

The GbetaE characteristic polynomial: log-correlated structure, the stochastic zeta function, and a curious Mellin transform

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The characteristic polynomial Φ_N of the Gaussian beta-ensemble encodes two very different limits at once: a microscopic Sine_{beta} point process governing its zeros, and a global log-correlated Gaussian field governing its modulus. I will present a single bulk identity for Φ_N , exact up to vanishing errors, that exhibits both structures simultaneously and shows them to be asymptotically independent. The mechanism is an explicit Gaussian approximation of the Dumitriu-Edelman tridiagonal recursion as a diffusively perturbed rotation, in which the only surviving non-Gaussianity is a single $O(1)$ stochastic-Airy constant. As corollaries we obtain convergence of microscopic ratios to a stochastic zeta function (new for the GbetaE) and a log-correlated CLT recovering the Bourgade-Mody-Pain result. Finally I will describe an "Airy-Mellin functional" $M_{\beta}(s)$ that governs all moments $E|\Phi_N|^s$, admits several equivalent representations, reduces to a Barnes-G closed form at $\beta=2$, and can conjecturally be expressed in terms of a beta-deformation of the Barnes-G function.

Joint work with Gaultier Lambert.