

Dilating Semigroup Representations to the Boundary Quotient

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We study which representations of semigroups can be dilated to representations of their boundary quotients, and what relations these dilations must have. By Sehnm's equivalence of the reduced boundary quotient with the C^* -envelope of the reduced semigroup operator algebra, the problem reduces to dilating the representations to the C^* -envelope in the group-embeddable case. Using dilation results by Dritschel and McCough, we prove that dilations to the boundary are always possible as long as we have a completely contractive representation of the reduced semigroup operator algebra. That is guaranteed as long as the semigroup is amenable and the representation is constructible, but also in a very wide variety of other cases. Among those other cases are several classical dilation theorems, which can thus be viewed as special cases of this very general dilation result. This also demonstrates that a completely contractive representation of the operator algebra dilates to a representation with additional relations among its generators, the new relations coming from the boundary quotient. The results can be further generalised beyond the group-embeddable setting, in the right LCM case by recent results due to Brix, Bruce, and Dor-On.