# The Role of Three-Wave Interactions in Faraday Wave Pattern Formation

Laura Pinkney

Patterns in nature are seen in a variety of scenarios, including stripes on a zebra and hexagons in a honeycomb to name a few simple examples. These are both examples of patterns on a single length scale. Stripes are composed of a single wave, whereas hexagons arise from three waves of the same wavenumber, 60 degrees apart, interacting with each other. This is the simplest example of a three-wave interaction (or resonant triad); the sum of two wavevectors equals the third one. Since there are pattern forming systems with more than one length scale, it is natural and interesting to look at problems with two critical wavenumbers.

Three-wave interactions (3WIs) can be used to explain pattern-forming behaviour in the Faraday wave experiment close to onset. The experiment involves periodically forcing a container of fluid up and down and observing the patterns formed on the surface. When the forcing exceeds some threshold, the flat state becomes unstable, which can lead to a variety of patterns. When the forcing contains a single frequency component, simple patterns such as stripes, squares and hexagons can be observed. However, if multiple-frequency forcing is used, 3WIs can form between waves with two critical wavenumbers, leading to more complex structures such as superlattices, quasipatterns and spatiotemporal chaos.

We consider problems with two critical wavenumbers, where resonant triads form between two waves of a larger wavenumber and a third wave of a smaller wavenumber. The inclusion of a second wavenumber allows for patterns on a rhombic lattice to form, in addition to those on a hexagonal lattice. The dynamics exhibited by these 3WIs can be represented by a system of ODEs. This system is generalised, such that any pattern-forming system exhibiting 3WIs (with two wavelengths) can be reduced to this system of equations. We present an analysis of the patterns formed by a model PDE, an adaptation of the Lifshitz—Petrich equation with additional nonlinear terms. We use weakly nonlinear theory to draw comparisons between the patterns predicted by a system of ODEs and those observed in the PDE.