Liquid crystals and poroelastic media: two problems for hierarchical systems in Mechanics

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Hierarchical systems are widely present in both biological structures and technological applications. The ability to account for not only the macroscopic or exterior behavior of an elastic medium, but also its internal or microscopic arrangement, allows for a more complete and realistic description.
Recently, I have studied two problems in which a non-trivial interaction originates between two contributions from two different scales or parts of a system. In the first problem, we proved the existence of an equilibrium configuration for a nematic film with surface tension using the Calculus of Variations. The key aspect is the competition between the Frank energy and the area functional. In the second problem, we derived the equations of Biot's poroelasticity from the microstructure in the case of incompressibility of both an isotropic, linear elastic solid and a low Reynolds' number Newtonian fluid flowing through its pores. By using the asymptotic homogenization technique, we obtain a macroscale system of PDEs, with corresponding cell problems at the pore-scale; moreover, we retrieve the equivalence between the change in volume of the porous solid and the volume of fluid exchanged.
References:
[1] G. Bevilacqua, C. Lonati, L. Lussardi, A. Marzocchi, A variational analysis of nematic axisymmetric films: the covariant derivative case, arXiv preprint, arXiv:2405.20154, submitted.
[2] G. Bevilacqua, C. Lonati, L. Lussardi, A. Marzocchi, Nematic axisymmetric films: nematic order aligned along parallels, in preparation.
[3] R. Penta, C. Lonati, L. Miller, A. Marzocchi, Poroelasticity derived from the microstructure for intrinsically incompressible constituents, submitted.