Lie methods for omega-categorical Engel groups

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A structure (group, Lie algebra, associative algebra, etc) M is omega-categorical if there is a unique countable model of its first-order theory, up to isomorphism. This model theoretic notion has a dynamical definition: M is omega-categorical if and only if there are only finitely many orbits in the component-wise action of Aut(M) on the cartesian power M^n, for all natural number n. In 1981, Wilson conjectured that any omega-categorical locally nilpotent p-group is nilpotent. If true, a quite satisfactory decomposition of every omega-categorical groups would follow. This conjecture is very much open more than 40 years later.

The analogue statement for Lie algebras (every locally nilpotent omega-categorical Lie algebra is nilpotent) is also open. Both statement can be reformulated as: is any omega-categorical Engel group/Lie algebra nilpotent. As such, those questions are connected to Burnside-type problems and the work of Higman, Kostrikin, Zelmanov, Vaughan-Lee, Traustason, etc. For instance, using a classical result of Zelmanov, the conjecture for Lie algebras is true asymptotically in the following sense: for each n, every n-Engel Lie algebra over F_p is nilpotent for all but finitely many p's. There is a similar statement for groups. The situation for small values of the pair (n,p) is highly characteristic-dependent, for instance, 4-Engel Lie algebras over a field of characteristic p are nilpotent except if p = 2, 3 or p = 5. I recently proved that omega-categorical n-Engel Lie algebras over a field of characteristic p are nilpotent for (n,p) = (3,5) and (n,p) = (4,3). In this talk I will explain how to use those results to deduce that omega-categorical 3-Engel p-groups and 4-Engel p-groups (with p odd) are nilpotent.