Free-to-Interacting Maps for Quasicrystalline Symmetry-Protected Topological Phases

Quasicrystals in Fundamental Physics | Edinburgh Futures Institute

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Outline

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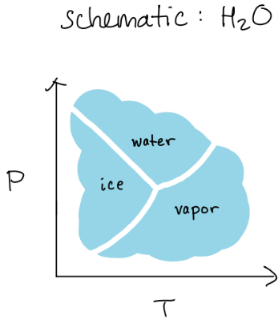
Symmetry-Protected Topological Phases

SPT phases are *invertible* phases of matter defined via...

- $\mathcal{M}$ = space of Hamiltonian operators with symmetries
- Δ = space of gapless Hamiltonians
- phase $\in (\mathcal{M} \setminus \Delta)/\sim$



SPT phases can be *free* or, more generally, *interacting*...

- *free* Hamiltonians are a subset $\mathcal{M}_{\text{free}} \subset \mathcal{M}_{\text{int}}$

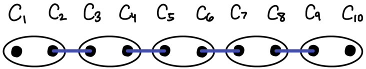
Question

How does $(\mathcal{M}_{\text{free}} \setminus \Delta)/\sim_{free}$ compare with $(\mathcal{M}_{\text{int}} \setminus \Delta)/\sim_{int}$?

Example: Free and Interacting Fermionic SPTs

The group of *free* $d = 1$ class BDI SPTs is $\mathbb{Z}$, generated by the TRS Majorana chain:

$$\hat{H} = \frac{i}{2}\left(\sum_{\ell} \hat{c}_{2\ell}\hat{c}_{2\ell+1}\right).$$



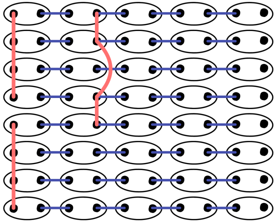
Interacting Trivialization [Fidkowski–Kitaev '10]

Eight copies of the TRS Majorana chain trivializes

$$\hat{H} = \sum_{\alpha=1}^{8} \hat{H}_\alpha \qquad \text{where} \qquad \hat{H}_\alpha = \frac{i}{2}\left(\sum_{\ell} \hat{c}^\alpha_{2\ell}\hat{c}^\alpha_{2\ell+1}\right)$$

with interaction term $\hat{c}^1_\ell \hat{c}^2_\ell \hat{c}^3_\ell \hat{c}^4_\ell + \hat{c}^5_\ell \hat{c}^6_\ell \hat{c}^7_\ell \hat{c}^8_\ell + \hat{c}^1_\ell \hat{c}^2_\ell \hat{c}^5_\ell \hat{c}^6_\ell + \dots$

$\implies$ The group of *interacting* BDI SPTs in $d = 1$ is $\mathbb{Z}/8$



Our [Debray–K.–Pacheco-Tallaj–Stehouwer] Approach

- Following [Freed–Hopkins, '16], we model the passage ($[H]_{free} \mapsto [H]_{int}$) from free to interacting SPTs as a homotopical map...

$$\begin{tikzcd}
\left\{\begin{array}{c}\text{“unstable”}\\ \text{free ”}\end{array}\right\} \arrow[r] & \left\{\begin{array}{c}\text{free fermion}\\ G\text{-SPTs}\end{array}\right\} \arrow[r] \arrow[d, "\simeq"] & \left\{\begin{array}{c}\text{interacting fermion}\\ G\text{-SPTs}\end{array}\right\} \arrow[r] \arrow[d, "\simeq"', "\text{ansatz}"] & \left\{\begin{array}{c}\text{intrinsically}\\ \text{interacting ”}\end{array}\right\} \\
\ker \arrow[r] & KO_G^{d-2,\tau}(X) \arrow[r, dashed, "F2I"] & \mho_\xi^{d+1}(X) \arrow[r] & \operatorname{coker} \\
 & \textit{K-theory} \arrow[u, squiggly] & \textit{invertible field theories} \arrow[u, squiggly] &
\end{tikzcd}$$

- Today: construct & compute *F2I* for tenfold-way weak phases & cut-and-project quasicrystals

Outline

Free Fermion SPTs: Tenfold Way

- *K*-theory classifies gapped free fermion Hamiltonians e.g. [Kitaev, '09, Ryu–Schnyder–Furusaki–Ludwig, '10] or spectral gap projections e.g. [Bellissard, '80s]
- **Ansatz**: If A is a superalgebra encoding symmetries, then *free fermion SPT phases in spatial dimension d* are classified by the group $K_{2-d}(A)$

Tenfold Way

Internal *symmetries* (time reversal, particle number conservation) for free fermion systems form representations of a Clifford algebra $C\ell_s$ e.g. [Altland–Zirnbauer, '97]:

$$K_{2-d}(C\ell_s) \cong KO^{d+s-2} \qquad \text{and} \qquad K_{2-d}(\mathbb{C}\ell_s) \cong KU^{d+s-2}$$

AZ Class	D	BDI	AI	CI	C	CII	AII	DIII	A	AIII
Algebra	$C\ell_0$	$C\ell_{+1}$	$C\ell_{+2}$	$C\ell_{+3}$	$C\ell_4$	$C\ell_{-3}$	$C\ell_{-2}$	$C\ell_{-1}$	$\mathbb{C}\ell_0$	$\mathbb{C}\ell_1$

Interacting SPTs I: Tangential Structures

Definition

A *stable tangential structure* is an object $(\xi \colon X \to BO) \in \mathsf{Spc}_{/BO}$.

A stable tangential structure on a manifold M is a lift...

$$\begin{tikzcd} & X \arrow[d, "\xi"] \\ M \arrow[ur, dashed] \arrow[r, "TM"'] & BO. \end{tikzcd}$$

Examples

- $X = BSO \rightsquigarrow$ orientations
- $X = B\mathrm{Spin} \rightsquigarrow$ spin structures
- $X = B\mathrm{Pin}^{\pm} \rightsquigarrow$ $\mathrm{pin}^{\pm}$ structures

Interacting SPTs II: Bordism

Definition

The bordism group of d-manifolds with ξ-structure is

$$\Omega_d^\xi = (\{\text{closed } d\text{-dim'l } \xi\text{-manifolds}\}/\text{bordism}, \sqcup).$$

Example

In dimension 2 with $\xi \colon B\mathrm{Pin}^- \to BO$, the group is $\Omega_2^{\mathrm{Pin}^-} \cong \mathbb{Z}/8$ generated by $[\mathbb{RP}^2]$.

Pontryagin–Thom

Bordism for ξ-manifolds is encoded by the *Madsen–Tillmann spectrum*:

$$\pi_d(MT\xi) \cong \Omega_d^\xi.$$

Interacting SPTs III: TQFTs

Definition [Atiyah, '88]

An n-dimensional TQFT is a symmetric monoidal functor

$$\mathcal{Z}\colon \mathrm{Bord}_n \to \mathrm{Vect}_{\mathbb{C}}$$

from the *bordism category* Bord_n of closed $(n-1)$-manifolds and bordisms to the category $\mathrm{Vect}_{\mathbb{C}}$ of complex vector spaces.

- closed $(n-1)$-manifold $M \longmapsto \mathcal{Z}(M)$ the *state space* of the theory on M
- closed n-manifold $W \longmapsto \mathcal{Z}(W) \in \mathbb{C}$ the value of the *partition function* of $\mathcal{Z}$

Ansatz [Freed–Hopkins, '21]

Interacting (fermionic) SPT phases of a given symmetry type are classified by the deformation class of their low energy field theory, which is a *fully extended reflection-positive invertible field theory* on manifolds with certain (twisted spin) tangential structure.

Interacting SPTs IV: Invertible Field Theories

Theorem [Freed–Hopkins '16, Grady '23]

Let ξ be a stable tangential structure.

$$\left\{\begin{array}{c}\text{fully extended reflection-positive}\\ (d+1)\text{-dim'l invertible } \xi\text{-theories}\end{array}\right\}/\text{deformation} \cong [MT\xi, \Sigma^{d+2} I_{\mathbb{Z}}] =: \mho_\xi^{d+1}$$

- There is a short exact sequence

$$0 \longrightarrow (\mathrm{Hom}(\Omega_{d+1}^{\xi}, \mathbb{C}^\times))_{\mathrm{tor}} \longrightarrow \mho_\xi^{d+1} \longrightarrow \mathrm{Hom}(\Omega_{d+2}^{\xi}, \mathbb{Z}) \longrightarrow 0$$

(where, recall, $\Omega_{d+1}^{\xi} = (\{\text{closed } (d+1)\text{-dim'l } \xi\text{-manifolds}\}/\text{bordism}, \sqcup)$)

Example [Debray–Gunningham, '18]

The interacting analog of the TRS Majorana chain is the Arf–Brown TQFT, which generates the group $\mho^3_{\mathrm{Pin}^-} \cong (\mathrm{Hom}(\Omega_2^{\mathrm{Pin}^-}, \mathbb{C}^\times))_{\mathrm{tor}} \cong \Omega_2^{\mathrm{Pin}^-} \cong \mathbb{Z}/8$.

Connecting Free and Interacting

We want a map comparing fermionic symmetry-protected topological phases of two types...

$$\left\{\begin{matrix}\text{free fermion}\\ G\text{-SPTs}\end{matrix}\right\} \longrightarrow \left\{\begin{matrix}\text{interacting fermion}\\ G\text{-SPTs}\end{matrix}\right\}$$

$$\underset{K\text{-theory}}{\rightsquigarrow} \qquad \underset{\text{invertible field theories}}{\rightsquigarrow}$$

... so we need a way to connect (bordism classes of) manifolds and K-theory.

Free-to-Interacting Maps

Main Ingredient for F2I Map

The Atiyah–Bott–Shapiro, or spin orientation, [ABS,'63] is a ring map

$$\Omega_n^{\mathrm{Spin}} \to KO_n$$

sending a (bordism class of) spin manifold to the *KO*-class of its Clifford-linear Dirac operator acting on sections of the spinor bundle

$$[M] \mapsto \not{D}_M \curvearrowright \overline{\mathcal{C}^\infty(\mathcal{S})}$$

- The *Anderson dual* of this map gives the class D free-to-interacting map

$$KO^{d-2} \to \mho^{d+1}_{\mathrm{Spin}}$$

- Use *generalized ABS maps* $\Omega_n^{H(s)} \longrightarrow KO_{n+s}$ to model the rest of the tenfold way [Freed–Hopkins, '21, Debray–K.–Stehouwer, '26]

Outline

Warm-Up: Weak Phases

Definition [see e.g. Kitaev, '09, Ewert–Meyer, '19]

Weak SPT phases are SPT phases that are protected by $\mathbb{Z}^d$ translation symmetry. Mathematically, if $\bar{S}^d \hookrightarrow \bar{\mathbb{T}}^d$ is the inclusion of the top cell of the Brillouin zone,

$$\{\text{free weak phases}\} := \ker(KR^{d+s-2}(\bar{\mathbb{T}}^d) \to KR^{d+s-2}(\bar{S}^d))$$

- Most famous example: TRS insulator in $3+1$d [Fu–Kane, Fu–Kane–Mele, '21]
- *Dislocations* can trap topological bound states and potentially host nonabelian anyons
- Open question: are weak phases robust to interactions?

Parameterizing Over Tori

Free Classification [Kitaev, '09, Ewert–Meyer, '19]

$$\{\text{free weak phases}\} = \ker\left(KR^{d+s-2}(\bar{\mathbb{T}}^d) \to KR^{d+s-2}(\bar{S}^d)\right)$$

- Here, $\bar{\mathbb{T}}^d$ is the *Brillouin zone* torus

Interacting Classification [Freed–Hopkins, '20, AC-D-K-PT-S-S, '24]

Let $\mho^{d+1}_{H(s)}$ denote the group of IFTs on $H(s)$-manifolds. Then

$$\{\text{interacting weak phases}\} = \ker\left(\mho^{d+1}_{H(s)}(\mathbf{T}^d) \to \mho^{d+1}_{H(s)}(S^0)\right).$$

- Here, $\mathbf{T}^d$ is the *unit cell* torus

T-Duality

- Need to exchange the ($\mathbb{Z}/2$-equivariant) *Brillouin zone torus* $\bar{\mathbb{T}}^d := \mathrm{Hom}(\mathbb{Z}^d, U(1))$ with the (non-equivariant) *unit cell torus* $\mathbf{T}^d := \mathbb{R}^d/\mathbb{Z}^d$

- T-duality [e.g. Kitaev, '09, Mathai–Thiang, '16] in K-theory achieves this:

$$KR^s(\bar{\mathbb{T}}^d) \xrightarrow{\cong} KO_{-s}(\mathbf{T}^d) \xrightarrow[\text{Poincaré}]{\cong} KO^{d+s}(\mathbf{T}^d)$$

- Here, the first isomorphism is

$$KR^s(\bar{\mathbb{T}}^d) \xrightarrow[\text{Gelfand}]{\cong} KO_{-s}(C^*_{\mathbb{R}}(\mathbb{Z}^d)) \xrightarrow[\text{Baum–Connes}]{\cong} KO_{-s}(\mathbf{T}^d)$$

Weak Free-to-Interacting Map

- Let $F2I_s\colon KO^{d+s-2} \to \mho^{d+1}_{H(s)}$ be Freed–Hopkins' free-to-interacting map for symmetry s
- Since we have a map of spectra, it immediately generalizes to a parameterized version:

Ansatz [Antolín Camarena–Debray–K.–Pacheco-Tallaj–Stehouwer–Sheinbaum,'24]

The free-to-interacting map for discrete translation-invariant phases (including weak phases) is

$$F2I_{weak}\colon KR^{s-2}(\overline{\mathbb{T}}^d) \xrightarrow{\text{T-duality}} KO^{d-s-2}(\mathbf{T}^d) \xrightarrow{F2I_s} \mho^{d+1}_{H(s)}(\mathbf{T}^d)$$

for symmetry type s.

Stable Splittings of Tori

James splitting

For any generalized cohomology theory E, the cohomology of the torus splits:

$$\tilde{E}^k(\mathbf{T}^d) \cong \bigoplus_{n=0}^{d} \binom{d}{n} E^{k-n}(\mathrm{pt}).$$

- Example: $\mathbf{T}^2 \simeq_{stably} S^2 \vee S^1 \vee S^1 \vee S^0$, so $\tilde{E}^0(\mathbf{T}^2) \cong E^{-2} \oplus (E^{-1})^{\oplus 2} \oplus E^0$

$\Longrightarrow$ Our parameterized free-to-interacting map also splits!

$\Longrightarrow$

$$F2I^d_{weak} = \bigoplus_{k=0}^{d} \binom{d}{k} F2I^{d-k}_{strong}$$

$$\ker F2I^d_{weak} = \bigoplus_{k=0}^{d} \binom{d}{k} \ker F2I^{d-k}_{strong}\,, \qquad \operatorname{coker} F2I^d_{weak} = \bigoplus_{k=0}^{d} \binom{d}{k} \operatorname{coker} F2I^{d-k}_{strong}$$

Example—Weak Time-Reversal-Symmetric Insulators (Class AII)

Colored summands are valence band numbers and strong phases:

Corollary 3.4 (Symmetry class AII, $s = -2$). *The free-to-interacting map for the groups of weak phases in Altland-Zirnbauer type AII is:*

(3.5)

d	$\ker(\mathrm{F2I}) \longrightarrow$	$KO^{d-4}(\mathbf{T}^d) \xrightarrow{\mathrm{F2I}}$	$\mho^{d+2}_{\mathrm{Pin}^{\tilde{c}+}}(\mathbf{T}^d) \longrightarrow$	$\operatorname{coker}(\mathrm{F2I})$
1	0	$\mathbb{Z}$	$\mathbb{Z}$	0
2	0	$\mathbb{Z} \oplus \mathbb{Z}_2$	$\mathbb{Z} \oplus \mathbb{Z}_2$	0
3	0	$\mathbb{Z} \oplus \mathbb{Z}_2^3 \oplus \mathbb{Z}_2$	$\mathbb{Z} \oplus \mathbb{Z}_2^3 \oplus \mathbb{Z}_2^3$	$\mathbb{Z}_2^2$

- In this case, *all weak phases persist* in the presence of interactions
- We compute [AC-D-K-PT-S-S, '24] that the strong phase in $d = 3$ coming from the free phase is detected on $[\mathbb{CP}^1 \times \mathbb{CP}^1] \in \Omega_4^{\mathrm{Pin}^{\tilde{c}+}}$, while the intrinsically interacting phases are detected on $[\mathbb{RP}^4]$ and $[\mathbb{CP}^2]$

Quasicrystal Classification From [Else–Prem–Huang–Gromov, '21]

Classification [(3), Else–Prem–Huang–Gromov, '21]

Invertible topological phases with internal symmetry G (and no point group symmetry) in a quasicrystalline system with D elastic modes are classified by

$$\bigoplus_{k=0}^{D} \mathcal{Q}_{d-k}^{\times\binom{D}{k}} \tag{1}$$

where $\mathcal{Q}_{d-k}$ is a classification of invertible topological phases with internal symmetry G in $d-k$ spatial dimensions

- This formula comes from parameterizing over the dual reciprocal lattice $\mathcal{L}^* \cong \mathbf{T}^D$
- I will take $\mathcal{Q}_{d-k}$ to be K-theory and invertible field theories, resp.

Quasicrystal Classification in Terms of Invertible Field Theories

- Combining the ansatz from [(3), Else–Prem–Huang–Gromov, '21] with the invertible field theory classification of [Freed–Hopkins, '21], we get...

Ansatz

Interacting cut-and-project quasicrystalline SPTs in d spatial dimensions with D elastic modes and symmetry type H are classified by

$$[MTH, \mathbf{T}^D \wedge \Sigma^{d+2} I_{\mathbb{Z}}] =: \mho_H^{d+1}(\mathbf{T}^D). \tag{2}$$

- That is, such SPTs are classified by $d+1$-dimensional invertible field theories on H-manifolds parameterized over the torus $\mathbf{T}^D$
- Calculable via the same methods as for weak phases

Quasicrystalline Free-to-Interacting Maps

- Let $F2I_s^k$ be the (strong) free-to-interacting map for symmetry type s in dimension k.
- By analogy with the weak phases case...

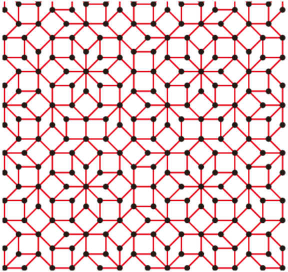
Ansatz

For a cut-and-project quasicrystalline SPT of symmetry type s in d spatial dimensions with D phason modes, we set

$$F2I := \bigoplus_{i=0}^{D} (F2I_s^{d-i})^{\oplus \binom{D}{i}}.$$

Ammann–Beenker Tiling I

- Consider fermionic SPTs on the Ammann–Beenker tiling
- This is a cut-and-project quasicrystal with $D = 4$ and $d = 2$
- See e.g. [Varjas et al, '19, Chen–Chen–Gao–Zhou–Xu, '20, Else–Huang–Prem–Gromov, '21, Yan et al, '25]
- For now, ignore the 8-fold symmetry



Ammann–Beenker Tiling II

For class D superconductors, using [Cor. 9.81, Freed–Hopkins, '21] (see also [Cor. 3.7, AC-D-K-PT-S-S, '24]) we compute

$$\bigoplus_{i=0}^{4}(KO^{-i})^{\oplus\binom{4}{i}} \xrightarrow{F2I} \bigoplus_{i=0}^{4}(\mho^{3-i}_{\mathrm{Spin}})^{\oplus\binom{4}{i}}$$

$$\begin{array}{lccc}
i=0 & \mathbb{Z} & \xrightarrow{\cong} & \mathbb{Z} \\
1 & (\mathbb{Z}/2)^4 & \xrightarrow{\cong} & (\mathbb{Z}/2)^4 \\
2 & (\mathbb{Z}/2)^6 & \xrightarrow{\cong} & (\mathbb{Z}/2)^6 \\
3 & 0 & & 0 \\
4 & \mathbb{Z} & \xrightarrow{\cong} & \mathbb{Z}
\end{array}$$

- These are all $\cong$'s because the spin orientation is 7-connected

Ammann–Beenker Tiling III

For class CI insulators, using [Cor. 9.101, Freed–Hopkins, '21] (see also [Cor. 3.16, AC-D-K-PT-S-S, '24]) we compute

$$\bigoplus_{i=0}^{4}(KO^{3-i})^{\oplus\binom{4}{i}} \xrightarrow{F2I} \bigoplus_{i=0}^{4}(\mho^{3-i}_{\mathrm{Pin}^{h+}})^{\oplus\binom{4}{i}}$$

$i=0$	0		0
1	0		$(\mathbb{Z}/2)^4$
2	0		0
3	$\mathbb{Z}^4$	$\xrightarrow{\text{mod } 2}$	$(\mathbb{Z}/2)^4$
4	$\mathbb{Z}/2$		0

- There's a new unstable free phase coming from $i = 4$

Intrinsically Quasicrystalline Phenomena

Ansatz (combining [(3), Else–Prem–Huang–Gromov, '21] + [Freed–Hopkins, '21])

Interacting cut-and-project quasicrystalline SPTs in d spatial dimensions with D elastic modes and symmetry type H are classified by $\mho_H^{d+1}(\mathbf{T}^D) = \bigoplus_{i=0}^{D} (\mho_H^{d+1-i})^{\oplus\binom{D}{i}}$.

- Bordism is *connective*; i.e. $\Omega_i^H = 0$ for $i < 0$, and so IFTs satisfy $\mho_H^i = 0$ for $i < -1$
- Therefore $\mho_H^{d+1-i} = 0$ for $i > d+2$, and the only new phase invariants we see using this ansatz are from $\mho_H^0$ and $\mho_H^{-1}$ (i.e. $\mathcal{Q}_{-1}$ and $\mathcal{Q}_{-2}$), no matter how large $D - d$ is

Fact

$$\mho_H^{-1} = \begin{cases} \mathbb{Z} & \text{if } H \to O \text{ factors through } SO \\ 0 & \text{if not} \end{cases}$$

- There are *new intrinsically interacting, intrinsically quasicrystalline phases* in classes AII and CII coming from $\mho_H^0$ [*This is a correction from what I said in the talk*]
- But there are *no* such phases coming from $\mho_H^{-1}$

Ammann–Beenker Tiling IV

For class C insulators (with charge conjugation $C^2 = -1$), using [Cor. 9.99, Freed–Hopkins, '21] (see also [Cor. 3.19, AC-D-K-PT-S-S, '24]), we compute

$$\bigoplus_{i=0}^{4} (KO^{4-i})^{\oplus\binom{4}{i}} \xrightarrow{F2I} \bigoplus_{i=0}^{4} (\mho^{3-i}_{\mathrm{Spin}^h})^{\oplus\binom{4}{i}}$$

$$\begin{array}{ccc}
i = 0 & \mathbb{Z} \xrightarrow{\cong} \mathbb{Z} \\
1 & 0 \qquad\quad 0 \\
2 & 0 \qquad\quad 0 \\
3 & 0 \qquad\quad 0 \\
4 & \mathbb{Z} \xrightarrow{\cong} \mathbb{Z}
\end{array}$$

- There is a new (non-crystalline) stable free phase coming from $i = 4$

Future Directions

- New symmetry types, especially mixed spatial and internal symmetries
- Beyond cut-and-project quasicrystalline SPTs
- Identifying bordism invariants and generators

Related Articles

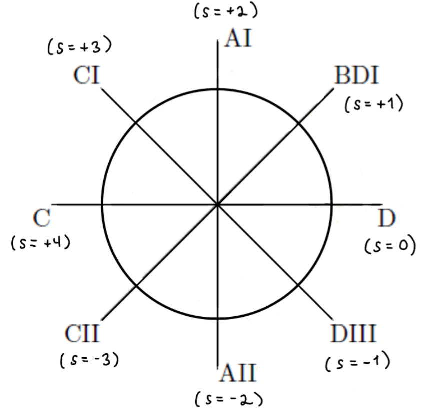
- Omar Antolín Camarena, Arun Debray, CK, Natalia Pacheco-Tallaj, Daniel Sheinbaum, and Luuk Stehouwer. *Weak Topological Phases in the Presence of Interactions*. arXiv: `2410.10031 [math-ph,cond-mat.str-el,hep-th]`.
- Arun Debray, CK, and Luuk Stehouwer. *Unraveling the Bott Spiral*. arXiv: `2605.00316 [math-ph,cond-mat.str-el,hep.th,math.AT]`
- CK. *Quasicrystals and Invertible Field Theories*. In progress.

Thank you!

Outline

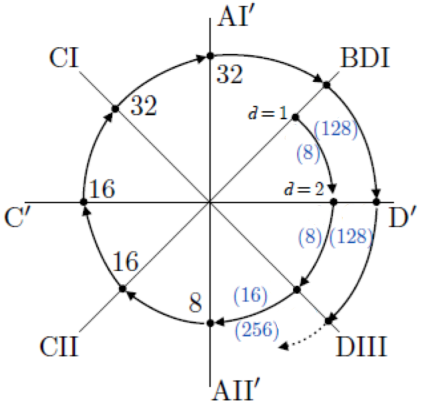
The "Bott Spiral" of Primed Phases [Queiroz–Khalaf–Stern, '16]

"Bott Clock" of Free SPTs



- (1,1)-periodic from KO^{d+s-2}

"Bott Spiral" of Interacting SPTs



- not periodic! $\mathbb{Z}/(2^n)$ for increasing n

Computational Results

Theorem [Debray–K.–Stehouwer, '26]

Let $\widetilde{m} := \lfloor (d-\ell-k+i+1)/8 \rfloor$, where $i \in \{0,1,2,3\}$ with $i \equiv \ell \mod 4$. For d large enough, the image of the type-$E_{\ell,k}$ free-to-interacting map

$$F2I_{\ell,k} \colon KO^{d+\ell-k-2} \to \mho^{d+1}_{\mathrm{Spin}}(ME_{\ell,k})$$

is

$$\operatorname{im}(F2I_{\ell,k}) \cong \begin{cases} \mathbb{Z}/2^{4+4\widetilde{m}-i} & d \equiv \ell+k-2i+2 \bmod 8 \\ \mathbb{Z}/2^{5+4\widetilde{m}-i} & d \equiv \ell+k-2i+6 \bmod 8 \\ \mathbb{Z}/2 & d \equiv \ell+k-2i \bmod 8 \text{ or } \ell+k-2i+1 \bmod 8. \end{cases}$$

- Here, $\widetilde{m}$ tracks the number of times we have gone around the spiral
- $ME_{\ell,k}$ is a Thom spectrum encoding twisted spin structures for a *discrete* version of the tenfold way symmetries

Example Free-to-Interacting Map

Example

Consider $d = 2$. The F2I map for the class D superconductor is an isomorphism:

$$\mathbb{Z} \cong KO^0(\mathrm{pt}) \longrightarrow \mathbb{Z} \cong \mho^3_{\mathrm{Spin}}.$$

The F2I map for the class D′ superconductor, meanwhile, is reduction mod 8:

$$\mathbb{Z} \cong KO^0(\mathrm{pt}) \longrightarrow \mathbb{Z}/(2^3) \cong \mho^3_{\mathrm{DPin}}.$$

Computational Methods

- Free-to-interacting maps are built out of twisted ABS maps

$$MT\mathrm{Spin} \wedge X^V \to \Sigma^s KO$$

where X^V is some Thom spectrum

- These are often computable by looking just at the

$$X^V \to \Sigma^s KO$$

component, since they are $MT\mathrm{Spin}$-module maps

- Anderson–Brown–Peterson give a 2-local splitting of $MT\mathrm{Spin}$ into

$$MT\mathrm{Spin} \simeq \bigvee_{i\geq 1} \Sigma^{a_i} ko \vee \bigvee_{j\geq 1} \Sigma^{b_j} \tau_{\geq 2} ko \vee \bigvee_{k\geq 1} \Sigma^{c_k} H\mathbb{Z}/2$$

- The homotopy groups of each piece of $MT\mathrm{Spin} \wedge X^V$ are often computable via the Adams spectral sequence