QUASICRYSTALLINE STRING LANDSCAPE

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Based on [2406.00129] and [2406.00185] with Houri-Christina Tarazi and Cumrun Vafa

STRING THEORY

- UV complete quantum gravity theory!
- Particles are excitations of small strings.

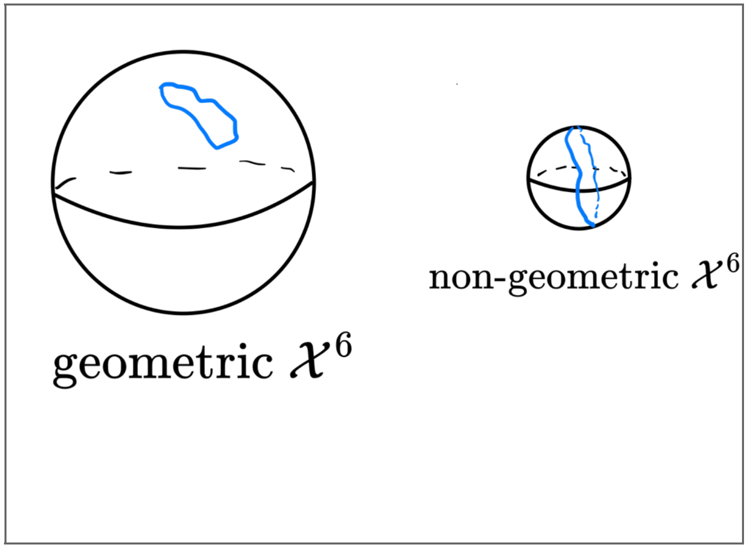
- Graviton $g^{\mu\nu}$ is always in the excitation spectrum.

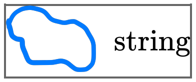
PROBLEMS

- String theory needs 1+9 dimensions. $\mathcal{M}^{1,9} = \mathbb{R}^{1,3} \times \boldsymbol{\mathcal{X}}^6$. The $\boldsymbol{\mathcal{X}}^6$ has to do too much:
    1. Non-supersymmetric
    2. Positive cosmological constant $\Lambda > 0$
    3. $SU(3) \times SU(2) \times U(1)$ gauge group
    4. Chiral fermions
    5. Stable

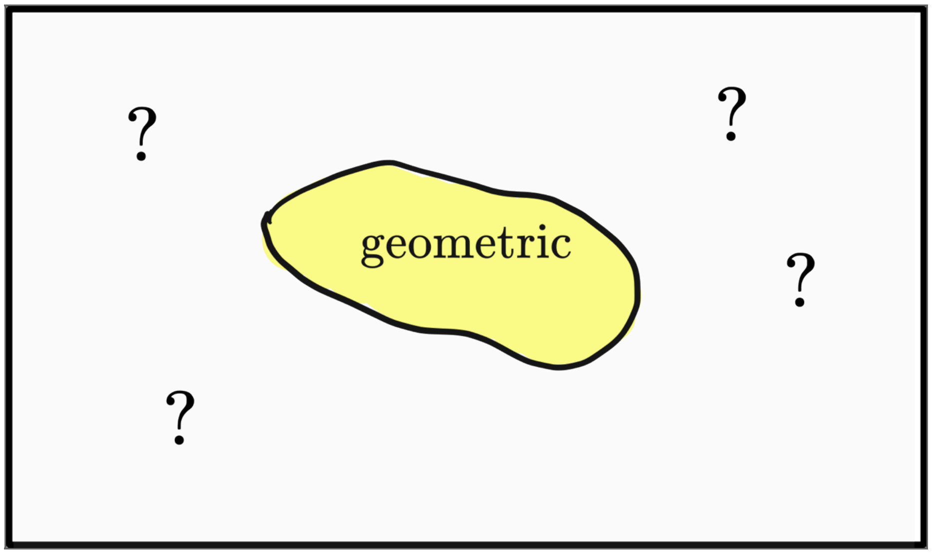
NON-GEOMETRIC SOLUTION

Geometric $\mathcal{X}^6$ cannot do all that. Find non-geometric $\mathcal{X}^6$.



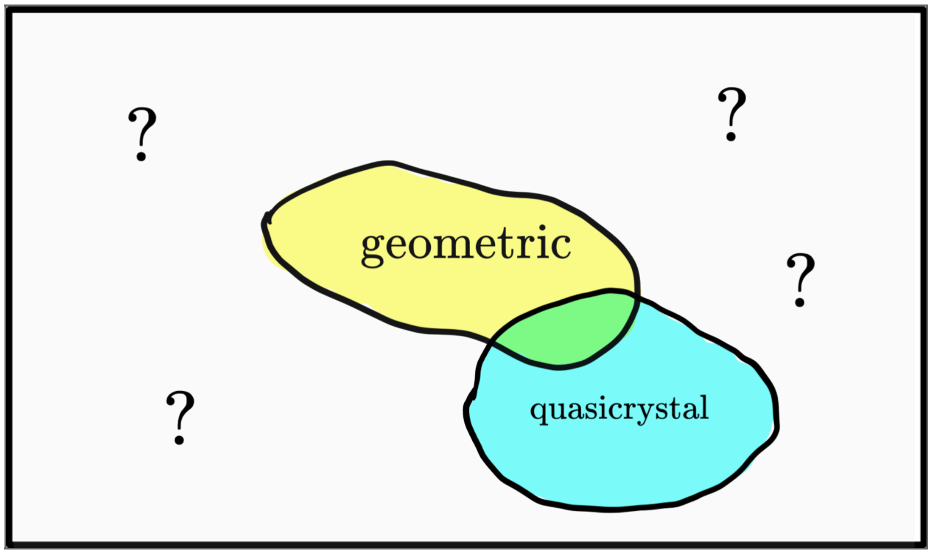


NON-GEOMETRIC LANDSCAPE IS UNCHARTED



QUASICRYSTALLINE LANDSCAPE

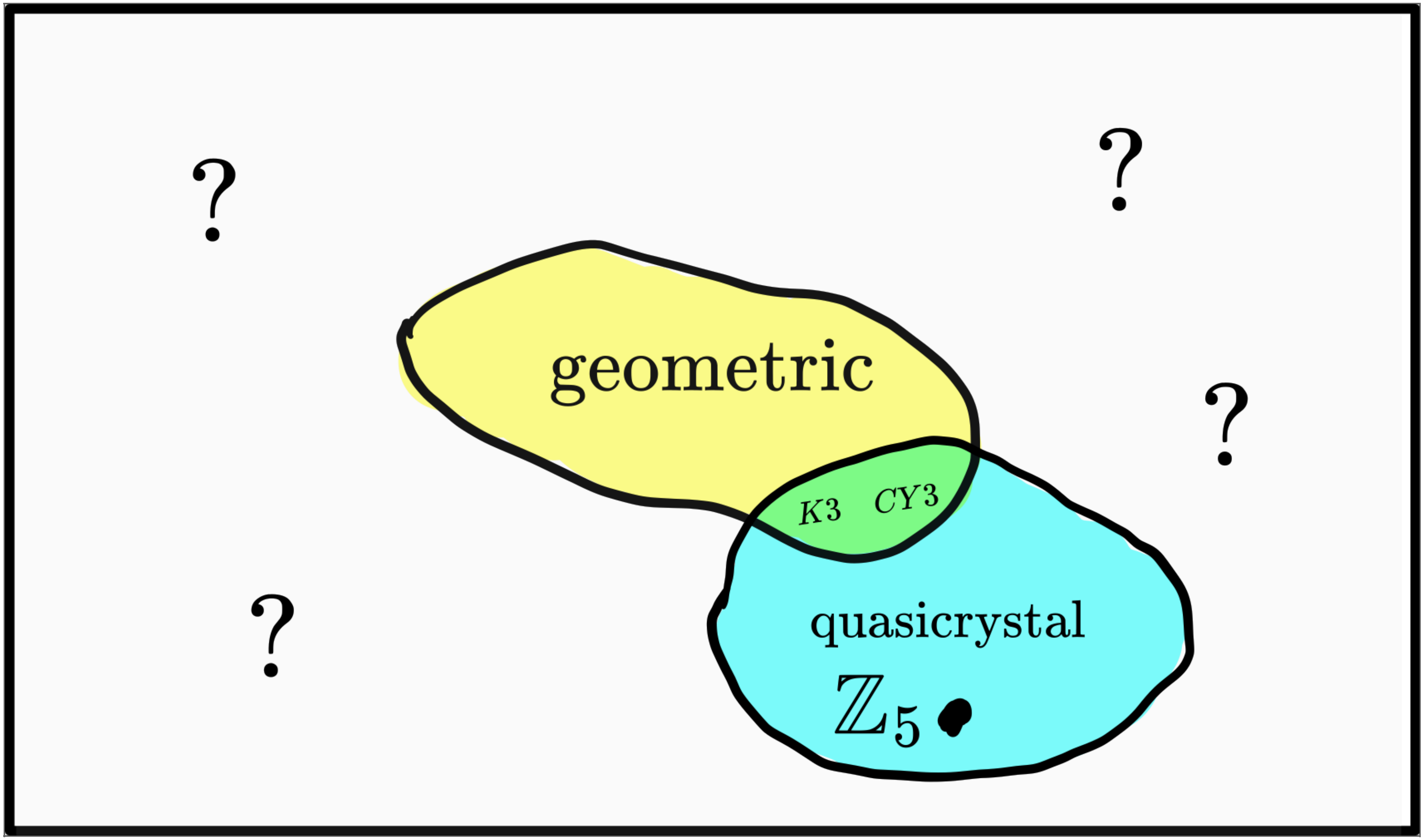
Quasicrystalline strings give us a controlled frontier into the non-geometric.



FINDINGS

In this unexplored territory, we find:

1. K3 and CY3 (recover geometry)
2. A $\mathbb{Z}_5$ quasicrystalline string nearly phenomenological
    - Non-supersymmetric ✓
    - $\Lambda > 0$ ✓
    - $SU(3) \times SU(2) \times U(1)$ ✓
    - Chiral fermions ✓
    - Unstable (by just one scalar) $\times$


?
?
geometric
?
$K3$
$CY3$
?
quasicrystal
$\mathbb{Z}_5$

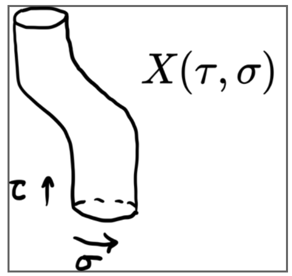
OUTLINE

1. String theory on a torus T^d
2. Quasicrystalline symmetries $\mathbb{Z}_N$
3. Quasicrystalline orbifolds $T^d/\mathbb{Z}_N$
   a. K3 and CY3
   b. Non-supersymmetric $\mathbb{Z}_5$ example

1. Torus T^d is encoded by a polarized lattice $\mathrm{II}^{d,d}$.
2. Projecting to the polarization surface gives quasicrystal.
3. Gauge the quasicrystal symmetry.

WHAT IS STRING THEORY?

String worldsheet $= 1+1$d SCFT ($c = 10$)



$$S = \underbrace{\int G_{\mu\nu} dX^\mu \wedge \star dX^\nu}_{\text{spacetime curvature} = \text{interacting CFT}} + \underbrace{\int B_{\mu\nu} dX^\mu \wedge dX^\nu}_{\text{like } A_\mu \text{ in QED}}.$$

Background is determined by $G_{\mu\nu}$ (symmetric), $B_{\mu\nu}$ (antisymmetric).

STRING EXCITATIONS

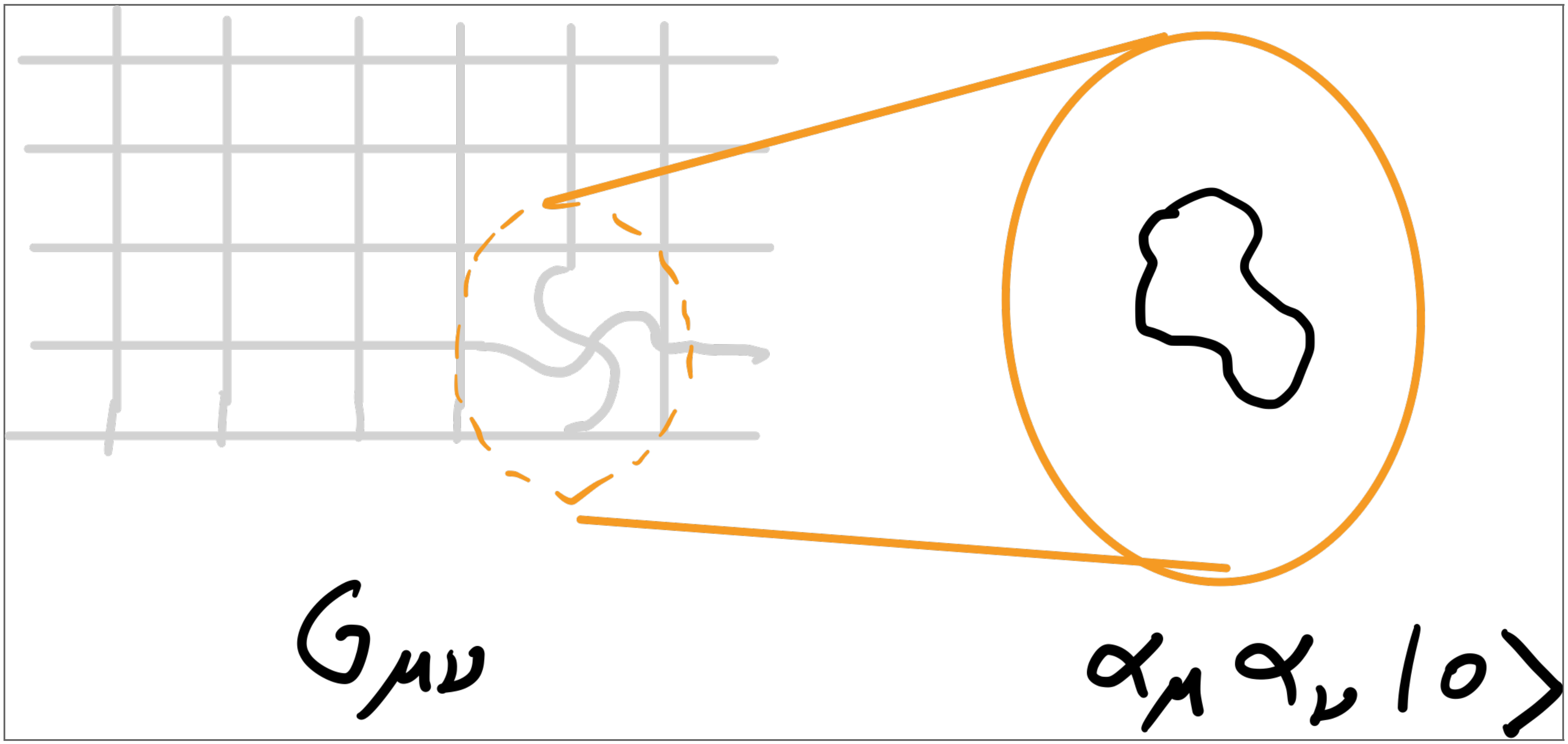
Quantize $X(\tau,\sigma)$

$\partial_+ X = \sum_n \alpha_n^\mu e^{2\pi i n(\tau+\sigma)}, \qquad \partial_- X = \sum_n \tilde{\alpha}_n^\mu e^{2\pi i n(\tau-\sigma)}$

The background $G_{\mu\nu}, B_{\mu\nu} \Longrightarrow$ define worldsheet. Worldsheet CFT $\Longrightarrow$ has $G_{\mu\nu}, B_{\mu\nu}$ excitations.

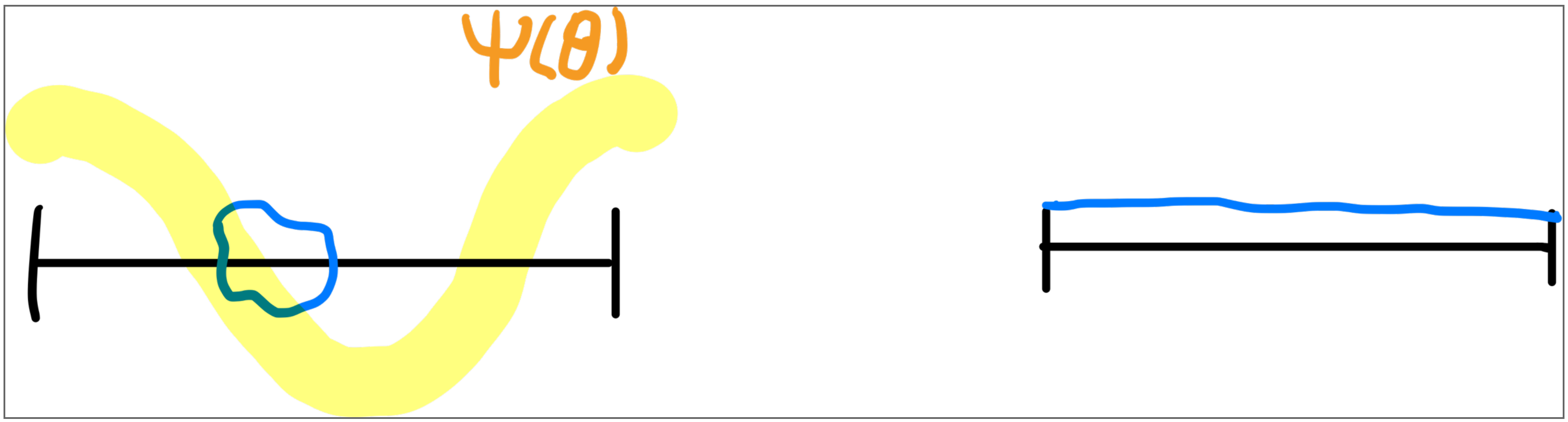
$G_{\mu\nu} : \alpha_{-1}^{(\mu}\alpha_{-1}^{\nu)}|0\rangle, \qquad B_{\mu\nu}\alpha_{-1}^{[\mu}\alpha_{-1}^{\nu]}|0\rangle.$

STRING EXCITATIONS



STRING THEORY ON S^1

$$G_{\theta\theta} = Rd\theta^2 \qquad B_{\theta\theta} = 0.$$

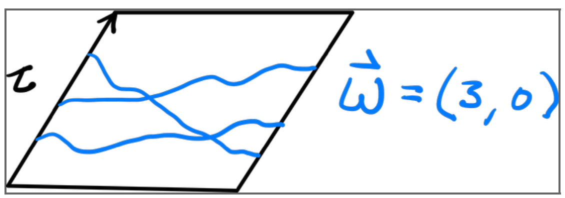


momentum n

winding $w\,|n, w\rangle$

STRING THEORY ON $T^2 = S^1 \times S^1$

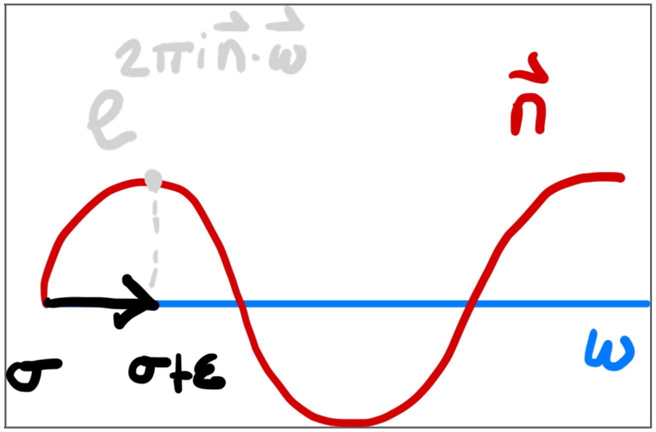
$$\tau = G_{\phi\phi} + iG_{\theta\phi}, \qquad \rho = B_{\theta\phi} + i\mathrm{Vol}$$



Momentum: $\vec{n} \in \mathbb{Z}^2$ Winding: $\vec{w} \in \mathbb{Z}^2$.

CONFORMAL SPIN

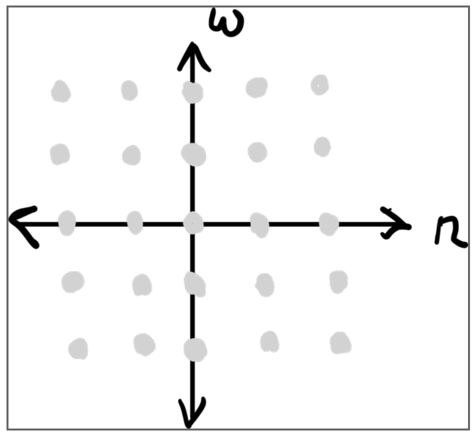
h: charge under worldsheet translations $\sigma \to \sigma + \epsilon. h = 2\vec{n} \cdot \vec{w}$



CHARGE LATTICE

Define norm whose value is the spin:

$$(\vec{n}, \vec{w}) \cdot (\vec{n}', \vec{w}') \equiv \vec{n} \cdot \vec{w}' + \vec{n}' \cdot \vec{w}.$$



$$\underbrace{\|(\vec{n}, \vec{w})\| \in 2\mathbb{Z}}_{\text{even}} + \underbrace{\exists(\vec{n}, \vec{w}) \cdot (\vec{n}', \vec{w}') = 1}_{\text{unimodular}} \implies \text{Unique II}^{d,d}$$

EVEN UNIMODULAR LATTICE IS UNIQUE

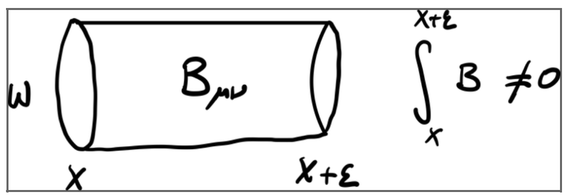
So the choice of background $G_{\mu\nu}, B_{\mu\nu}$ is not encoded in the charge lattice $\mathrm{II}^{d,d}$.
It's because spin doesn't depend on G, B, but mass does.
What lattice data computes mass?

CHARGE LATTICE AND MASS

The mass of a momentum-winding state $|\vec{n}, \vec{w}\rangle$ is

$$\text{mass} = \underbrace{(\vec{n} + B\vec{w})G^{-1}(\vec{n} + B\vec{w})}_{E=p^2} + \underbrace{\vec{w}G\vec{w}}_{\text{stretched string tension}}$$

. If $B \neq 0$, winding w supplies translation charge (momentum)



ENCODING G, B AS LATTICE POLARIZATION

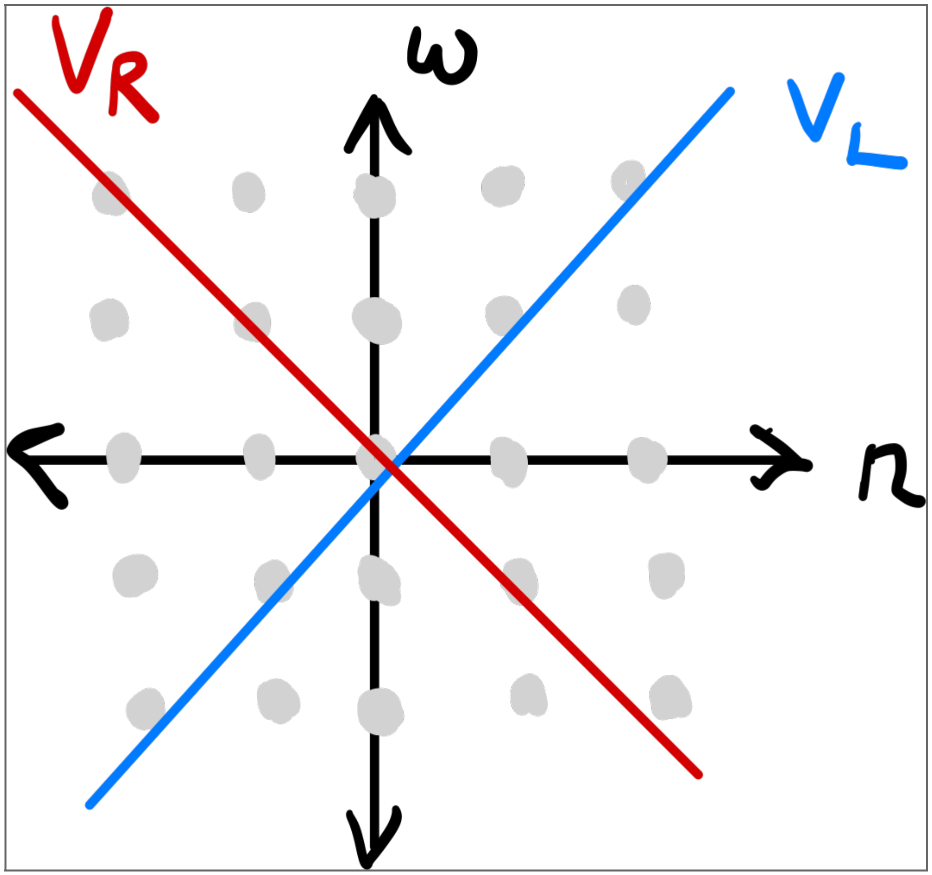
$$\mathrm{mass} = (\vec{n} + B\vec{w})G^{-1}\underbrace{(\vec{n} + B\vec{w})}_{\equiv p_L - p_R} + (G\vec{w})^T G^{-1}\underbrace{(G\vec{w})}_{\equiv p_L + p_R}$$

$$\mathrm{mass} = p_L^2 + p_R^2, \qquad \mathrm{spin} = p_L^2 - p_R^2.$$

In physics we choose G, B.

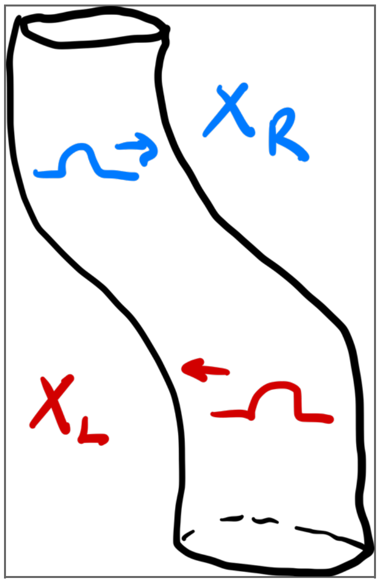
In lattice theory we choose polarization $\underbrace{\mathbb{R}^{d,0}}_{p_L} \oplus \underbrace{\mathbb{R}^{0,d}}_{p_R} \hookrightarrow \mathrm{II}^{d,d}$.

LATTICE POLARIZATION



POLARIZATION AS LEFT & RIGHT MOVERS

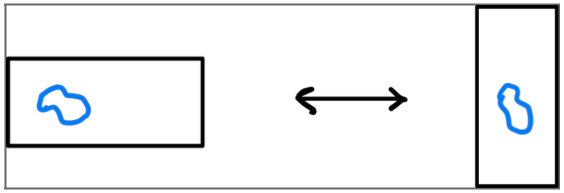
$$X(\tau,\sigma) = X_L(\tau+\sigma) + X_R(\tau-\sigma)$$
$$\text{momentum} = p_L + p_R, \qquad \text{winding} = p_L - p_R.$$



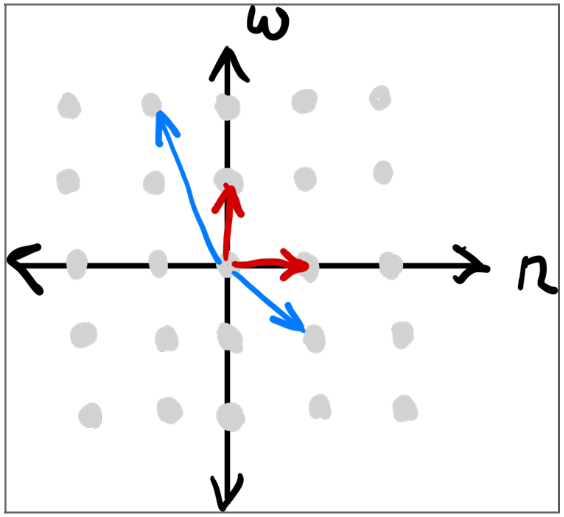
1. String on $T = \mathrm{II}^{d,d}$ with polarization
2. ?
3. ?
   a. ?
   b. ?

DUALITIES

Physics: different theory, same observables.

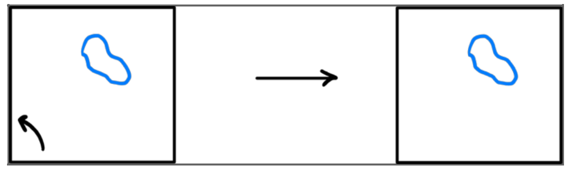


Lattice: Automorphisms $O(d, d, \mathbb{Z})$ of lattice $\mathrm{II}^{d,d}$.

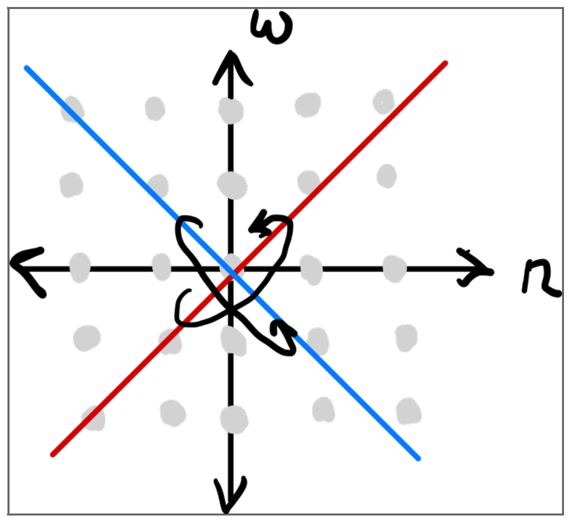


SYMMETRIES

Physics: duality fixes the theory.



Lattice: Duality fixes polarization $O(d) \times O(d) \subset O(d, d, \mathbb{Z})$.

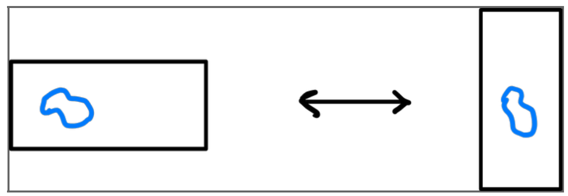


KINDS OF DUALITIES

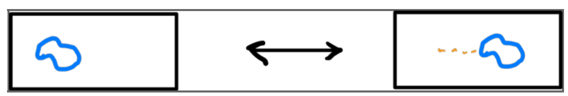
1. Geometric
    - Shift
    - Rotation
2. Non-geometric
    - T-duality
    - B-field gauge transformation

GEOMETRIC

Rotations:



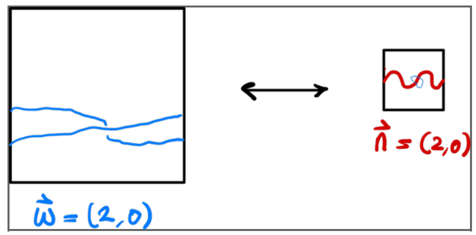
Shifts:



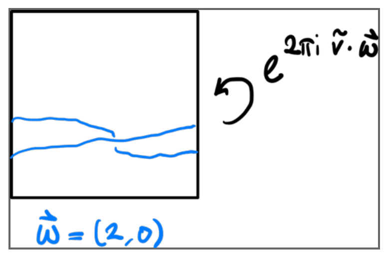
$$|p_L, p_R\rangle \mapsto e^{2\pi i v\cdot(p_L+p_R)}|\theta p_L, \theta p_R\rangle.$$

NON-GEOMETRIC

T-dualities:



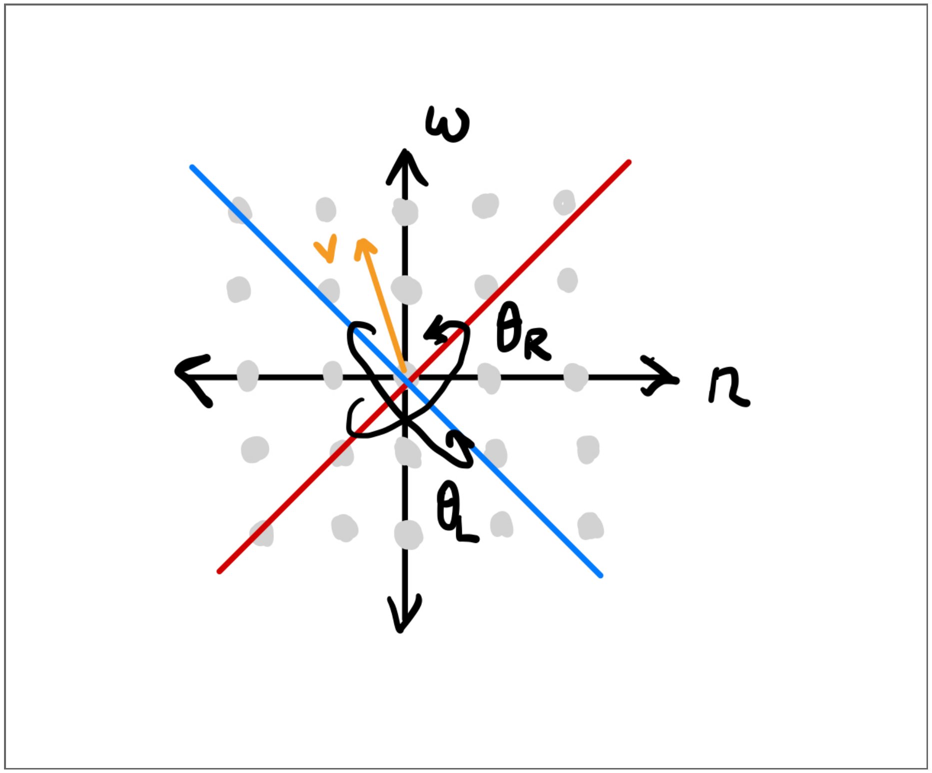
B-gauge transformation:



$$|p_L, p_R\rangle \mapsto e^{2\pi i \tilde{v}\cdot(p_L - p_R)} |\theta_L p_L, p_R\rangle.$$

GENERAL DUALITY TRANSFORMATION

$$(\theta_L, \theta_R) \in O(d) \times O(d) \subset O(d, d, \mathbb{Z}),$$
$$v = (v_L, v_R) \in \mathrm{II}^{d,d} \otimes \mathbb{R}.$$



ACTION ON HILBERT SPACE

On charged states:

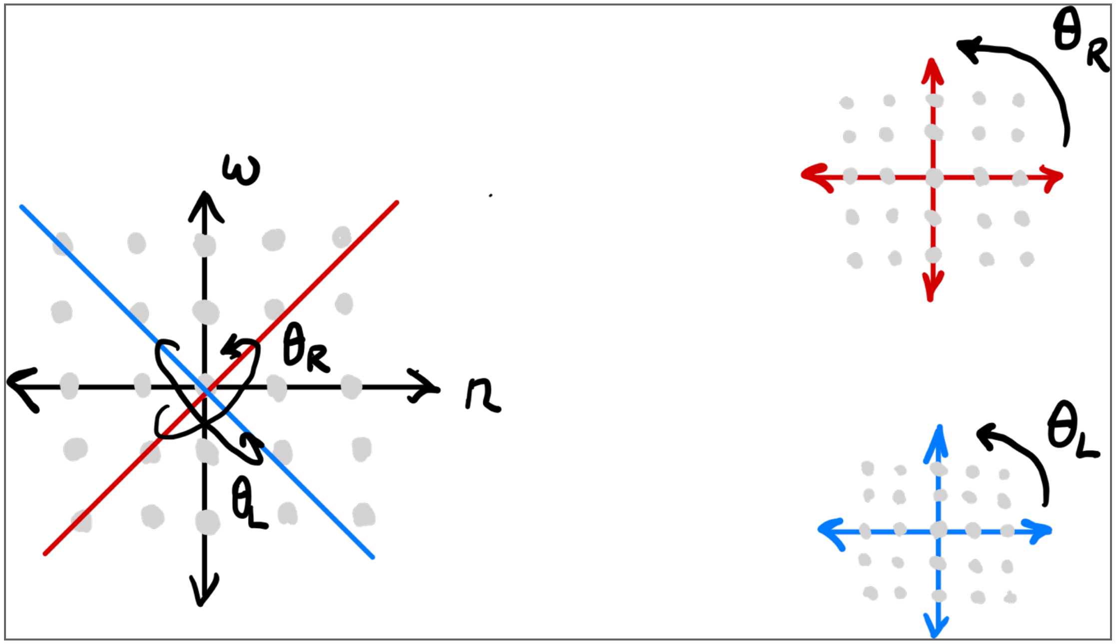
$$|p_L, p_R\rangle \mapsto e^{2\pi i (v_L, v_R)\cdot(p_L, p_R)} |\theta_L p_L, \theta_R p_R\rangle.$$

On excitations:

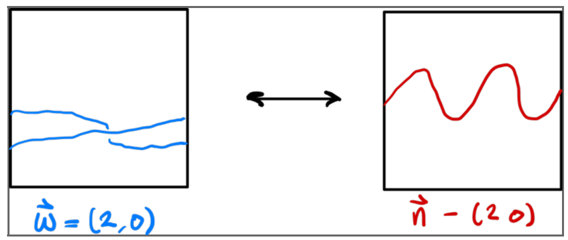
$$\alpha_L \alpha_R |0\rangle \mapsto (\theta_L \alpha_L)(\theta_R \alpha_R)|0\rangle.$$

CRYSTALLINE SYMMETRIES

Polarization along sublattice. θ_L and θ_R act as blocks.



EXAMPLE: A_1^2



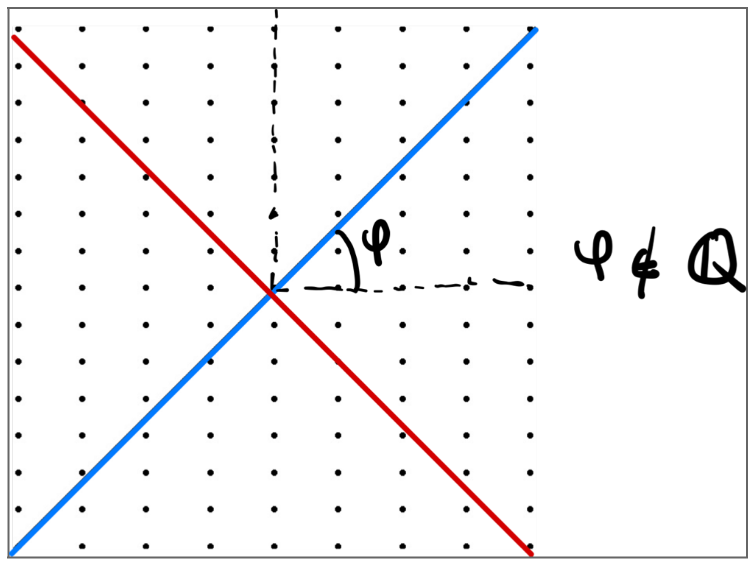
Geometric $\mathbb{Z}_4$: $\tau = G_{\phi\phi} + iG_{\theta\phi} = i$

T-duality $\mathbb{Z}_4$: $\rho = B_{\theta\phi} + i\mathrm{Vol} = i$

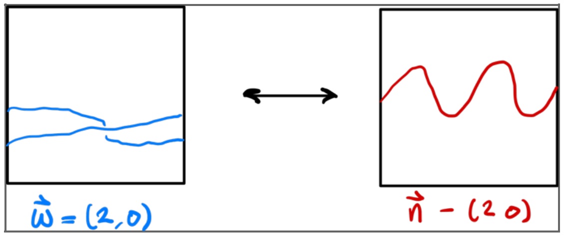
No one symmetry fixes all parameters.

QUASICRYSTALLINE SYMMETRY

Polarization along an irrational slope. (θ_L, θ_R) act on the whole lattice.



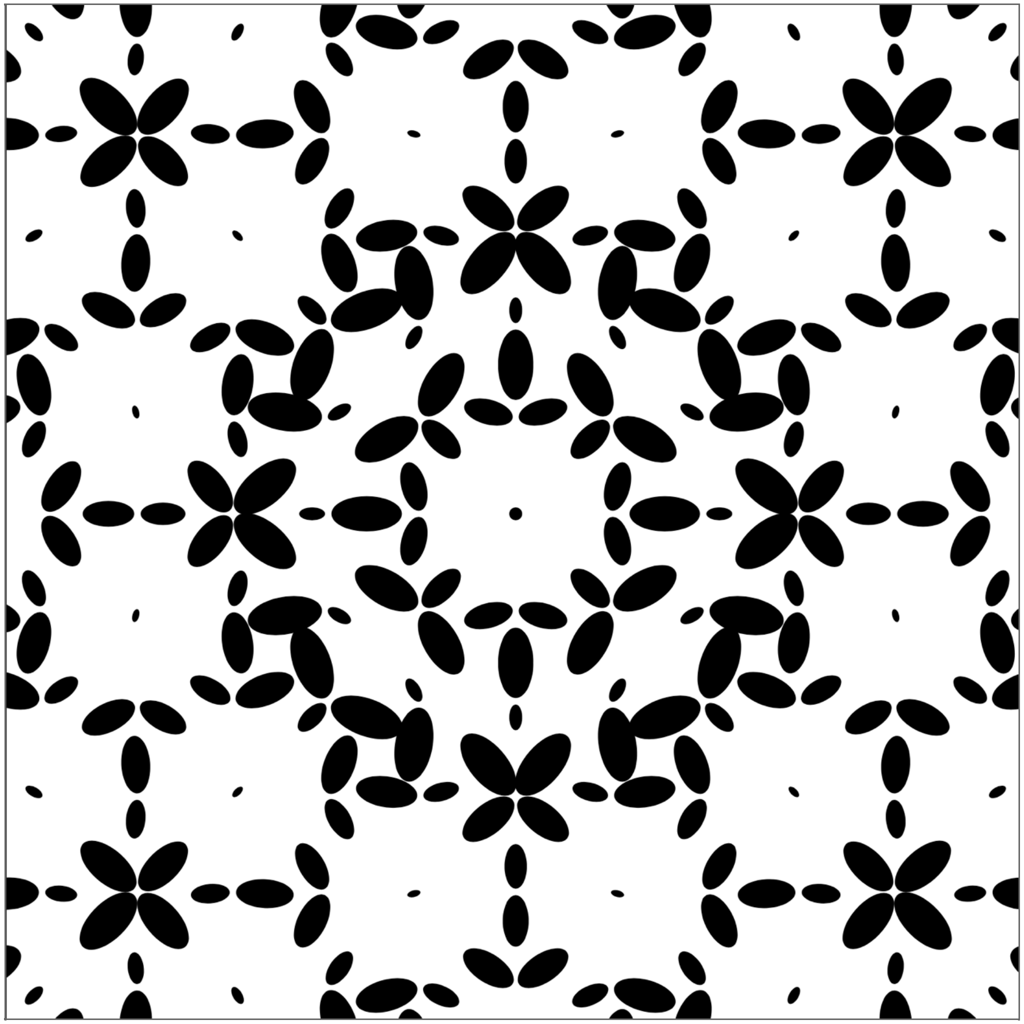
EXAMPLE: $\mathbb{Z}_{12}$



Geometric $\mathbb{Z}_4$: $\tau = G_{\phi\phi} + iG_{\theta\phi} = i$

T-duality $\mathbb{Z}_3$: $\rho = B_{\theta\phi} + i\mathrm{Vol} = e^{2\pi i/3}$

$\mathbb{Z}_{12} = \mathbb{Z}_3 \times \mathbb{Z}_4$ fixes all parameters.



(p_L, p_R) projected to p_L; where p_R gives ellipses.

QUASICRYSTALS FIX G, B

$$\alpha_L \alpha_R |0\rangle \mapsto (\theta_L \alpha_L)(\theta_R \alpha_R)|0\rangle$$

$\theta_L \neq \theta_R^{-1}$ for quasicrystals.

For $\mathbb{Z}_{12}$,

$$\theta_L = \mathrm{Rot}\left(\tfrac{1}{12}\right), \qquad \theta_R = \mathrm{Rot}\left(\tfrac{5}{12}\right).$$

QUASICRYSTAL ROTATION PLANES

In fact, quasicrystals always have the form $\theta = (\theta_L, \theta_R) = \left(\mathrm{Rot}\left(\frac{1}{N}\right), \cdots, \mathrm{Rot}\left(\frac{p}{N}\right), \cdots\right)$ for all p coprime with N. Also write

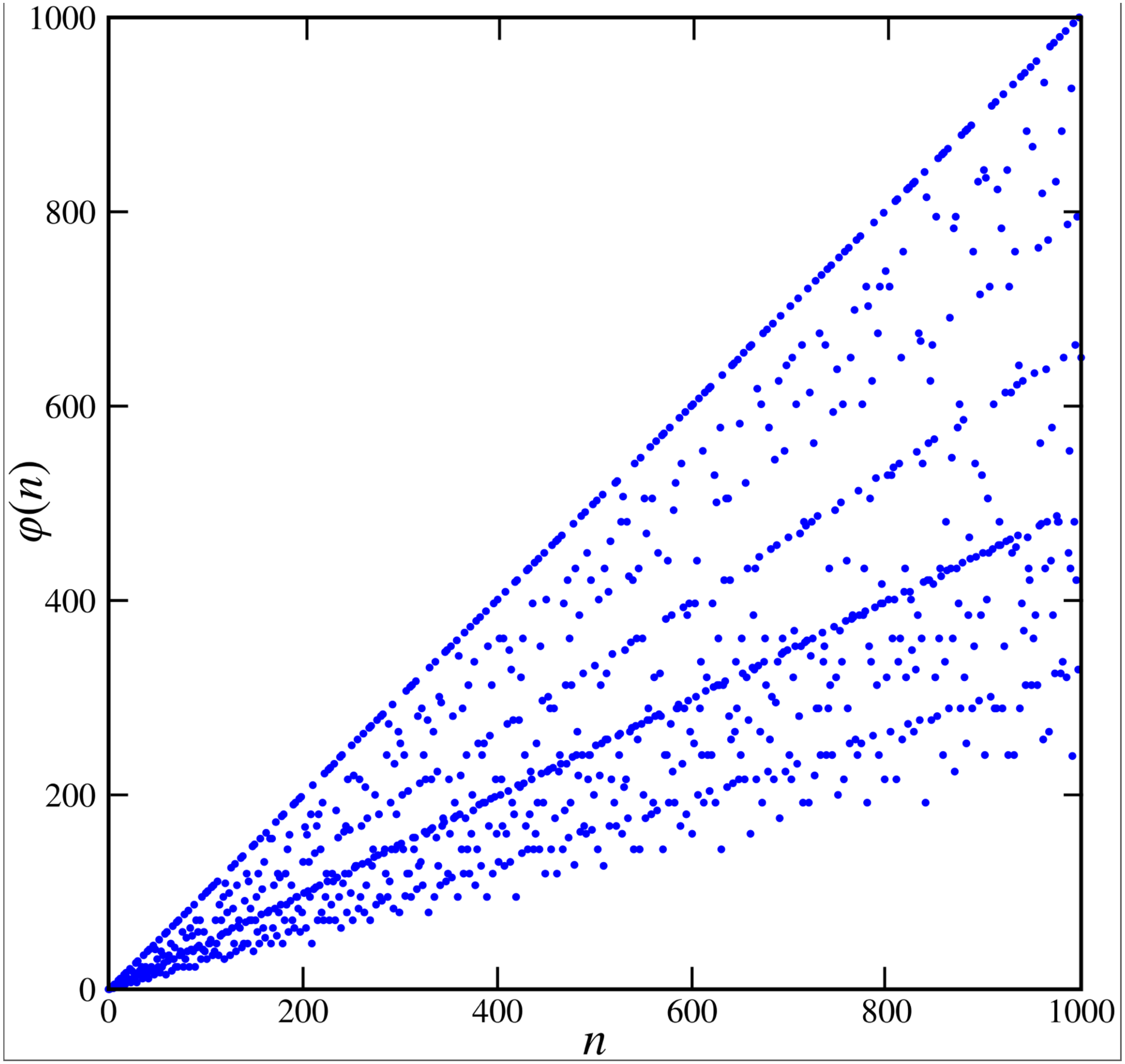
$$\mathrm{Twist} = \left(\frac{1}{N}, \cdots, \frac{p}{N}, \cdots\right).$$

EULER TOTIENT FUNCTION

$$\phi(N) = \# \text{ of coprime } p.$$

The dimension of the charge lattice $\mathrm{II}^{d,d}$ must be greater than $\phi(N)$.

$$2d \geq \phi(N).$$



1000
800
600
400
200
0
$\varphi(n)$
0
200
400
600
800
1000
n

QUASICRYSTALS IN 2D

For $\mathrm{II}^{2,2}$:

$$\phi(N) = 4 \implies N = 5, 8, 10, \mathbf{12}$$

How about $5, 8, 10$?

UNIMODULAR QUASICRYSTAL THEOREM

$$\mathbb{Z}_N \text{ quasicrystal is in } \mathrm{II}^{d,d}$$

iff

$$N \neq p^r \text{ and } N \neq 2p^r \quad (p \text{ prime})$$

UNIMODULAR QUASICRYSTALS

Lattice rank $\phi(m) = r + s$	Signature $(r; s)$	Symmetry order m
4	$(2; 2)$	12
8	$(4; 4)$	$15, 20, 24, 30$
12	$(6; 6), (10; 2)$	$21, 28, 36, 42$
16	$(8; 8), (12; 4)$	$40, 48, 60$
20	$(14; 6), (18; 2)$	$33, 44, 66$
24	$(16; 8), (20; 4)$	$35, 39, 45, 52, 56, 70, 72, 78, 84, 90$
28	$(22; 6)$	$\varnothing$
32	$(24; 8)$	$51, 68, 80, 96, 102, 120$

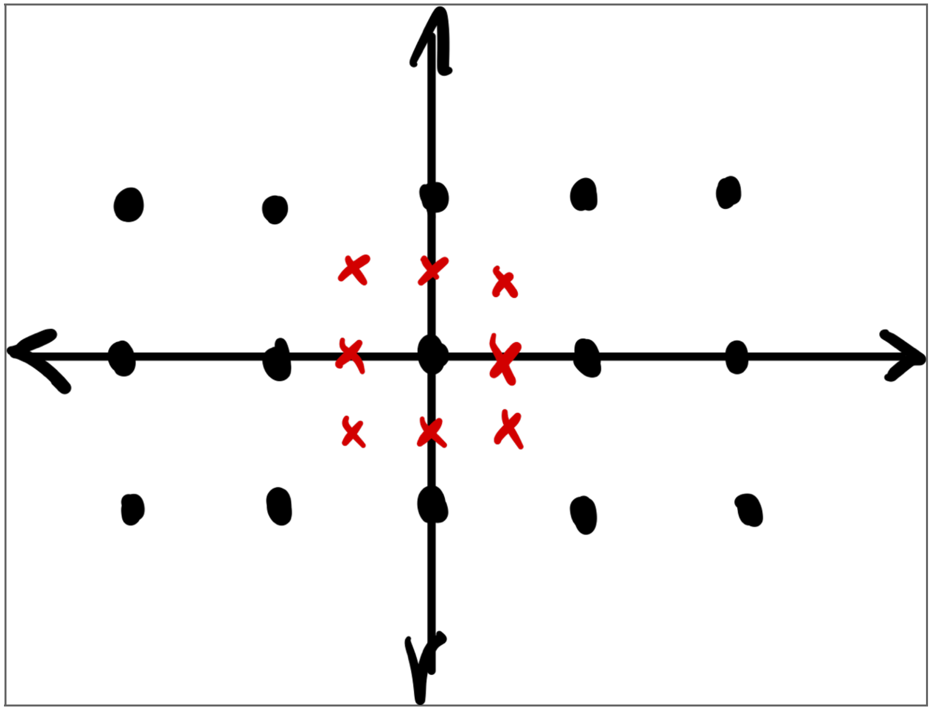
For $(2, 2)$,

$$N = 5 \qquad \text{prime}$$

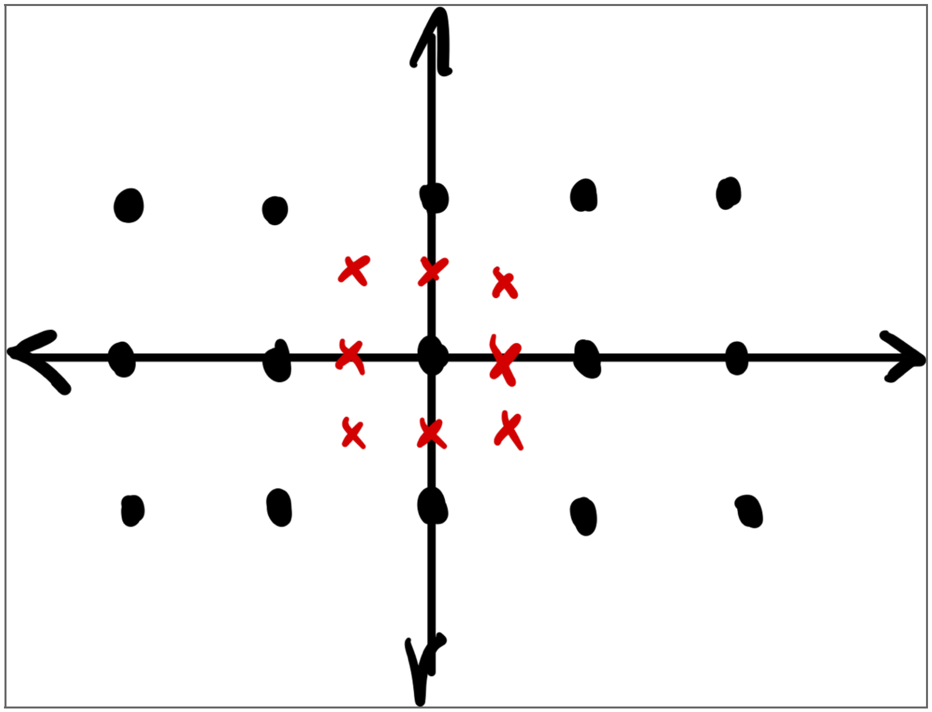
$$N = 2^3, N = 2 \cdot 5 \qquad \text{(double) prime power}$$

these live in a non-unimodular lattice $\Lambda^{2,2}$

$$\nexists (p_L, p_R) \cdot (p'_L, p'_R) = 1.$$



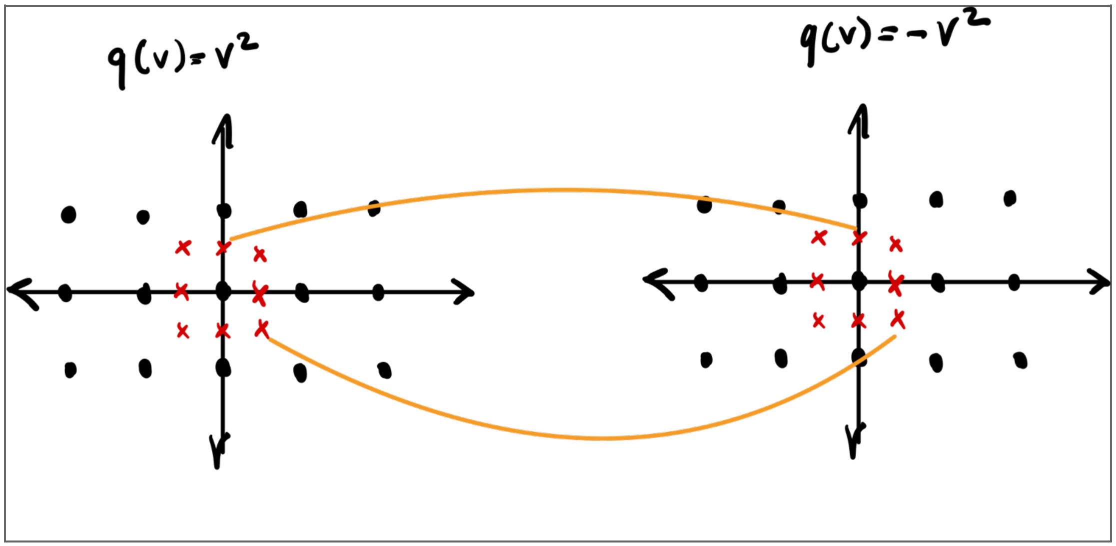
Red crosses denote the missing vectors $v' \cdot v = 1$.



Can't just include them: $v'^2 \notin \mathbb{Z}$.

GLUING CONSTRUCTION

Take another copy with negative norm. Glue along missing vectors.



GLUING CONSTRUCTION

$$\mathrm{II}^{d,d} = \Lambda \# \Lambda^{(-)} = \bigoplus_{r \in \Lambda^\vee/\Lambda} (\Lambda + \boldsymbol{r}, \Lambda^{(-)} + \boldsymbol{r})$$

Can always obtain a unimodular lattice $\mathrm{II}^{d,d}$ by doubling a lattice Λ.

GLUED QUASICRYSTALS

For $\mathrm{II}^{2,2}$ $N = 5, 8, 10$:

$\nexists \mathbb{Z}_N$ quasicrystalline string. (Non-unimodular)

$\exists \mathbb{Z}_N \times \mathbb{Z}_N$ quasicrystalline string. (Unimodular by gluing)

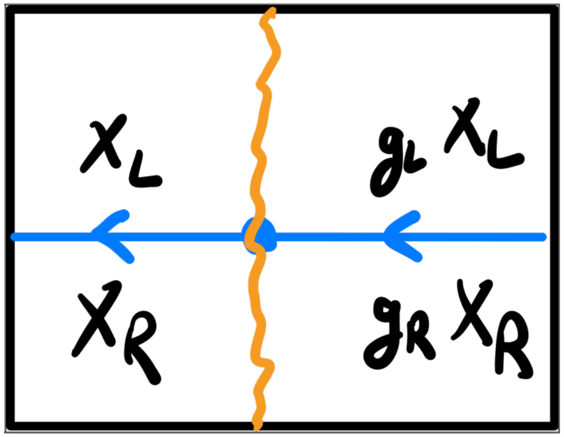
1. String on $T = \mathrm{II}^{d,d}$ with polarization
2. Irrational polarization $\implies$ Quasicrystal
3. ?
    a. ?
    b. ?

GAUGING/ORBIFOLDING

Continuous symmetry: add gauge field $A_\mu \in H^1(M, U(1))$.

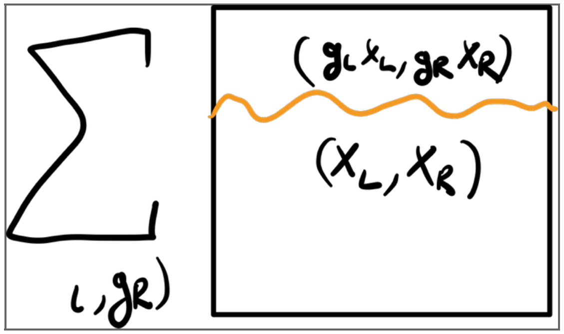
Discrete symmetry: add discrete gauge field $A \in H^1(M, \mathbb{Z}_N)$.

Equivalently, $\mathbb{Z}_N$ monodromy when you go around a cycle.



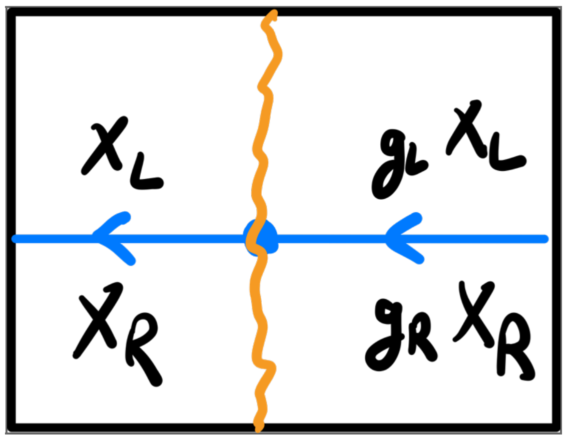
TEMPORAL DEFECTS

Implement projection to invariant states $|\psi\rangle \mapsto \sum_{g \in G} g|\psi\rangle$



SPATIAL DEFECTS

Introduce twisted sector Hilbert spaces $X(\sigma + 2\pi) = g \cdot X(\sigma)$



GAUGING QUASICRYSTALLINE SYMMETRY

There is no more spacetime description.
Only 2d SCFT obtained by quasicrystalline gauging.

$Q = 16$ Quasicrystalline Orbifolds				
Dimension	Lattice	Twist	IIA	IIB
6	$\Gamma_5^{2,2}\Gamma_5^{2,2}[11]$ $\Gamma_8^{2,2}\Gamma_8^{2,2}[11]$ $\Gamma_{10}^{2,2}\Gamma_{10}^{2,2}[11]$ $2\Gamma_{12}^{2,2}$	$\mathbb{Z}_5 \ : (1,1;2,2)/5$ $\mathbb{Z}_8 \ : (1,1;3,3)/8$ $\mathbb{Z}_{10} : (1,1;3,3)/10$ $\mathbb{Z}_{12} : (1,1;5,5)/12$	$\mathcal{N} = (1,1)$ $G + 20V$	$\mathcal{N} = (2,0)$ $G + 21T$

K3

Quasicrystalline orbifold	$\mathcal{Q}$ charges	Co_0 class	$\Gamma^{4;20}$	Symmetries
$\mathbb{Z}_5$	$(1^5, 2^5)/5$	$5C$	HM122	$5^{1+2} : \mathbb{Z}_4$
$\mathbb{Z}_8$	$(1^2, 2^3, 3^2, 4^3)/8$	$8H$	HM143	$\mathbb{Z}_8.\mathbb{Z}_2^3$
$\mathbb{Z}_{10}$	$(1, 2^3, 3^1, 4^3, 5^2)/10$	$10F$	HM159	D_{20}
$\mathbb{Z}_{12}$	$(1, 2, 3^2, 4^3, 5, 6^2)/12$	$12N$	HM157	D_{24}

CY3

4d Type IIB $\mathcal{N} = 2$ Quasicrystalline Orbifolds				
$\Gamma^{6,6}$	Twist	Gauge group	Spectrum	$(h_{2,1}, h_{1,1})$
$3\Gamma_{12}^{2,2}$	$(1,1,2;5,5,10)/12$	$U(1)^{11}$	$G + 11V + 12H_0$	$(11,11)$, [39]
$3\Gamma_{12}^{2,2}$	$(1,2,3;5,10,15)/12$	$U(1)^{22}$	$G + 22V + 11H_0$	$(22,10)$, [40]
$3\Gamma_{12}^{2,2}$	$(1,3,4;5,15,20)/12$	$U(1)^{14}$	$G + 14V + 27H_0$	$(14,26)$, [40]
$3\Gamma_{12}^{2,2}$	$(1,4,5;5,20,1)/12$	$U(1)^{29}$	$G + 29V + 6H_0$	$(29,5)$, [40]
$\Gamma_{24}^{4,4} + \Gamma_{12}^{2,2}$	$(1,7,8;5,11,16)/24$	$U(1)^{8}$	$G + 8V + 21H_0$	$(8,20)$
$\Gamma_{24}^{4,4} + \Gamma_{12}^{2,2}$	$(1,11,10;5,7,2)/24$	$U(1)^{13}$	$G + 4V + 17H_0$	$(4,16)$, [39]

1. String on $T = \mathrm{II}^{d,d}$ with polarization
2. Irrational polarization $\implies$ Quasicrystal
3. Gauge quasicrystal symmetry
    a. We get K3 and CY3
    b. ?

4D NON-SUSY QUASICRYSTALLINE STRING

- Non-supersymmetric ✓
- $\Lambda > 0$ ✓
- $SU(3) \times SU(2) \times U(1)$ ✓
- Chiral fermions ✓
- Unstable (by just one scalar) $\times$

CHARGE LATTICE

Complicated $\mathbb{Z}_5$ quasicrystal charge lattice

$$\mathrm{II}^{22,6} = \Lambda_{E_8} \oplus \Lambda_5 \# \Lambda_{\mathbb{Z}_5} \oplus \Lambda_{\mathbb{Z}_5} \# \Lambda_{\mathbb{Z}_5}.$$

Orbifold by the diagonal $\mathbb{Z}_5$ quasicrystalline symmetry and shift

$$v = \frac{1}{5}(2, 0, 3, 0, 1, \cdots).$$

QUASICRYSTALLINE ORBIFOLD RESULT

Gauge group: $SO(10) \times SU(5) \times SU(3) \times SU(2) \times U(1)^4$.
Massless spectrum: $G + 84V + 561F_c + 9F_0 + 354S_c + 1S_0$.
$\#\text{Fermions} > \#\text{Bosons} \implies \Lambda > 0.$

	$SO(10)\times SU(5)\times SU(3)\times SU(2)\times U(1)^4$ reps	
Sector	Complex scalars	Left handed Weyl fermions
Untwisted	$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})_{0,0,0,0}$	$9(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})_{0,0,0,0}$
	$3(\mathbf{16},\mathbf{1},\mathbf{1},\mathbf{1})_{0,0,0,-15}$	$(\mathbf{16},\mathbf{1},\mathbf{1},\mathbf{1})_{0,0,0,-15}$
	$3(\mathbf{10},\mathbf{1},\bar{\mathbf{3}},\mathbf{1})_{0,0,0,10}$	$(\mathbf{10},\mathbf{1},\bar{\mathbf{3}},\mathbf{1})_{0,0,0,10}$
	$3(\mathbf{1},\mathbf{5},\mathbf{1},\mathbf{1})_{-2,-10,-2,0}$	$(\mathbf{1},\mathbf{5},\mathbf{1},\mathbf{1})_{-2,-10,-2,0}$
	$3(\mathbf{1},\overline{\mathbf{10}},\mathbf{1},\mathbf{2})_{-1,5,-1,0}$	$(\mathbf{1},\overline{\mathbf{10}},\mathbf{1},\mathbf{2})_{-1,5,-1,0}$
	$3(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{2})_{5,-5,5,0}$	$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{2})_{5,-5,5,0}$
		$3(\mathbf{16},\mathbf{1},\mathbf{3},\mathbf{1})_{0,0,0,5}$
		$3(\mathbf{1},\mathbf{10},\mathbf{1},\mathbf{1})_{-4,0,-4,0}$
		$3(\mathbf{1},\mathbf{1},\bar{\mathbf{3}},\mathbf{1})_{0,0,0,-20}$
		$3(\mathbf{1},\mathbf{5},\mathbf{1},\mathbf{2})_{3,5,3,0}$
		$3(\mathbf{1},\bar{\mathbf{5}},\mathbf{1},\mathbf{1})_{2,-10,2,0}$
$\hat{g}+\hat{g}^4$		$15(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{2})_{-1,-3,-1,12}$
		$15(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})_{-2,7,1,12}$
		$15(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})_{0,7,-3,12}$
		$5(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{2})_{1,-3,-5,12}$
		$5(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{2})_{-3,-3,3,12}$
		$5(\mathbf{1},\mathbf{5},\mathbf{1},\mathbf{1})_{2,2,2,12}$
		$5(\mathbf{1},\bar{\mathbf{5}},\mathbf{1},\mathbf{1})_{2,-3,-1,12}$
		$5(\mathbf{1},\bar{\mathbf{5}},\mathbf{1},\mathbf{1})_{0,-3,3,12}$
		$5(\mathbf{1},\mathbf{1},\bar{\mathbf{3}},\mathbf{2})_{-1,-3,-1,-8}$
		$5(\mathbf{1},\mathbf{1},\bar{\mathbf{3}},\mathbf{1})_{0,7,-3,-8}$
		$5(\mathbf{1},\mathbf{1},\bar{\mathbf{3}},\mathbf{1})_{-2,7,1,-8}$
$\hat{g}^2+\hat{g}^3$	$15(\mathbf{1},\mathbf{1},\bar{\mathbf{3}},\mathbf{1})_{4,-1,1,4}$	$5(\mathbf{1},\mathbf{1},\bar{\mathbf{3}},\mathbf{1})_{4,-1,1,4}$
	$15(\mathbf{1},\mathbf{1},\bar{\mathbf{3}},\mathbf{1})_{2,-1,5,4}$	$5(\mathbf{1},\mathbf{1},\bar{\mathbf{3}},\mathbf{1})_{2,-1,5,4}$
	$15(\mathbf{1},\mathbf{1},\bar{\mathbf{3}},\mathbf{1})_{-2,-6,-2,4}$	$5(\mathbf{1},\mathbf{1},\bar{\mathbf{3}},\mathbf{1})_{-2,-6,-2,4}$

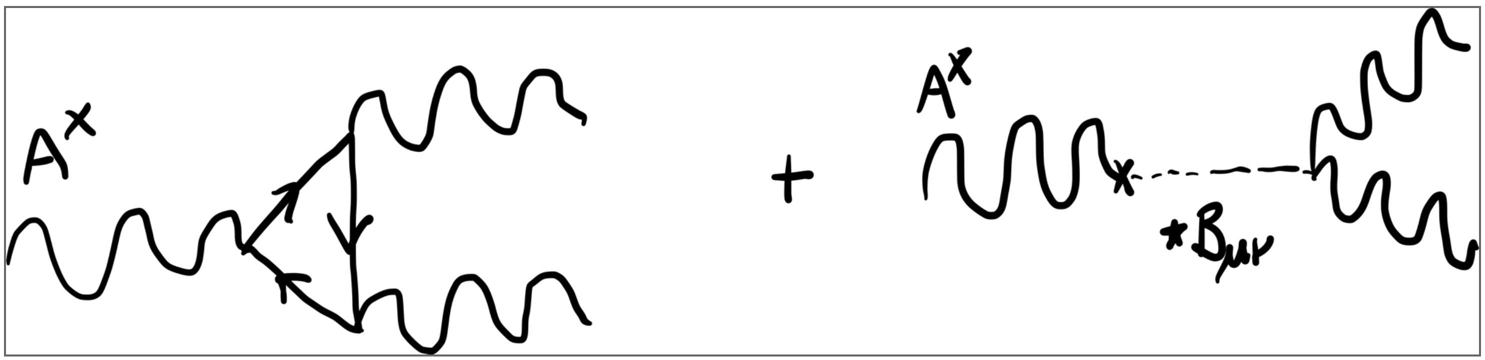
INSTABILITIES

Charged scalars can't have a linear instability $\lambda\varphi \not\subset V(\varphi)$.

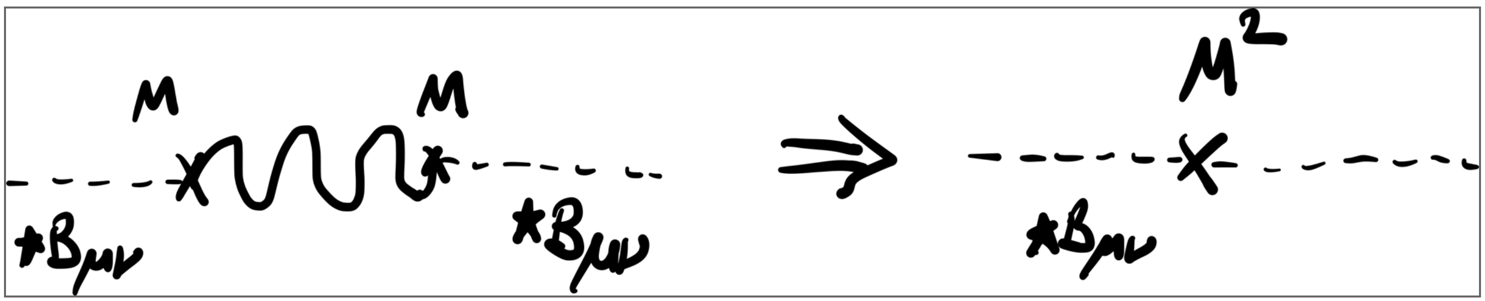
Neutral scalars can:

1. Axion $\star B_{\mu\nu}$,
2. Dilaton ϕ.

AXION IN GREEN-SCHWARTZ

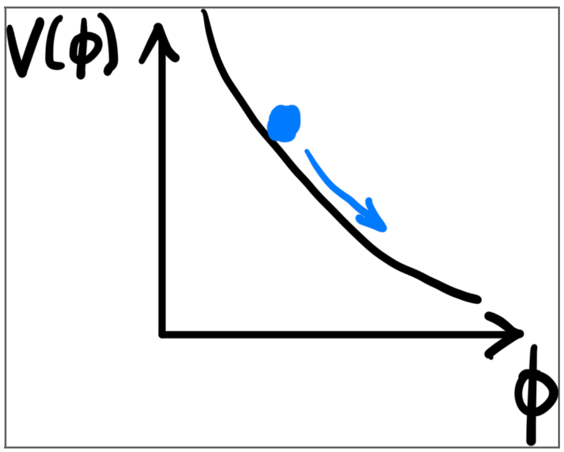


AXION MASSES UP



DILATON ROLLS

The only instability in the theory $V(\phi) = \Lambda e^{-2\sqrt{2}\phi}$



1. String on $T = \mathrm{II}^{d,d}$ with polarization
2. Irrational polarization $\implies$ Quasicrystal
3. We gauge quasicrystal symmetry
   a. K3 and CY3
   b. Non-susy $\mathbb{Z}_5$ with only dilaton instability

QUASICRYSTALS IN THE FUTURE

To also fix the dilaton, we'd need quasicrystalline M-theory orbifolds.
Work in progress: can symmetries of M-theory be gauged?

THANK YOU!