Dimension theories from groupoids to classifiable C*-algebras, and back again

Kang Li

The motivation comes from the spectacular breakthrough in the Elliott classification program for simple nuclear C*-algebras: the class of all separable, simple, finite nuclear dimensional C*-algebras satisfying the UCT is classified by their Elliott invariants. Shortly after, Xin Li proved that those classifiable C*-algebras have a twisted étale groupoid model (G, Σ). A natural question is which twisted étale groupoid C*-algebras have finite nuclear dimension. Very recently, Bönicke and I have extended the previous results to show that their nuclear dimensions are bounded by the dynamic asymptotic dimension of the underlying groupoid G and the covering dimension of its unit space G_0, and are actually independent of Σ. The essential flaw is that dynamic asymptotic dimension cannot be consistent with nuclear dimension for simple C*-algebras because every simple C*-algebra with finite nuclear dimension has nuclear dimension either zero or one. Therefore, we (together with Liao and Winter) introduced the so-called diagonal dimension for an inclusion ($D \subseteq A$) of C*-algebras. In this talk, I will explain how the diagonal dimension of ($C_0(G_0) \subseteq C_r^*(G,\Sigma)$) is indeed consistent with dynamic asymptotic dimension of G and the covering dimension of G_0. Moreover, we compute the diagonal dimension and the dynamic asymptotic dimension for Xin Li's groupoid model. This is joint work with Christian Bönicke, as well as Zehong Huang and Hang Wang.