Loxodromic Sequences of Tangent Circles in Polytopal Packings

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In the 1960s, Coxeter initiated a study into infinite sequences of circles in the plane having the property that every 4 consecutive elements of a sequence are mutually tangent, that is they form a Descartes quadruple. He went on to show that that if their bends fall into geometric progression, then there is a unique such sequence and the ratio between consecutive bends is equal to $\varphi+\sqrt{\varphi}$, where φ is the golden ratio. This spiral sequence of circles is mapped onto itself by a dilative-rotation whose dilation factor is $\varphi+\sqrt{\varphi}$ and angle of rotation is related to the degree 4 equation satisfied by $\varphi+\sqrt{\varphi}$. Some 50 years later, the theory of polytopal circle packings was developed, whereas Descartes configurations correspond to the tetrahedron. In this talk, we will extend Coxeter's notion of loxodromic sequences of tangent circles to the polytopal setting, show that for each regular polytope there is a unique sequence where the bends fall into geometric progression, and provide explanations to some of Coxeter's observations which are obscured by the elegant algebraic properties of the golden ratio.