Characterization of eternal solutions of the KPZ fixed point

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The KPZ fixed point is a Markov process on the space of upper semi-continuous functions. It is the conjectured universal scaling limit of the height function evolution for models in the KPZ universality class and has been shown to be such for many solvable (and even some unsolvable) models. The directed landscape provides a coupling for the growth of the KPZ fixed point starting from all initial conditions. Under this coupling, starting from an initial condition, the forward evolution of the KPZ fixed point can be described by a variational problem involving the directed landscape. It is an interesting question to characterize all eternal solutions of the KPZ fixed point (i.e. functions defined for both forward and backward times and satisfying the variational formula at all times). In this talk we give a full characterization of these eternal solutions.

Directed landscape can be thought of as a random metric space and geodesics and semi-infinite geodesics are well studied objects. Semi-infinite geodesics in different directions can be studied via corresponding Busemann functions, and [Busani, Sepp¨al¨ainen, Sorensen, 2024] showed that Busemann functions associated to different directions are examples of eternal solutions. We show that a general eternal solution can be described by patching up different Busemann functions along certain interfaces. We will see that these interfaces show some interesting geometric behaviors, and describe how their geometry uniquely characterizes the eternal solution.

The talk is based on joint works with Ofer Busani and Evan Sorensen.