

The Column-Diamond Resolution over Quadratic Monomial Rings

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Free resolutions of modules over monomial quotient rings often encode rich combinatorial structure, yet explicit constructions can be difficult to obtain. In this talk, we introduce the Column-Diamond construction, a recursive method for building infinite periodic free resolutions of modules obtained by quotienting a quadratic monomial quotient ring by one of its variables. The construction proceeds by iteratively attaching two elementary types of building blocks, called column complexes and diamond complexes, determined by the combinatorics of the defining monomial relations.

We prove that this construction yields an exact complex. As an application, we obtain a combinatorial description of the Betti numbers: the i -th Betti number is equal to the number of valid words of length i in a graph naturally associated with the defining monomial ideal. This interpretation leads to a proof that the generating function of the Betti sequence is rational.

This talk is based on joint work with Daniel Bravo and Federico Castillo.