

Arveson's extension theorem for selfadjoint operator spaces

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Operator systems, i.e., closed, selfadjoint subspaces of $B(H)$ containing its unit, have a central role in the theory of Operator Algebras. Lately, the community has been actively considering selfadjoint operator subspaces, but which need not be unital. Those appear naturally, for example when comparing operator systems up to their stabilisations as in the work of Connes and van Suijlekom. The absence of a unit disrupts the link between the norm and the matrix order structure, and thus many of the standard results from the unital setting no longer apply. This creates significant difficulties in obtaining even fundamental theorems like the celebrated Arveson's Extension Theorem. Towards this end, Werner introduced the notion of the partial unitisation that encapsulates the appropriate information for matrix ordered operator space to be represented as a selfadjoint subspace of some $B(H)$. It appears that the injective morphisms in this category are those that lift to unital completely isometric maps on the unitisation, and not just the completely isometric complete order embeddings. In this talk I will present why this is the right notion, and (old and new) characterisations of such maps in terms of extensions and gauge isometries (in the sense of Russell), with a focus on approximately positively generated or singly generated spaces. We will show, in action and with examples, that the question whether a map is an embedding is a norm-preservation question. This is joint work with Alexandros Chatzinikolaou, Se-Jin (Sam) Kim, and Apollonas Paraskevas.