Retreat for women in applied mathematics 2026

UNIVERSITY OF CAMBRIDGE

Geometric Field Theory for Elastohydrodynamics of Cosserat Rods

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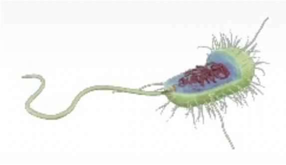
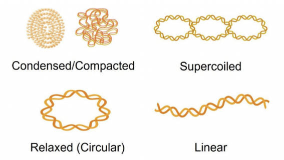
Joint work with M. Warda, B. Németh, L. Kikuchi and R. Adhikari **arXiv:2510.18097**

Problem Set-up

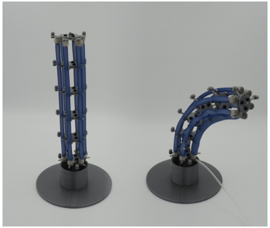
Cosserat rods everywhere



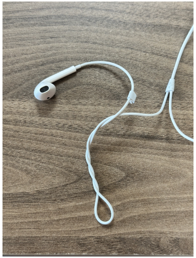
(a) Bacteria flagellum [1]



(b) DNA supercoiling [2]



(c) Soft robot arm [3]



(d) My earphones

Figure: Slender structures as **Cosserat rods** are ubiquitous in biology and soft robotics.

Development of Methodology

3-step flowchart

Vectorial expressions

(Traditional vector mechanics obscures geometric structure)

↓

Moving frame representation

(Frame-invariant components reveal spatial symmetries)

↓

Lie group and differential forms formulation

(Geometric field theory with manifest coordinate independence)

The Geometric Insight

Configuration living on $SE(3)$

Each cross-section's configuration $\varphi(u,t)$ is an element of $SE(3)$ with **position** $\mathbf{r}(u,t)$ and **orientation** $\mathbf{e}_i(u,t)$.

- $\varphi(u,t) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ \mathbf{r} & \mathbf{e}_1 & \mathbf{e}_2 & \mathbf{e}_3 \end{pmatrix} \in SE(3)$

- Lie group kinematics:

$$\partial_u \varphi = \varphi E, \quad \partial_t \varphi = \varphi V,$$

$$E = \begin{pmatrix} 0 & 0 & 0 & 0 \\ h_1 & 0 & -\Pi_3 & \Pi_2 \\ h_2 & \Pi_3 & 0 & -\Pi_1 \\ h_3 & -\Pi_2 & \Pi_1 & 0 \end{pmatrix} \in \mathfrak{se}(3) : \text{generalised strain,}$$

$$V = \begin{pmatrix} 0 & 0 & 0 & 0 \\ v_1 & 0 & -\Omega_3 & \Omega_2 \\ v_2 & \Omega_3 & 0 & -\Omega_1 \\ v_3 & -\Omega_2 & \Omega_1 & 0 \end{pmatrix} \in \mathfrak{se}(3) : \text{generalised velocity}$$

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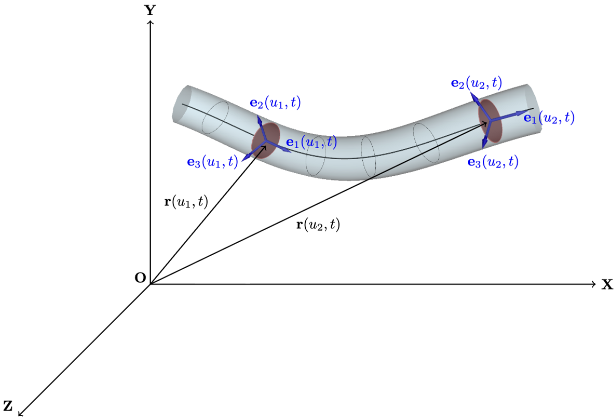


The Geometric Field Theory

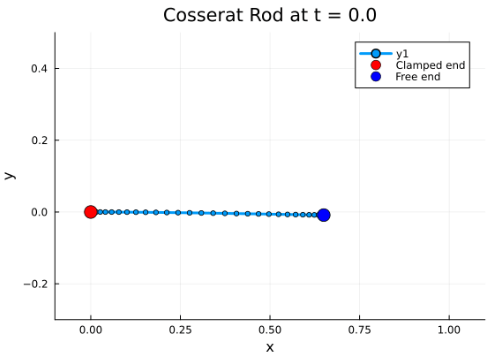
Unified framework

Key Results (in the overdamped regime):

- **Kinematics:** $d\varphi = \varphi\xi, \quad \xi = Edu + Vdt$
- **Compatibility condition:** $d\xi + \xi \wedge \xi = 0$
- **Dynamics:** $\mathcal{D}^*\Sigma + j = 0, \quad \mathcal{D}^* := \partial_u + \mathrm{ad}^*_E$
- **Constitutive laws:** $\underline{\Sigma} = \underline{\Sigma}(E), \quad \underline{j} = -\underline{\underline{\Gamma}} \cdot \underline{V} + \underline{j}(\varphi)$
- **Geometrised elastohydrodynamics equation:**
  $\dot{\varphi} = \varphi V, \quad \underline{V} = \underline{\underline{\Gamma^{-1}}}[\mathcal{D}^*\underline{\Sigma}(\varphi^{-1}\varphi') + \underline{j}(\varphi)].$

Applications

- Structure-preserving geometric numerical methods
- Actuated/active motion of rods



Takehome Messages

We have developed a **coordinate-independent, geometrically exact field theory** to study the non-linear stretching, shearing, bending and twisting motion of a rod in a viscous fluid,

using **structure-preserving geometric numerical integration**,

with applications in **actuated soft robotics**.

Thank you for your attention!

This talk is based on the arxiv preprint
Geometric Field Theory for Elastohydrodynamics of Cosserat Rods [4] (currently under peer review)

References

[1] Visible Body. *Visible Body*. Accessed 07/09/2025. URL: https://www.visiblebody.com/learn/biology/cells/prokaryotic-cells.

[2] Lohra M. Miller, Luke Hawkins, and Martin F. Jarrold. “Compaction, Relaxation, and Linearization of Megadalton-Sized DNA Plasmids: DNA Structures Probed by CD-MS”. In: *Journal of the American Society for Mass Spectrometry* 35.8 (2024). PMID: 39013154, pp. 1969–1975. DOI: 10.1021/jasms.4c00222.

[3] Gina Olson et al. “An Euler–Bernoulli beam model for soft robot arms bent through self-stress and external loads”. In: *International Journal of Solids and Structures* 207 (2020), pp. 113–131. ISSN: 0020-7683. DOI: https://doi.org/10.1016/j.ijsolstr.2020.09.015.

[4] Mingjia Yan et al. *Geometric Field Theory for Elastohydrodynamics of Cosserat Rods*. 2025. arXiv: 2510.18097 [cond-mat.soft]. URL: https://arxiv.org/abs/2510.18097.