Realistic Modelling of Human Head Tissues Exposure to Electromagnetic Waves

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Background

The following case study describes a research project conducted by the French National Institute for Computer Science and Automatics (INRIA) in collaboration with the French telecommunications company Orange, published in 2006. This project focused on the numerical modelling of the propagation of electromagnetic waves emitted from mobile phones through the human head [2]. As the popularity of mobile phones increased in the early 2000s, as did the concern for possible consequences of electromagnetic radiation on human health, due to the transmitting antenna of the mobile phone being positioned very close to the user's head when in use, thus absorbing a substantial portion of the radiated power. This project focuses on combining MRI images of the human head with finite element methods on unstructured grids to accurately numerically model the interaction of electromagnetic waves emitted by mobile phones on biological tissues.

Mathematical Approach

Mobile phones communicate by transmitting radio waves through a network of fixed antennas. These radiowaves are a type of low-frequency electromagnetic radiation and are non-ionising. However, the possible thermal biological effects of this type of radiation must be investigated, as they may be harmful. The propagation of electromagnetic waves is modelled by Maxwell's equations. Let $\Omega \in \mathbb{R}^3$ denote a bounded connected Lipschitz domain with connected boundary $\Gamma := \partial\Omega$, and let $\mathbf{x} \in \Omega$ denote the spatial variable. The governing system for the electric field intensity $\boldsymbol{E}$ and magnetic field strength $\boldsymbol{H}$ are given by

$$\epsilon(\mathbf{x})\frac{\partial \boldsymbol{E}}{\partial t} = \boldsymbol{\nabla} \times \boldsymbol{H} - \sigma(\mathbf{x})\mathbf{E}, \tag{1}$$

$$\mu(\mathbf{x})\frac{\partial \boldsymbol{H}}{\partial t} = -\boldsymbol{\nabla} \times \boldsymbol{E}, \tag{2}$$

where ϵ, μ, σ respectively denote the electric permittivity, magnetic permeability, and electrical conductivity of the medium through which the wave propagates, and t denotes time. The domain Ω represents the human head and is heterogenous. We assume that the material parameters, ϵ, μ, σ, are constant for each tissue type, and thus these parameters are piecewise-constant across Ω.

Domain Segmentation and Mesh Generation

The most common method used at the time to solve the time-domain Maxwell equations (1) was the Finite Difference Time-Domain (FDTD) method. In this approach, the computational domain is discretised using a structured Cartesian grid, which can be directly derived from MRI data. However, this method has well-known limitations in terms of accuracy. In particular, cell-size constraints and the use of a rectilinear grid make it difficult to accurately resolve the complex geometry of head tissues and the detailed structure of the mobile phone.

The aim of this work was therefore to develop a more accurate discretisation of the human head based on an unstructured grid of tetrahedral elements. Starting from MRI data, the head is first segmented into distinct tissue regions by assigning each voxel to a single material type. Tissue regions are then defined as connected components of voxels sharing the same label. The interfaces between adjacent tissue regions are reconstructed and discretised using unstructured triangular surface meshes. These surface meshes serve as inputs to a volumetric mesh generator, which produces a fully unstructured tetrahedral discretisation of the tissue volumes. The exterior of the head is also meshed, extending to an artificial far-field boundary where absorbing boundary conditions are imposed. This procedure yields a material-conforming tetrahedral mesh in which parameter discontinuities are aligned with element interfaces, enabling more accurate numerical discretisations of Maxwell's equations on heterogeneous media. We denote this mesh by $\mathcal{T}_h$.

Numerical Methods for Solving the Governing System

The governing system (1) is solved using a Discontinuous Galerkin Time-Domain (DGTD) finite element method on the unstructured tetrahedral mesh. Let $\mathbf{E}_h$ and $\mathbf{H}_h$ denote the numerical approximations of the electric and magnetic fields, respectively. Within each element $\tau \in \mathcal{T}_h$, the fields are approximated by vector-valued polynomials of degree at most k,

$$\mathbf{E}_h|_\tau, \mathbf{H}_h|_\tau \in \mathbb{P}_k(\tau)^3,$$

with no continuity enforced across element interfaces. A local weak formulation is constructed on each element, and inter-element coupling is achieved through numerical fluxes defined on element faces. The solver employs centered numerical fluxes, corresponding to arithmetic averages of the electromagnetic fields across interfaces. This choice yields a non-dissipative scheme that conserves electromagnetic energy at the discrete level in the absence of material losses. Time integration is carried out using an explicit leap-frog scheme, with the electric and magnetic fields staggered in time. For $k = 0$, the method reduces to a finite-volume time-domain scheme, while higher-order approximations improve spatial accuracy at the expense of stricter stability constraints.

Results

The numerical experiments consider a simplified head model composed of four tissue types: skin, skull, cerebrospinal fluid (CSF), and brain. A simplified mobile phone model is also included, consisting of a metallic casing and an internal antenna excited by a dipole source operating at a central frequency of 1.8 GHz.

A representative numerical result is shown in Figure 1, which displays the distribution of the Specific Absorption Rate (SAR) on the surface of the head. The SAR is the primary dosimetric quantity used in international exposure guidelines and is defined as

$$\mathrm{SAR} = \frac{\sigma}{\rho}|\mathbf{E}|^2, \tag{3}$$

where ρ denotes the tissue density.

The results indicate that electromagnetic energy absorption is highly localised in the region closest to the phone, with peak SAR values occurring near the ear. Lower SAR levels are observed elsewhere on the head, reflecting the rapid spatial attenuation of the electromagnetic field within biological tissue.

The authors further compare results obtained using a finite-volume time-domain discretisation ($k = 0$) on a fine mesh with those obtained using a first-order DGTD discretisation ($k = 1$) on a coarser mesh, for comparable numbers of degrees of freedom. While global absorption levels are similar, noticeable differences in the local SAR distributions are observed, highlighting the influence of discretisation order and mesh resolution on dosimetric accuracy.

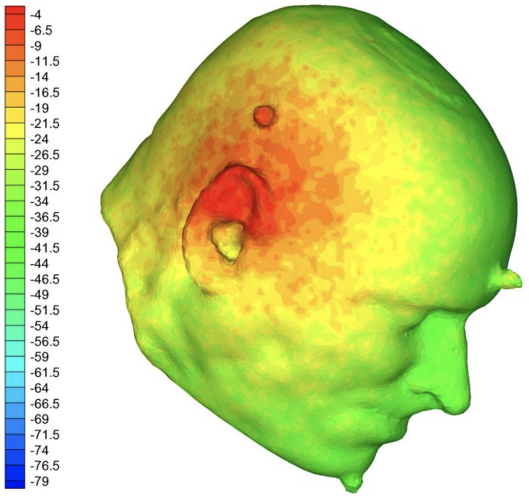

Figure 1: Distribution of the normalised Specific Absorption Rate SAR/SAR$_{\text{max}}$ (in dB) on the surface of the head. Source: Scarella et al. [2].

Industrial Outcomes

This research partnership between INRIA and Orange has led to the development of a new modelling approach based on a highly accurate discontinuous finite element method, which is particularly well suited to the discretisation of heterogeneous propagation media and that can easily handle locally refined unstructured meshes [1]. This new technique can be applied not just to radiation exposure from mobile phones, but to any application where electromagnetic waves propagate through non-uniform mediums.

References

[1] Thibaut Lery et al. *European success stories in industrial mathematics*. Springer Science & Business Media, 2011.

[2] Gilles Scarella et al. "Realistic numerical modelling of human head tissue exposure to electromagnetic waves from cellular phones". In: *Comptes Rendus Physique* 7.5 (2006), pp. 501–508.