

Resonant triad interactions of gravity waves in variable-depth containers

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Confining a liquid to a partially filled basin can lead to the spontaneous formation of resonant triads of gravity waves, in which three standing wave modes interact and continuously exchange energy. This internal resonance mechanism induces waves of relatively large magnitude, representing a potential hazard in many industrial settings, such as the transport of liquid cargo. However, the formation of these triads depends critically on the dimensions of the container and the depth to which it is filled. We demonstrate herein that resonant triads may, perhaps surprisingly, be excited when the depth of a rectangular tank varies in space, and that the time scale of the energy exchange shortens as the bathymetry becomes more pronounced. We combine asymptotic analysis and numerical computation to identify the topographies that induce the strongest triad interactions, paying particular attention to the limits of small- and large-amplitude depth variations.

I will discuss two classes of triad: weakly correlated triads forming for small variations in depth, and strongly correlated triads forming across deep basins connected by shallow ridges. I will then present more results into the existence and evolution of resonant triads in cylindrical and wedge confining basins. This demonstrates that triad is ubiquitous. Our study highlights a new mechanism for exciting internal resonances in variable-depth basins. It may enable the development of strategies to mitigate hazardous resonances in industrial settings.