Numerical analysis of first order Mean Field Games under displacement monotonicity — Part 2

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In a recent joint work with A. Meszaros (Durham University), we introduced a particle method for the numerical approximation of time-dependent first-order Mean Field Games (MFGs) systems with non-separable, displacement monotone Hamiltonians and terminal costs, for arbitrary time-horizons and (possibly) singular initial player distributions. The numerical scheme is based on an implicit Euler discretisation in time and sampling in space of the characteristic Hamiltonian/Pontryagin system associated with the continuous MFGs system. We prove convergence of the approximations of the player distribution uniformly in time in the 2-Wasserstein metric and the approximations for the gradient of the value function along optimal trajectories uniformly in time in a suitable L^2-norm as the number of spatial samples tends to infinity jointly with the temporal time-step vanishing. The error bound that we establish for this convergence further implies rates of convergence of the scheme for a range of spatial sampling techniques. Provided that the Lagrangian and terminal costs are additionally locally Lipschitz continuous, we also establish an asymptotic error bound uniformly in time in the L^1-norm for the approximations of the value function along optimal trajectories. We illustrate the performance of the scheme in numerical experiments for a range of initial agent distributions, time horizons, and space dimensions. In this Part 2, we present elements of the analysis of the numerical scheme and the corresponding simulations, while Part 1 with focus on the analysis of the continuous problem will be presented in the talk of A. Meszaros.