In the second part we concentrated on a specific very general type of discrete groups, and showed that for each such group one can associate a fusion system, A discrete group G is said to be 1) locally finite, if every finitely generated subgroup is finite, 2) a p-group if every element in it has a p-power order, and 3) artinian if it satisfies the descending chain condition. A group is said to be p-artinian if every p-subgroup of it is artinian. If G is locally finite and p-artinian, then every p-subgroup of G is locally finite and artinian, and hence is discrete p-toral by standard results. We show how to associate a fusion system to any locally finite p-artinian group G in such a way that the classifying space of the fusion system is homotopy equivalent to the p-completed classifying space of G. In doing so we need to deal with several nontrivial problems, one of which is that our groups need not have a Sylpw p-subgroups in the usual sense. To prove the equivalence of the classifying spaces of the group and the fusion system we make use of a theorem of Gonzalez which expresses the cohomology of the classifying space of fusion system over a discrete p-toral group as the stable elements in the cohomology of the Sylow subgroup. In this talk I will start with the very essential Dave Benson connection. After introducing the basics I will concentrate on the second part of the project.