Walks in the quadrant, differential Galois theory and elliptic curves

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The study of walks confined in a cone is a central subject in enumerative combinatorics. Indeed, this combinatorial class is in bijection with numerous classes of objects in discrete mathematics (permutations, trees, planar maps, etc.) in statistical physics (polymers, etc.) or even in probability (random walks, Brownian movements, etc.). In recent years, the work of numerous researchers in combinatorics, computer algebra, probability and differential Galois theory has made it possible to fully classify small-step walks and to develop robust strategies for studying large-step ones.

In this presentation, I will try to survey the main ingredients of this classification and show how the study of small steps is linked to that of elliptic curves on function fields and their Mordell-Weil lattice. If time permits, I will show how the classification of large steps walks in the so-called finite orbit case can be tackled using classical Galois theory.