Understanding strong Allee effects in diffusive population models with a moving boundary

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We examine travelling wave solutions of a reaction-diffusion equation that incorporates strong Allee effects in its population growth function. Additionally, we employ a Stefan-like moving boundary condition that governs the rate at which the edge of the population advances. A key novelty of our work is the combination of a strong Allee effect with a Stefan-like moving boundary condition in a diffusive population model. We investigate these sharp-front travelling wave solutions using both direct numerical simulation and phase plane analysis, which also enables us to obtain the relationship between the wave speed and the parameter associated with the moving boundary condition. Our results show that this model admits sharp-front travelling waves with wave speed $c<c^*$, where c^* is a critical speed that depends on the Allee threshold. This is a distinction between our model and the classical population models, which admit travelling wave solutions over $\mathbb{R}$ with speed $c\ge c^*$. We also present a systematic method for determining c^* for particular choices of strong Allee population growth models, where we apply asymptotic analysis to construct sharp-front travelling wave solutions in the limit $c\to c^{*-}$.