Quasicrystals in discrete-holographic tensor networks

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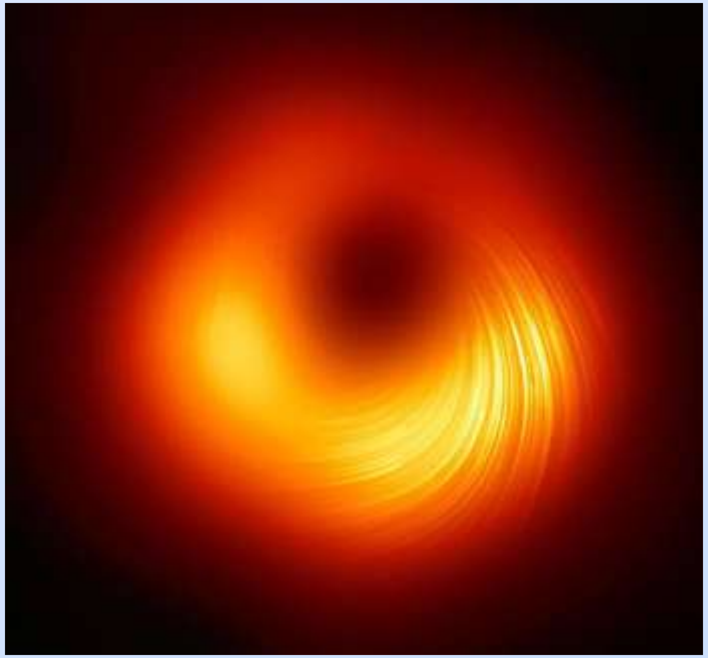
Quasicrystals in fundamental physics

Edinburgh, July 22, 2026

Introduction: Holographic Dualities

Physical systems whose fundamental degrees of freedom scale with the area of the volume considered.



Example: Black holes

$$S_{\mathrm{BH}} = \frac{A_{\mathrm{hor}}}{4G}$$

[Bekenstein 1973, Hawking 1975]

A general feature of gravity?

[‘t Hooft 1993, Sussking 1994]

Introduction: Holographic Dualities

Concrete setting: The AdS/CFT correspondence [Maldacena 1998, Witten 1998]

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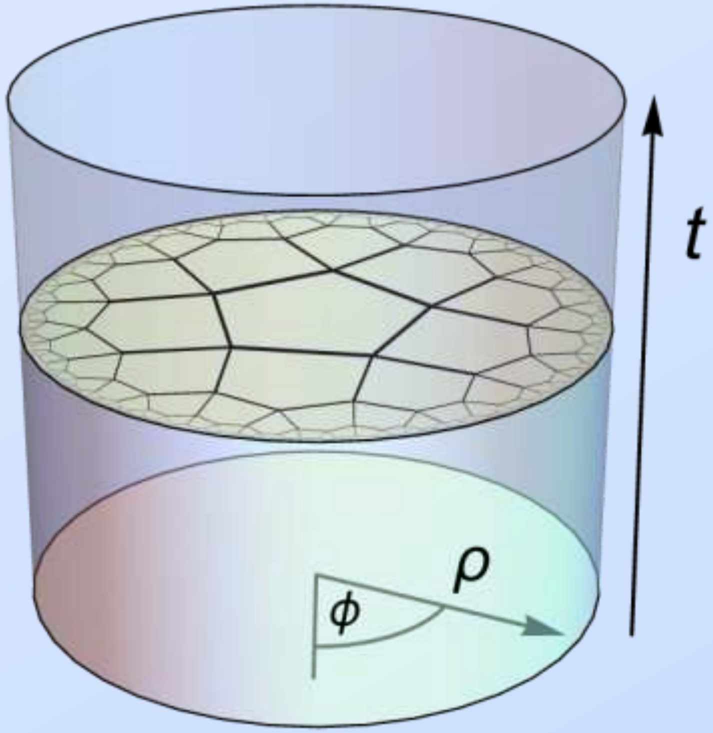
Anti-de Sitter spacetime



A spacetime with global negative curvature, asymptotic boundary

Introduction: Holographic Dualities

Concrete setting: The AdS/CFT correspondence [Maldacena 1998, Witten 1998]

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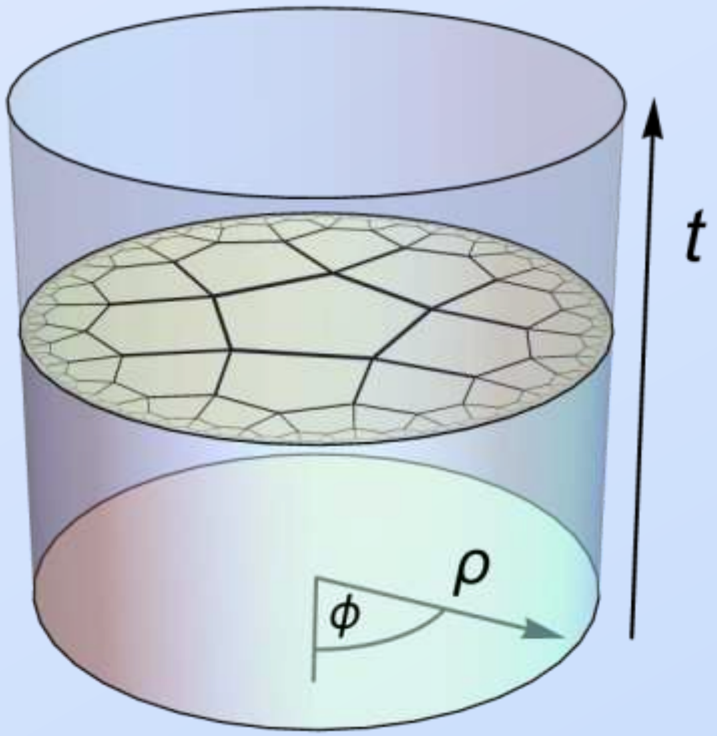
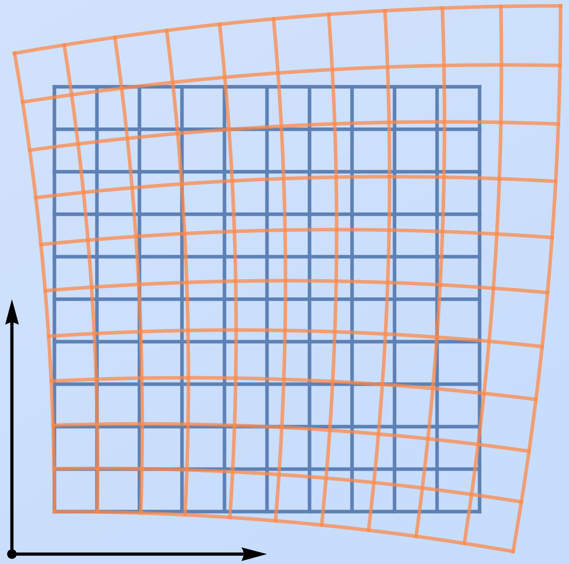


Duality

Conformal field theory



A spacetime with global negative curvature, asymptotic boundary

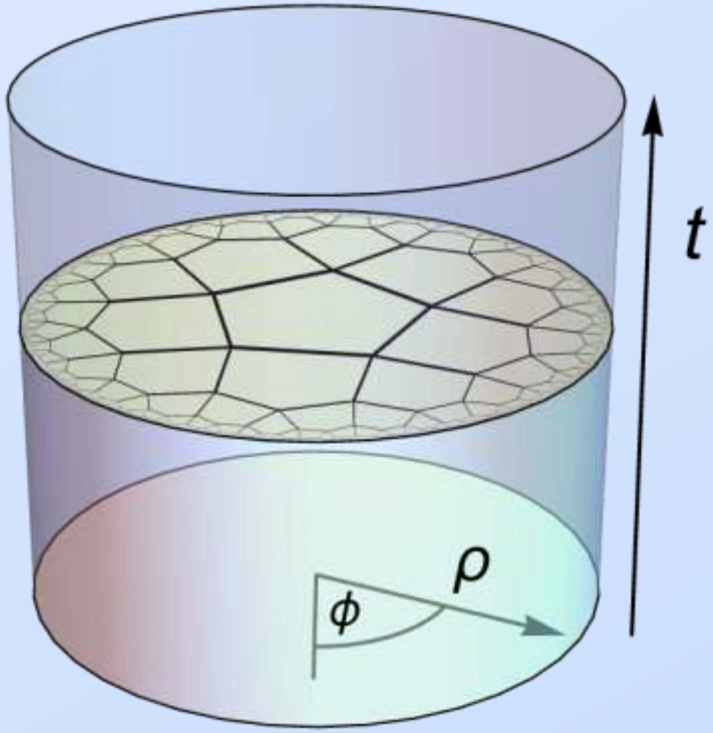
A quantum field theory invariant under all local and global changes of length.

Introduction: Holographic Dualities

Concrete setting: The AdS/CFT correspondence

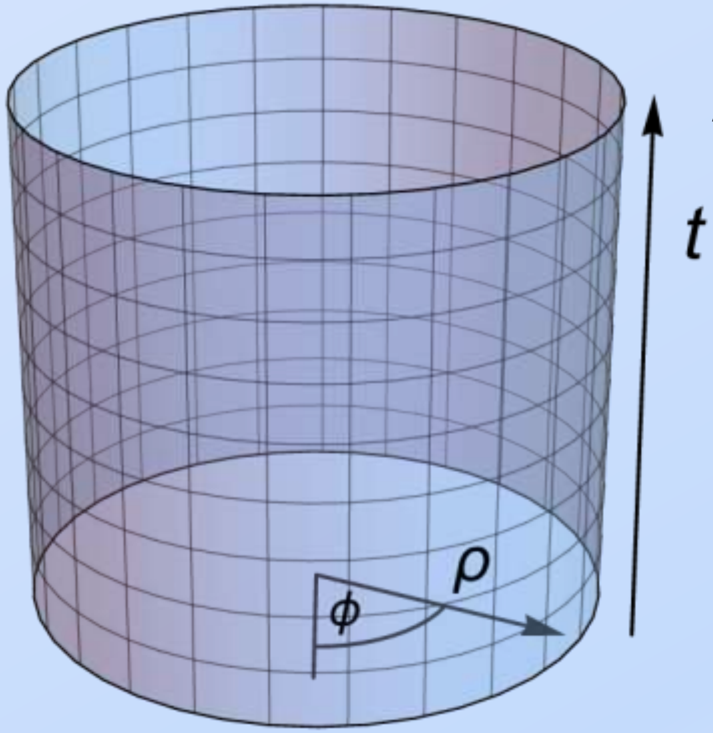
[Maldacena 1998, Witten 1998]

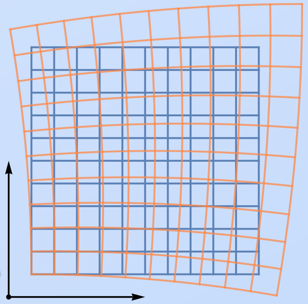
Bulk: Anti-de Sitter spacetime



Duality

Boundary: Conformal field theory





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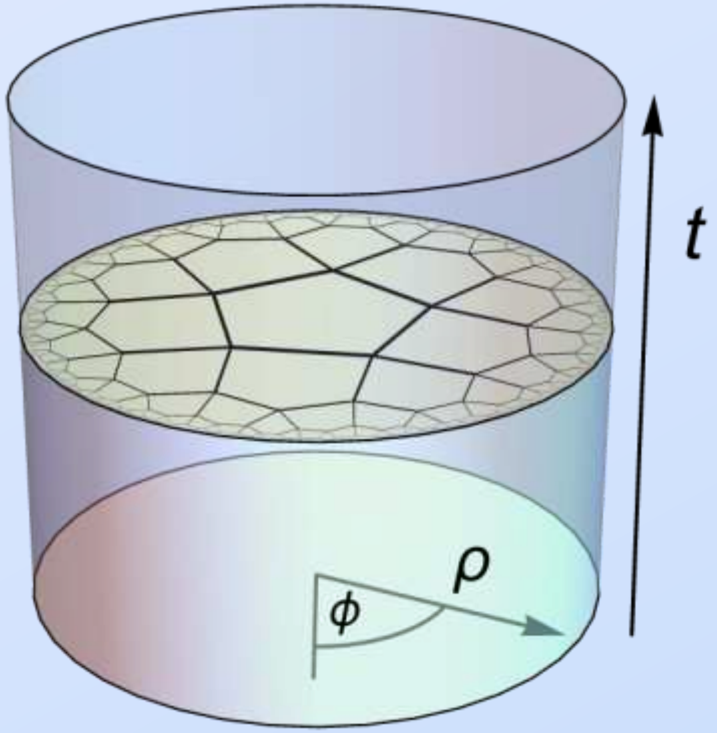

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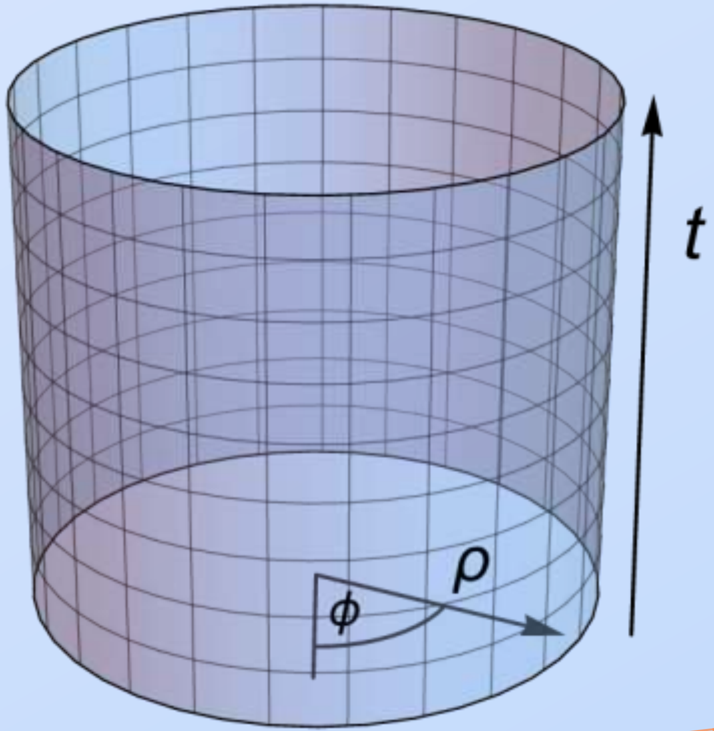
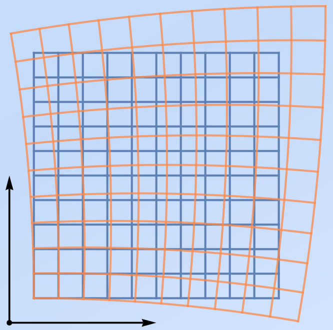
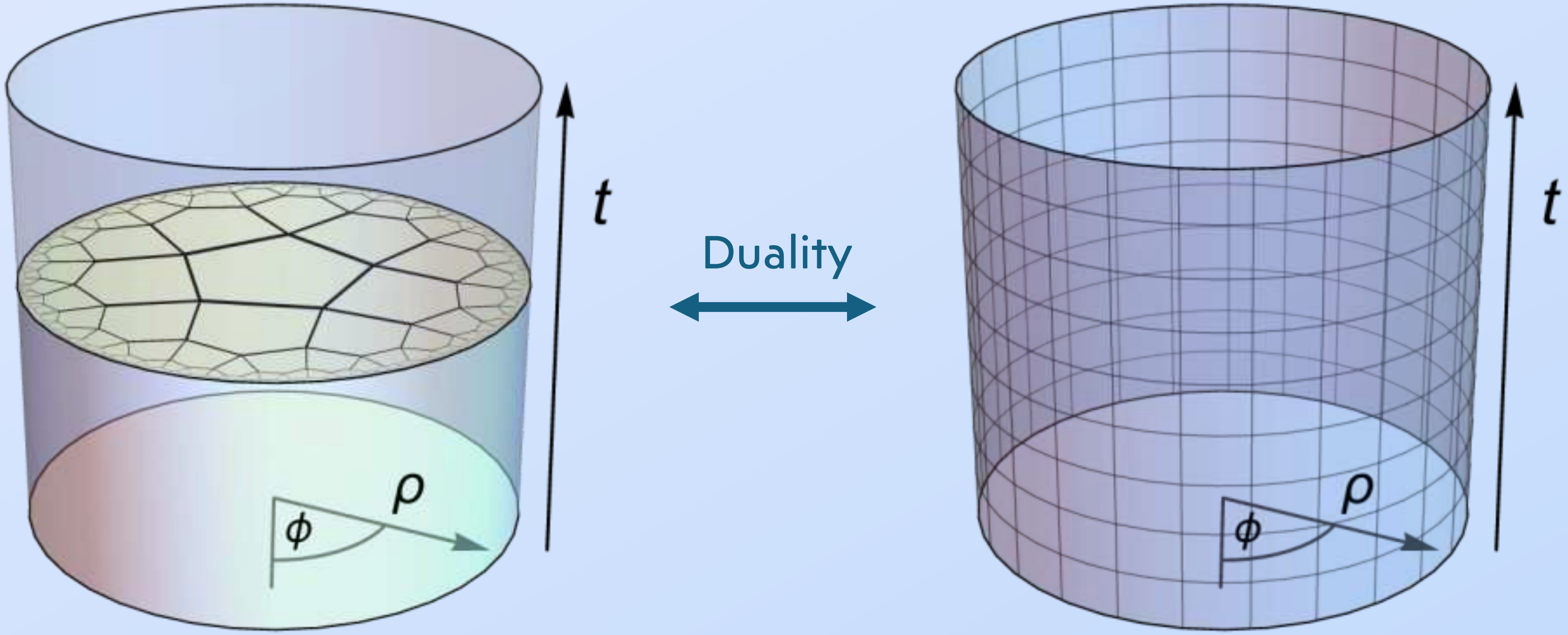


Follows the symmetries of a hyperbolic embedding in $\mathbb{R}^{d,2}$ spacetime → Symmetry group $SO(d,2)$ ← Follows conformal + Lorentz symmetries



[Brown/Henneaux, 1986]

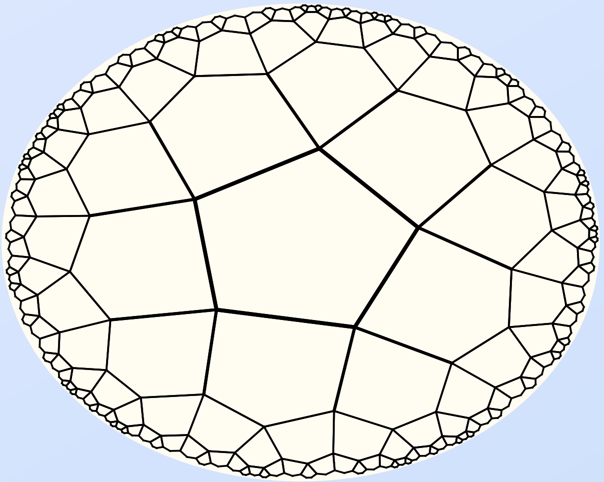
Introduction: Holographic Dualities



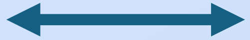
Motivation: Discrete, finite-dimensional instances of AdS/CFT?

Introduction: Holographic Dualities

Time-slice picture:



Hyperbolic bulk



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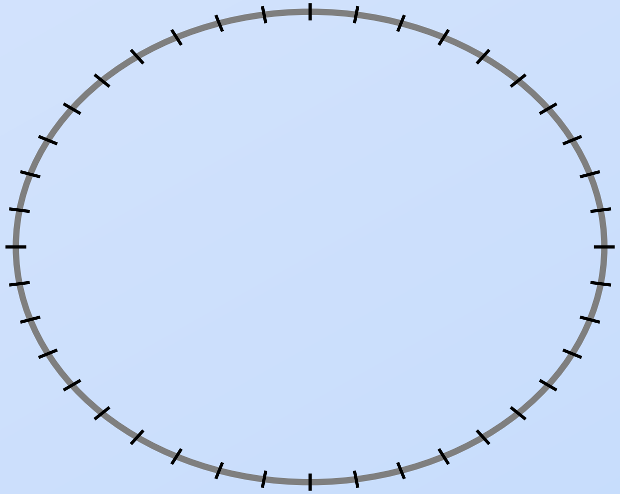

Flat boundary

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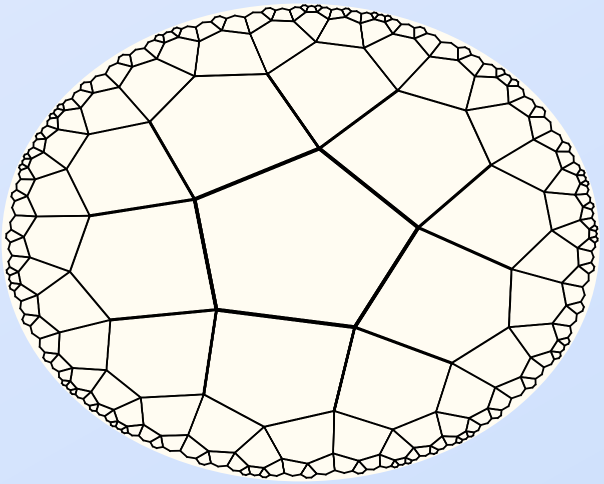

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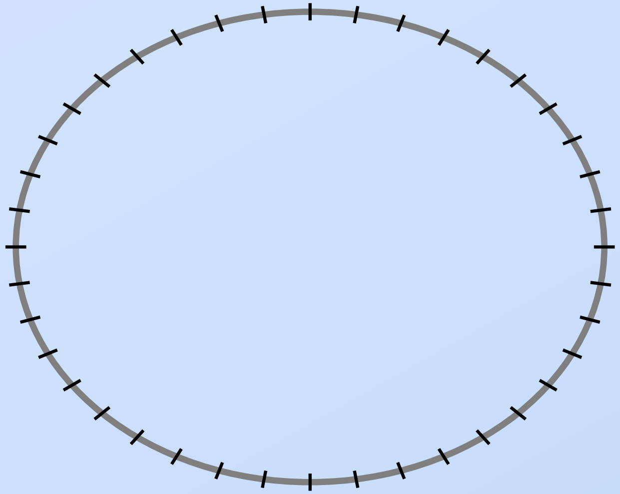


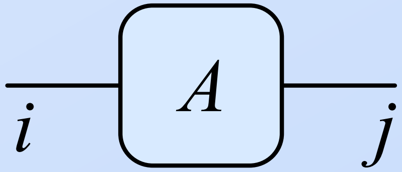
Flat boundary

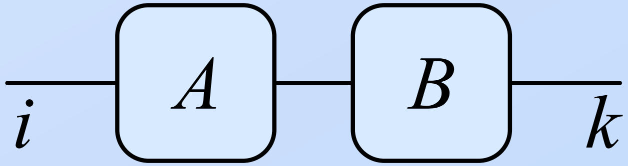
Main tool: *Tensor networks*, which represent the entanglement geometry of quantum states.

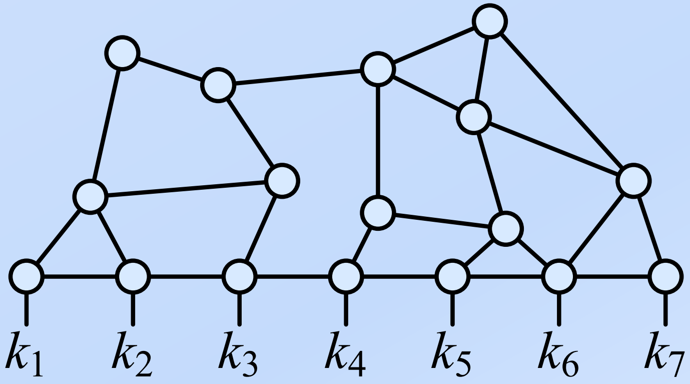
Introduction: Tensor Networks

What?

1. Tensor network: A generalization of matrix multiplication to tensors, expressed as a graph.

$A_{i,j} =$ 

$(AB)_{i,k} = \sum_j A_{i,j} B_{j,k} =$ 

$T_{k_1,k_2,\ldots,k_7} =$ 

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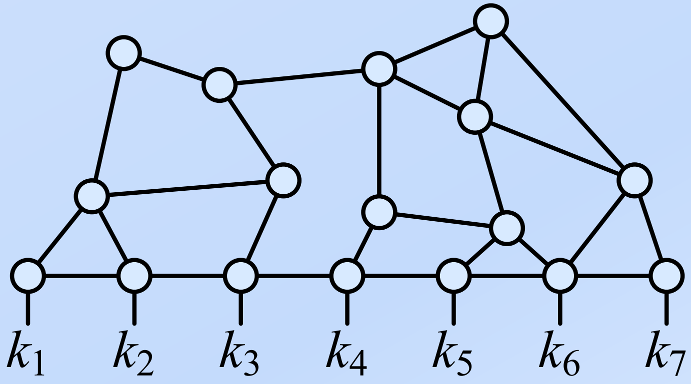
1. Tensor network: A generalization of matrix multiplication to tensors, expressed as a graph.
2. Use in quantum physics: Complex-valued tensors $T \in \mathbb{C}^n$ describe quantum states

$$|\psi\rangle = \sum_{k_1,k_2,\dots,k_n} \mathrm{T}_{k_1,k_2,\dots,k_n} |k_1\rangle \otimes \cdots \otimes |k_n\rangle$$

$$T_{k_1,k_2,\dots,k_7} =$$



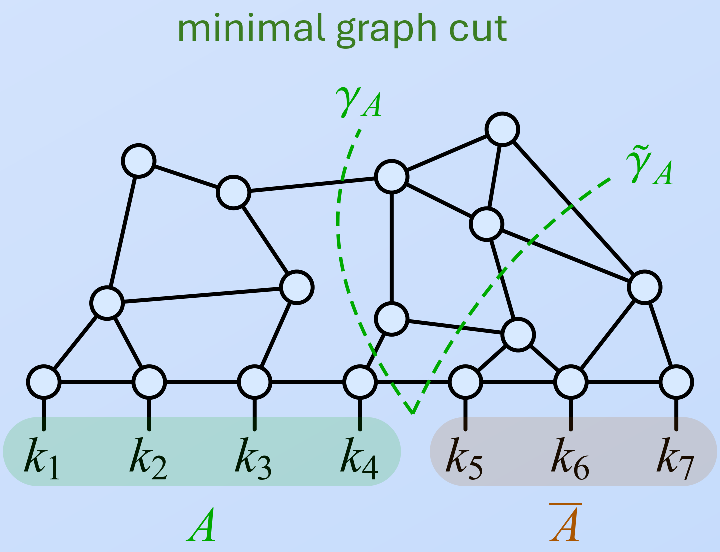
Introduction: Tensor Networks

Why?

1. Tensor networks are effectively *entanglement graphs*, making them useful ansatz classes.

Entanglement bound:

$$S_A = -\,\mathrm{Tr}\,[\rho_A \log \rho_A] \leq |\gamma_A|\ \log d$$



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minimal graph cut

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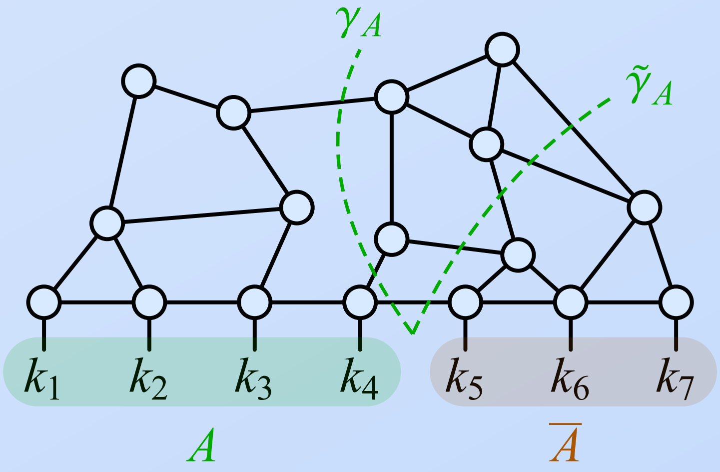

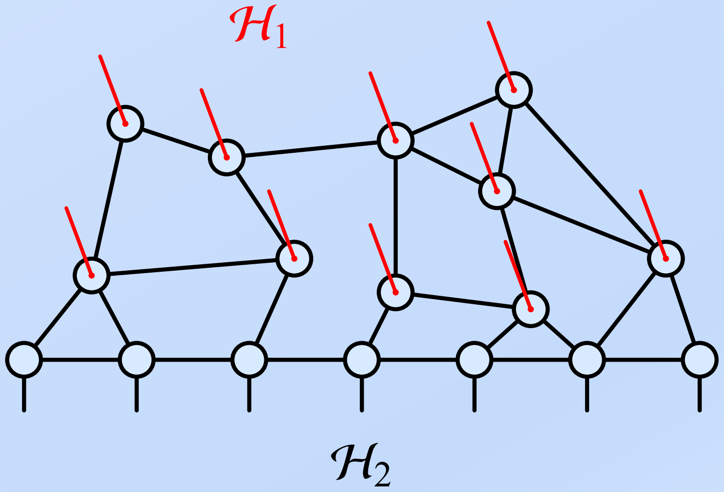

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1. Tensor networks are effectively *entanglement graphs*, making them useful ansatz classes.
2. Rather than states, they can also describe *linear maps*.
3. They make great toy models for bulk-boundary dualities!

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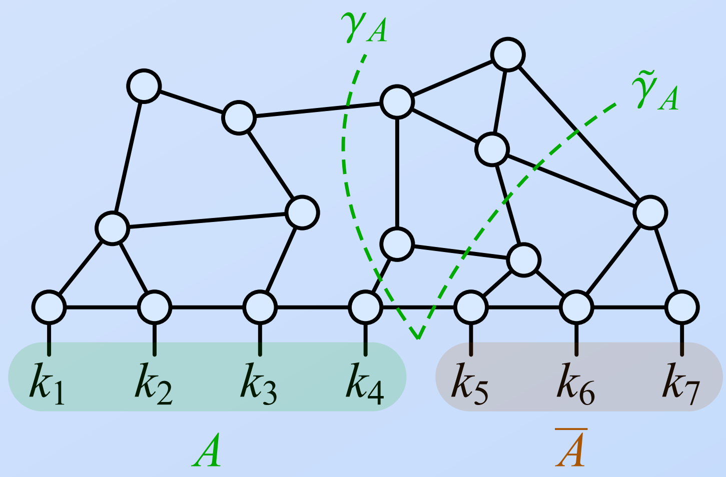
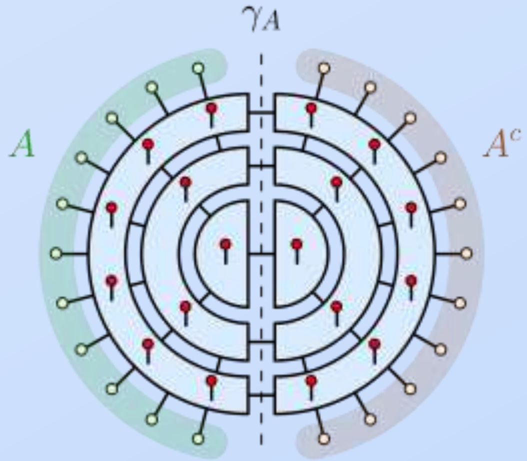
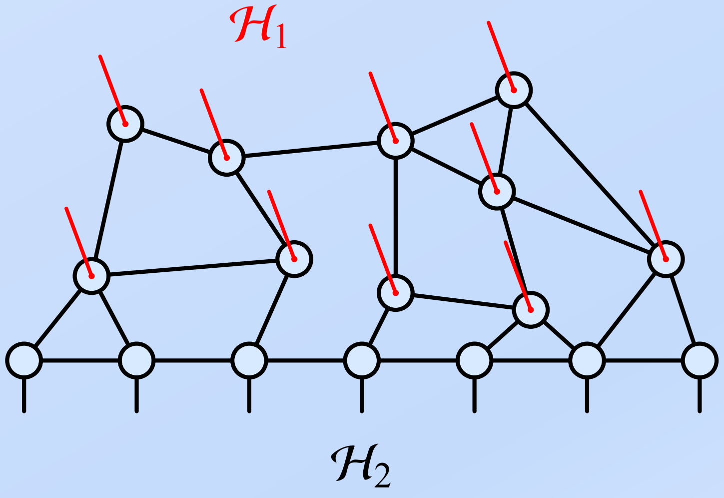


Holographic scaling:

$$S_A \sim |\gamma_A|$$

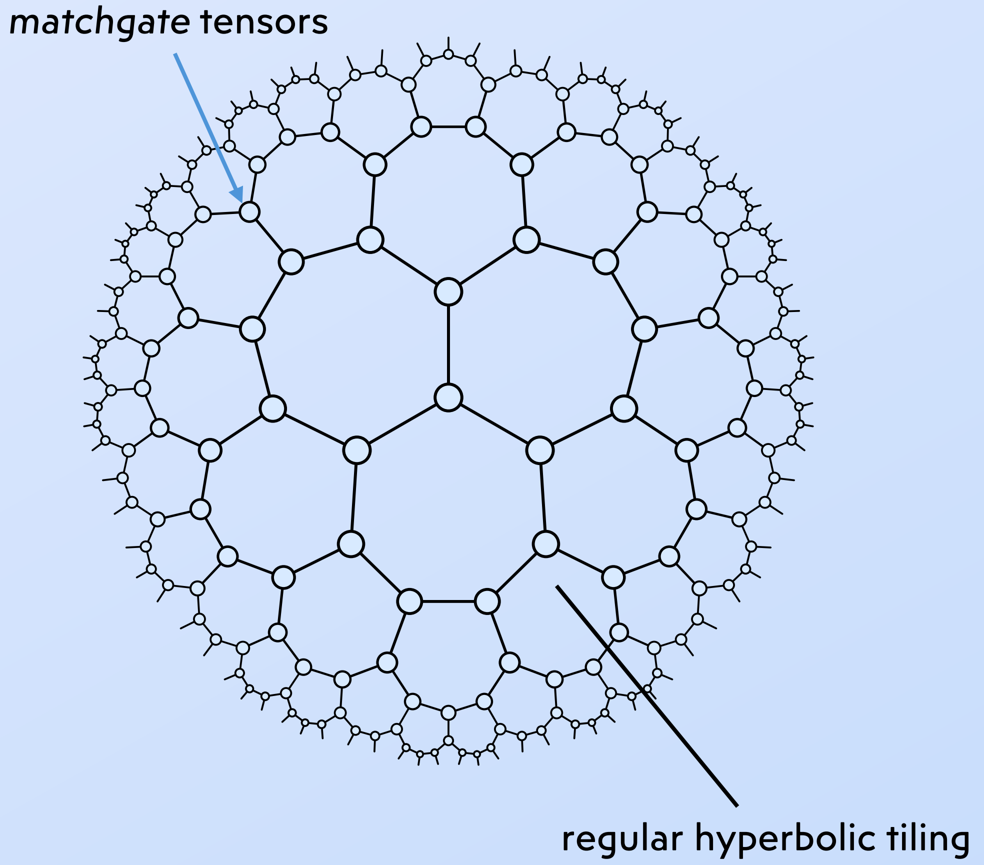




Introduction: Tensor Networks

Earlier research question:

What are the boundary states of hyperbolic tensor networks?



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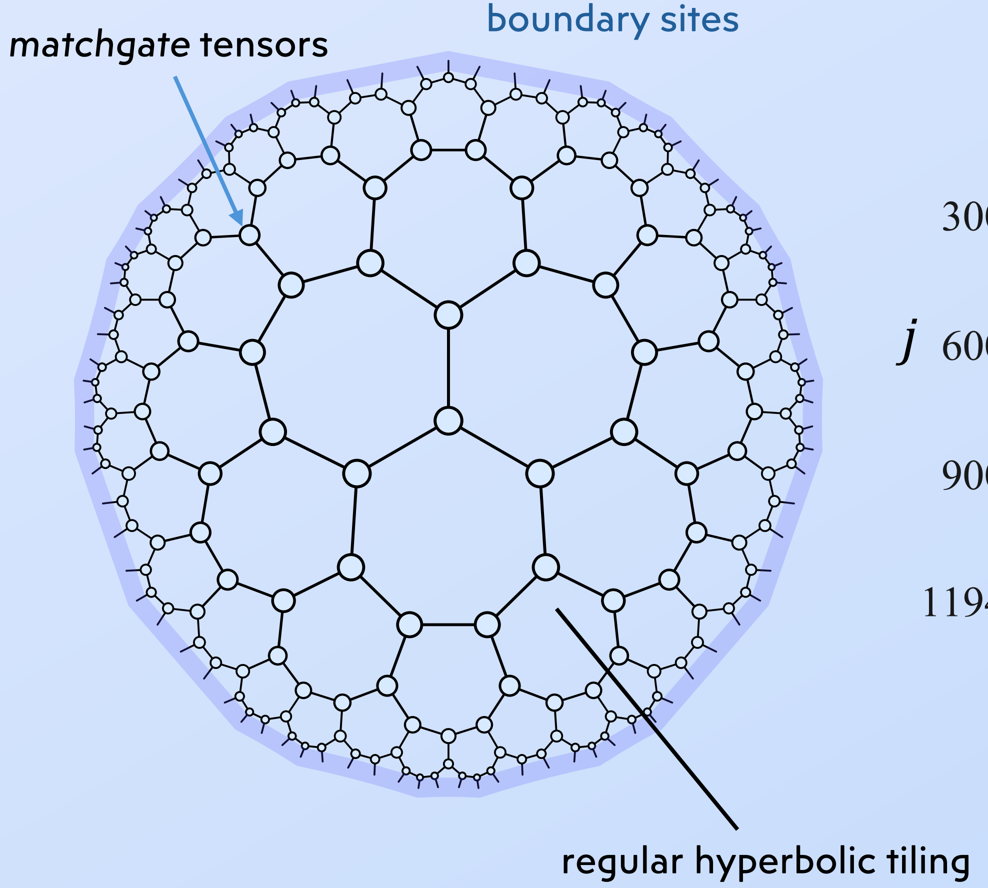
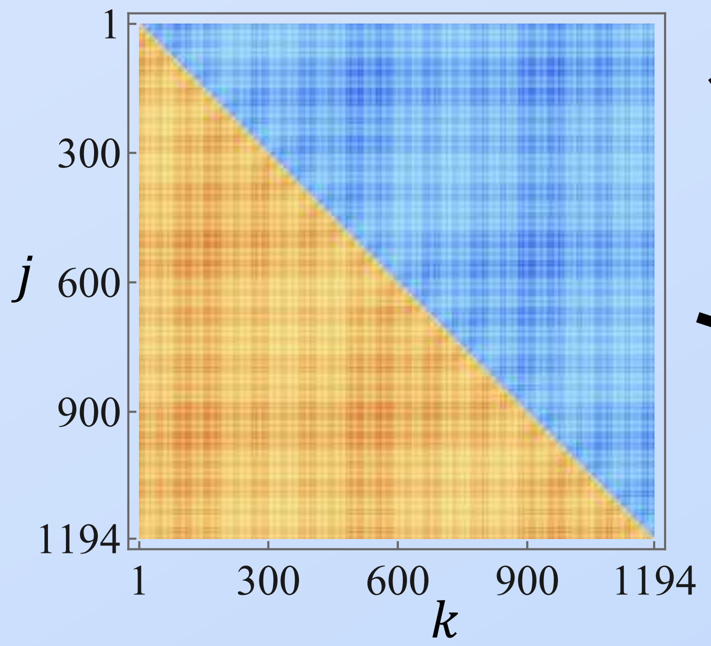
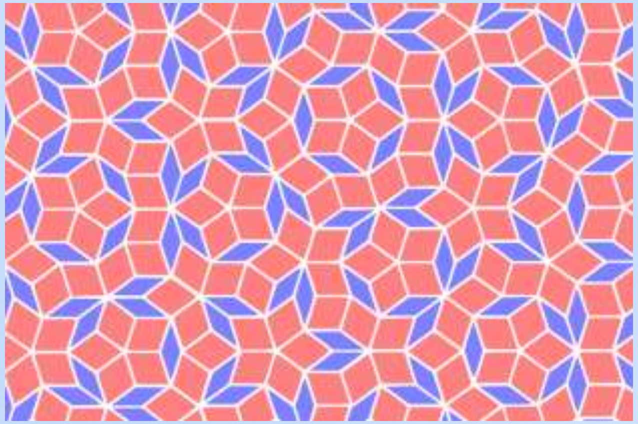




$$\frac{i}{2}\langle[c_j, c_k]\rangle \cdot |\mathrm{j} - \mathrm{k}|$$

?

!

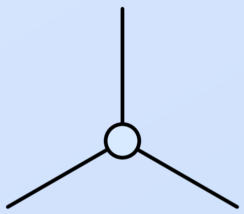


Quasicrystals

[Boyle/Dickens/Flicker 2018]

Holographic tensor network states

For $q = 3$:

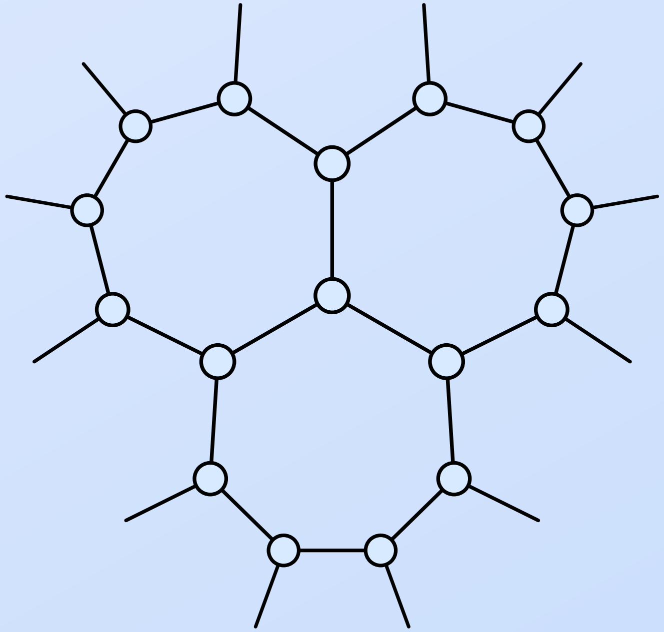


Method:

1. Pick a rotationally symmetric q-leg tensor $T_{i_1,i_2,\ldots,i_q} = T_{i_q,i_1,\ldots,i_{q-1}}$

Holographic tensor network states

For $q = 3$, $p = 7$, $L = 1$:



Method:

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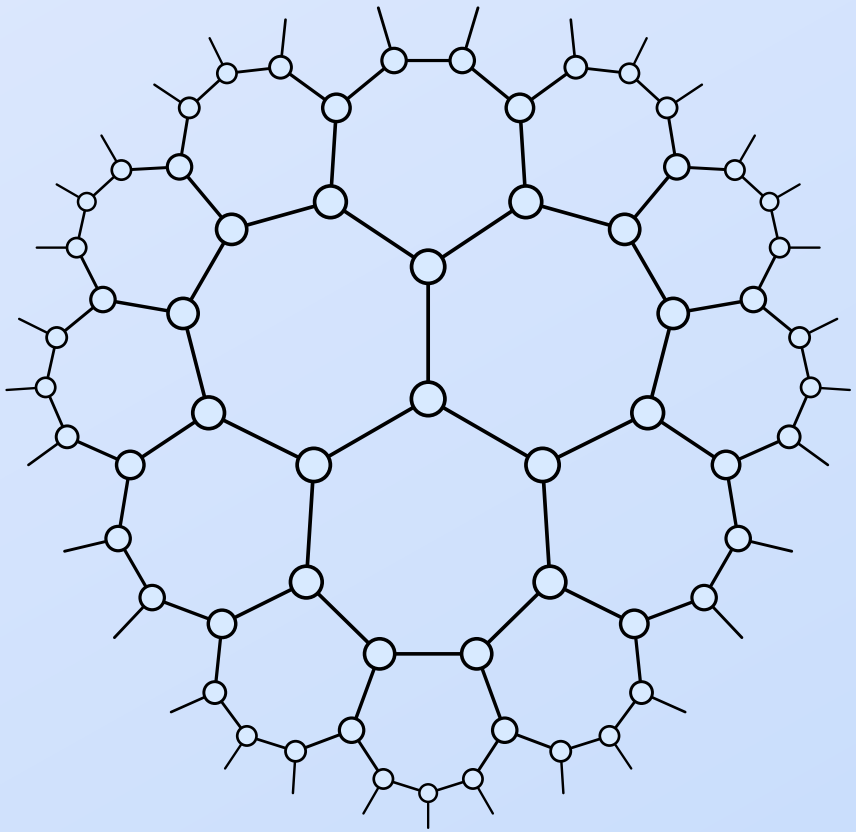
Holographic tensor network states

For $q = 3$, $p = 7$, $L = 2$:



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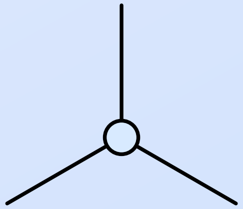
2. Choose a regular hyperbolic $\{p, q\}$ tiling and add layers of tensors through contraction
3. Study scaling properties of boundary state as L increases

Holographic tensor network states

Which tensors to choose?

$T_{i,j,k} =$ 

$$|\psi\rangle = \sum_{i,j,k=0}^{1} T_{i,j,k}\,|i\rangle|j\rangle|k\rangle$$

Our approach: ***Matchgate tensors*** representing free-fermionic ("*Gaussian*") states.

Capable of describing:

A. Code states of *holographic codes*
B. Ground states of *critical* free-fermion Hamiltonians

Explicitly: $|\psi\rangle = N\,\exp\!\left(\frac{1}{2}\sum_{m,n=1}^{3} A_{m,n} f_m^\dagger f_n^\dagger\right)|\emptyset\rangle$

generating matrix

fermionic vacuum

Holographic tensor network states

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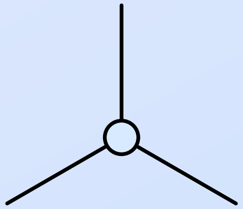
$T_{i,j,k} =$ 

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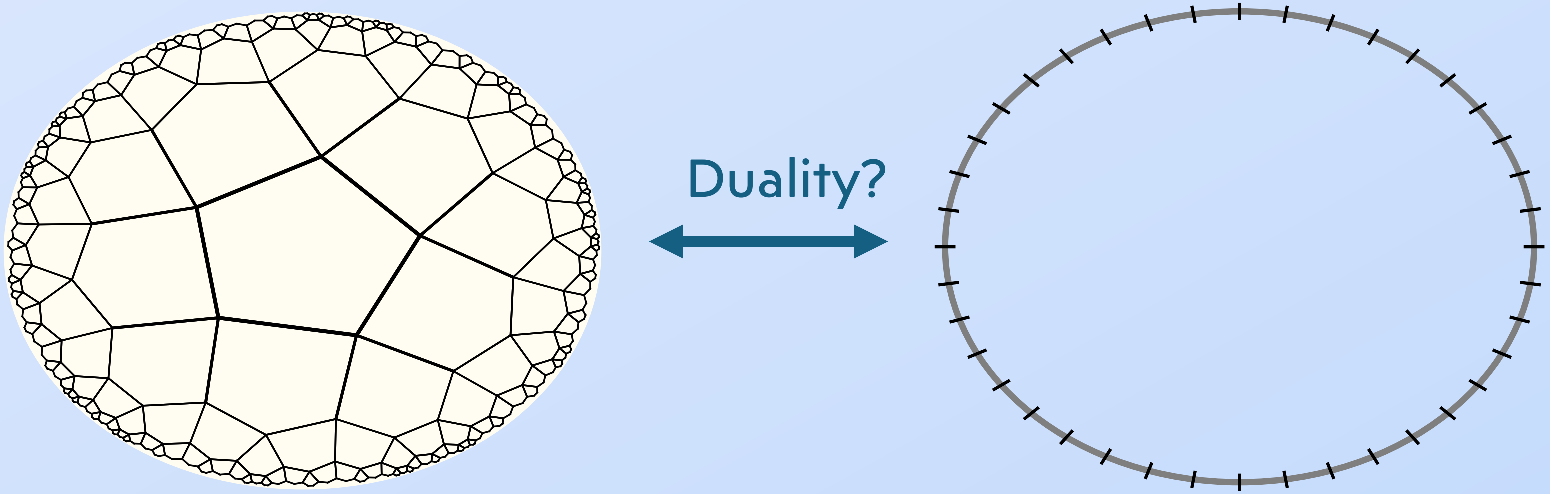
Holographic tensor network states

A. Matchgate tensors → Holographic code states

Time-slice picture of holography:



Hyperbolic bulk

Flat boundary

Holographic tensor network states

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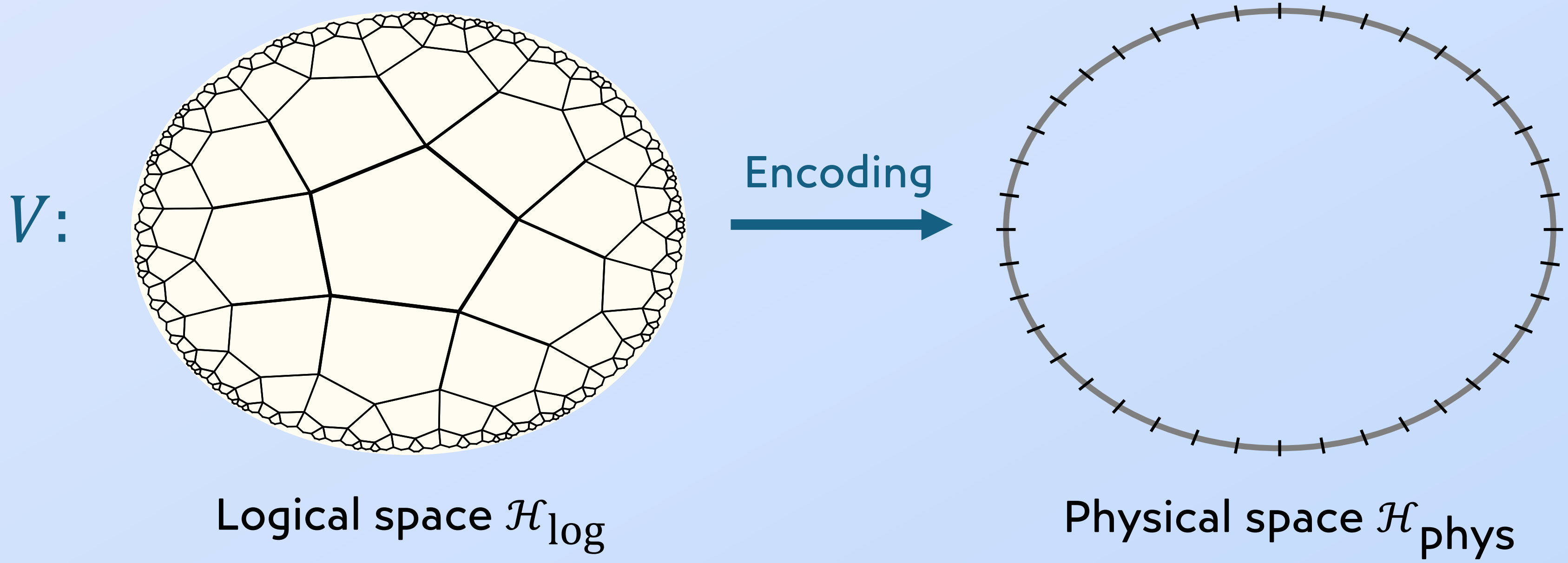
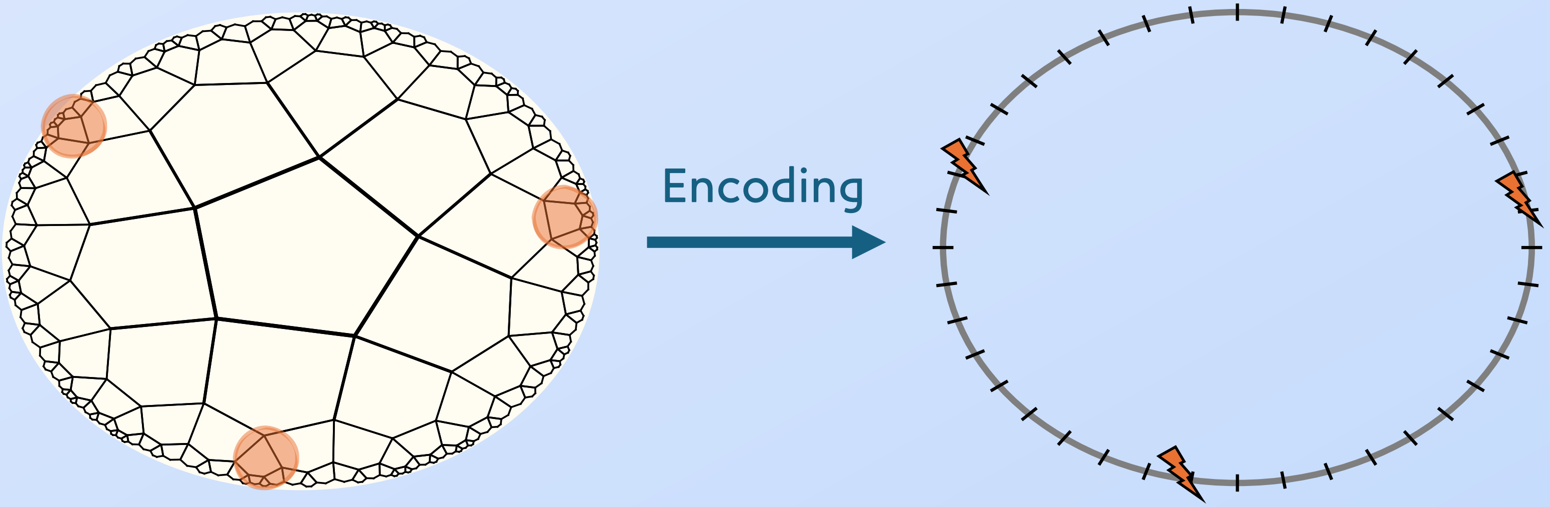


Holographic tensor network states

A. Matchgate tensors → Holographic code states

Time-slice picture of holography:

$V:$



Logical space $\mathcal{H}_{\log}$

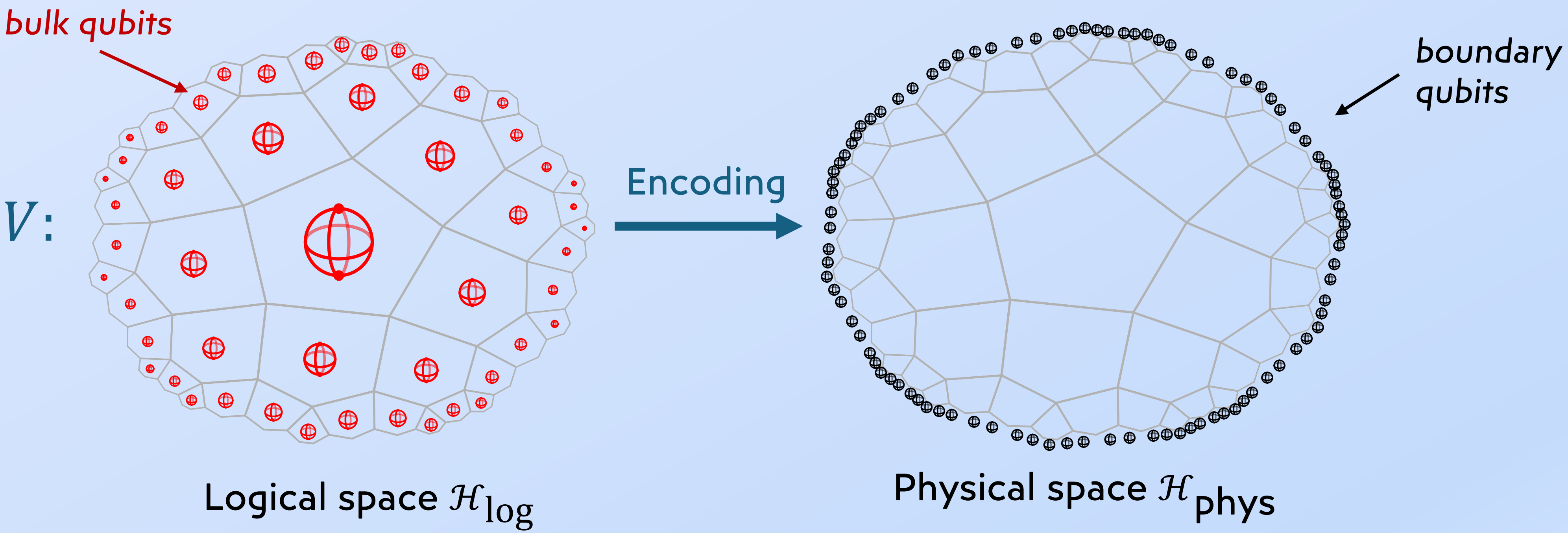
Physical space $\mathcal{H}_{\text{phys}}$

Holographic codes are inherently *error-correcting.* [Almheiri/Dong/Harlow 2014]

Holographic tensor network states

A. Matchgate tensors → Holographic code states

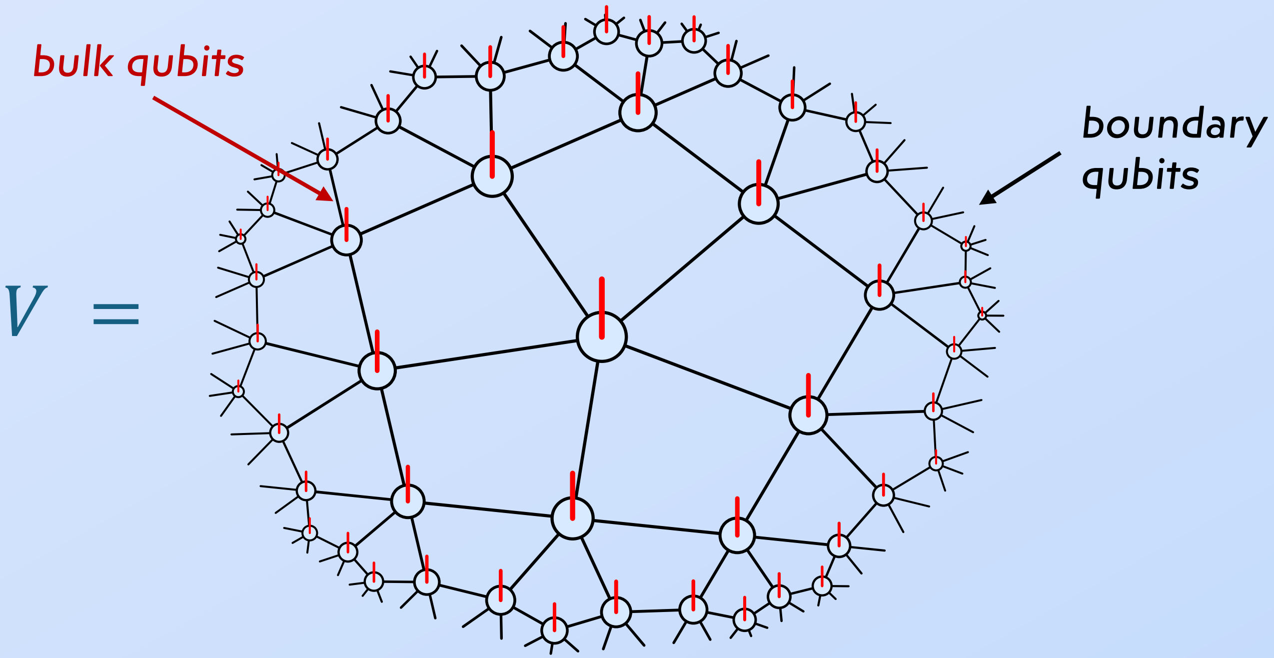
Time-slice picture of holography:



Holographic tensor network states

A. Matchgate tensors → Holographic code states

Reproduced using *tensor network maps*:

$V =$ 

These qubit codes reproduce the same *geometrical recovery features* as continuum holography.

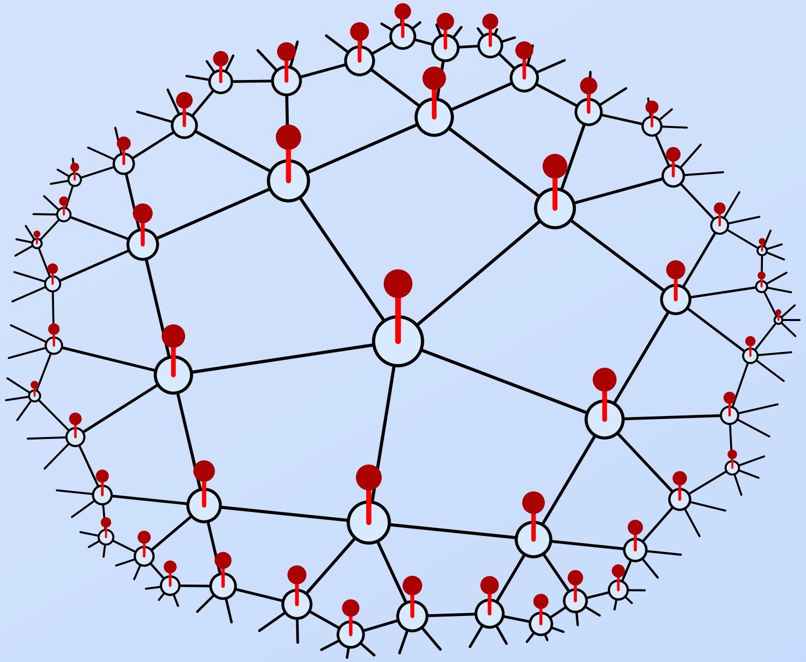
[Pastawski/Yoshida/Harlow/Preskill 2015]

Holographic tensor network states

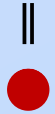
A. Matchgate tensors → Holographic code states

Reproduced using *tensor network maps*:

$$|\psi\rangle_{\mathrm{bdy}} = V|\psi\rangle_{\mathrm{bulk}} =$$



for $|\psi\rangle_{\mathrm{bulk}} = |\phi\rangle^{\otimes n}$



For logical basis states $|\phi\rangle = |0\rangle$ or $|1\rangle$,
the tensor network becomes a matchgate!

[AJ/Gluza/Pastawski/Eisert 2017]

Holographic tensor network states

A. Matchgate tensors → Holographic code states

In fermionic matchgate language, holographic code states have a simple *dimer* structure:

[AJ/Gluza/Pastawski/Eisert 2019]

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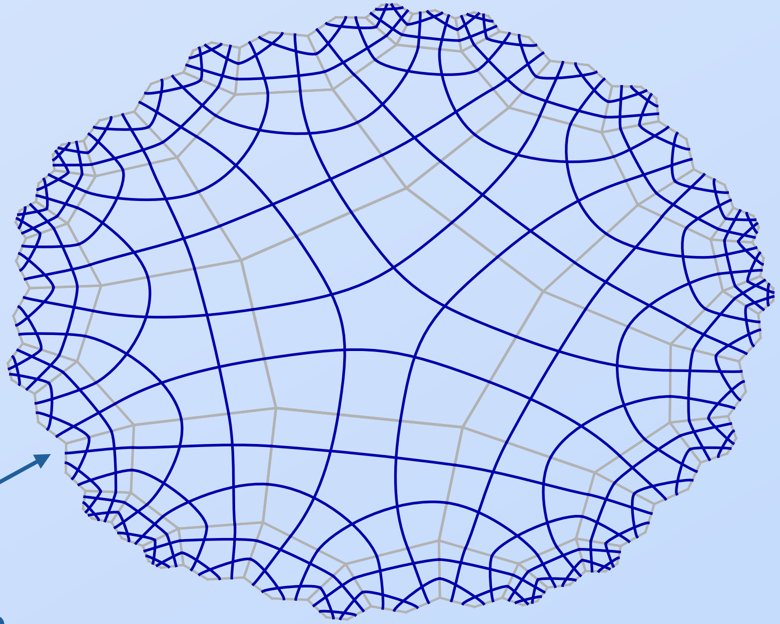

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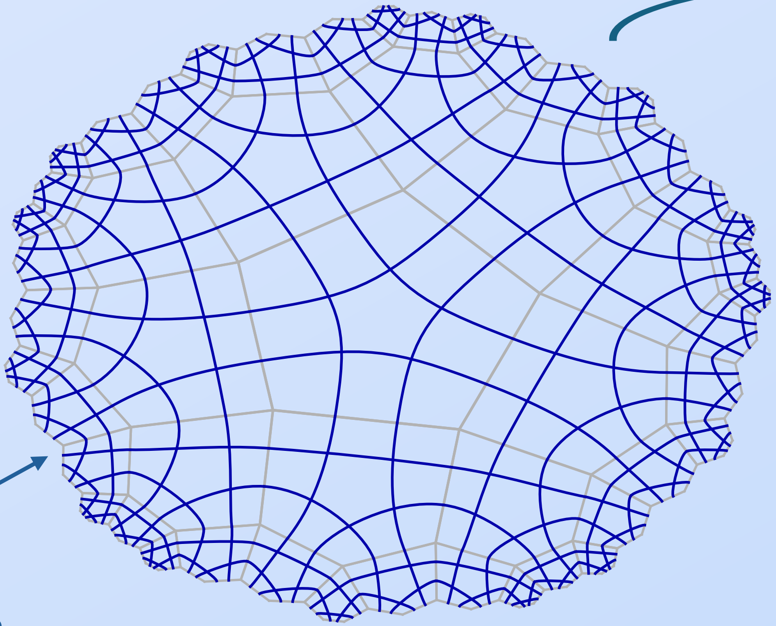
unroll



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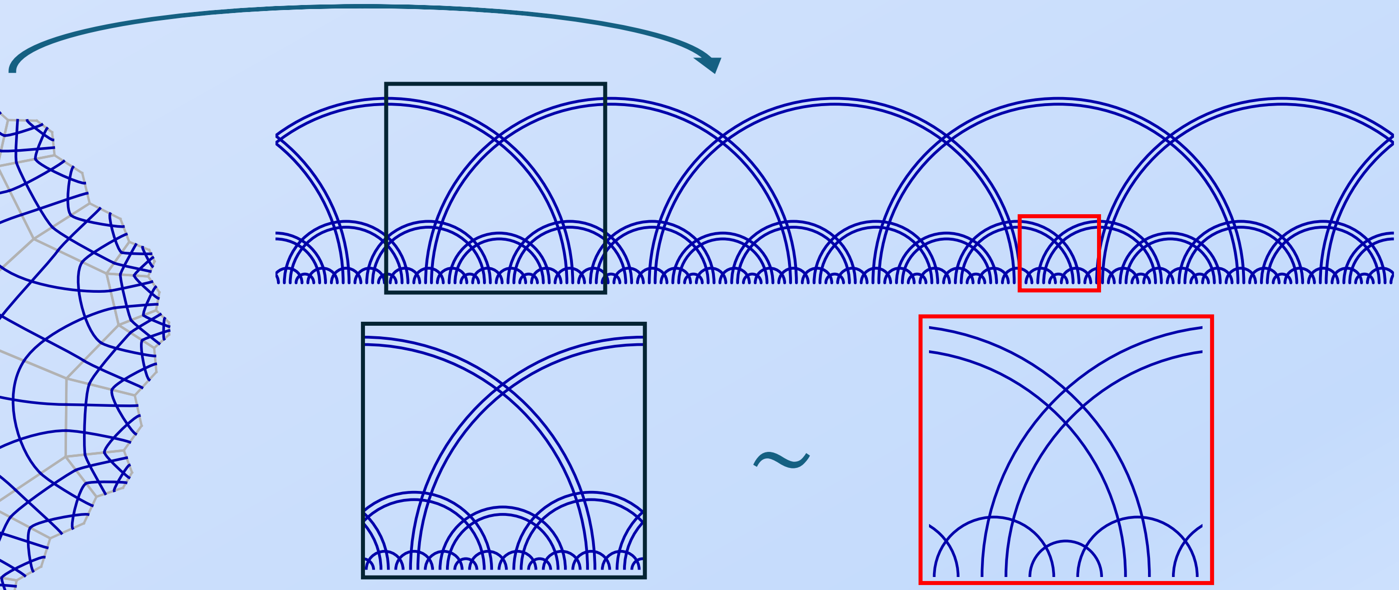


$\mathbb{Z}_5 \times$ quasiperiodic symmetries!

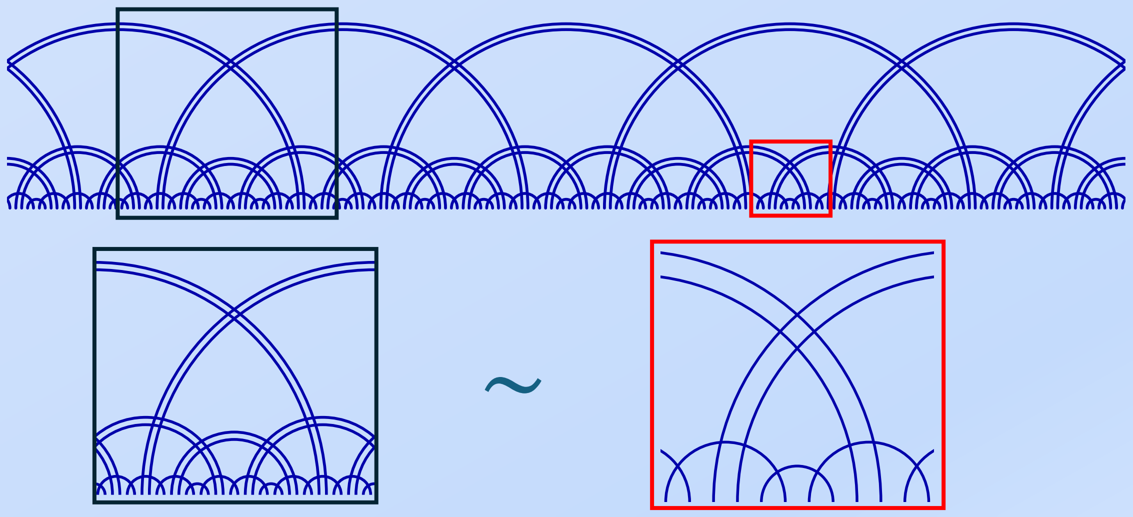
Holographic tensor network states

A. Matchgate tensors → Holographic code states

This quasiperiodicity has interesting consequences:

[AJ/Gluza/Pastawski/Eisert 2019]

1. Analytical dimer state from inflation rules
2. Long-range "critical" (poly.-decaying) correlations
3. Entanglement entropy growth
   $S_A \propto \log|A|$



$\mathbb{Z}_5 \times$ quasiperiodic symmetries!

In a CFT, conformal symmetries enforce correlation function scaling and entanglement.

Here, discrete self-similarity does the same!
→ ***Quasiperiodic CFT* (qCFT)**

[AJ/Zimborás/Eisert 2020]

Holographic tensor network states

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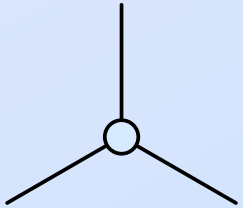
$T_{i,j,k} =$ 

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generating matrix

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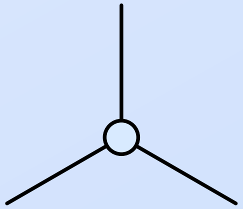
Holographic tensor network states

B. Matchgate tensors → Critical ground states

Given a boundary Hamiltonian H_{bdy}, find $\boldsymbol{A}$ with lowest energy.

Critical Ising model: $H_{\text{Ising}} = -J\sum_k (X_k X_{k+1} + Z_k) \leftrightarrow J\sum_k \left(f_k f_k^{\dagger} - f_k^{\dagger} f_k + (f_k^{\dagger} - f_k)(f_{k+1}^{\dagger} + f_{k+1})\right)$



$$= T_{i,j,k} \qquad\qquad |\psi\rangle = \sum_{i,j,k=0}^{1} T_{i,j,k}\, |i\rangle |j\rangle |k\rangle$$

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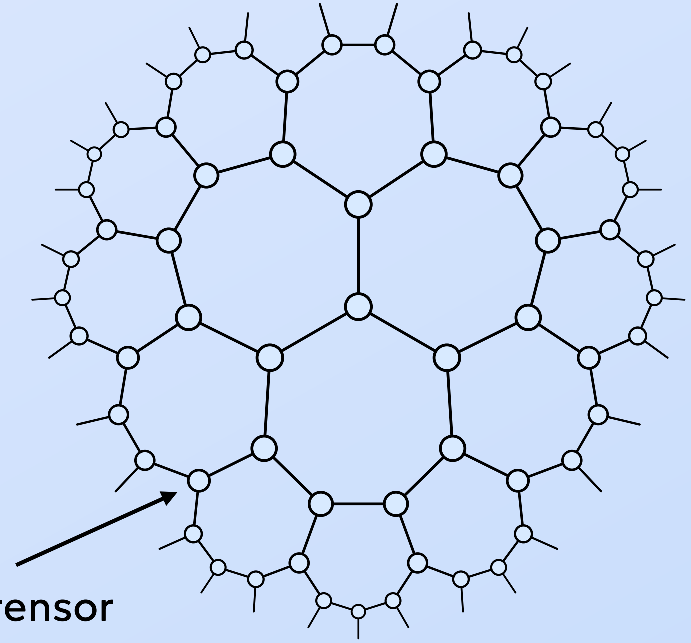
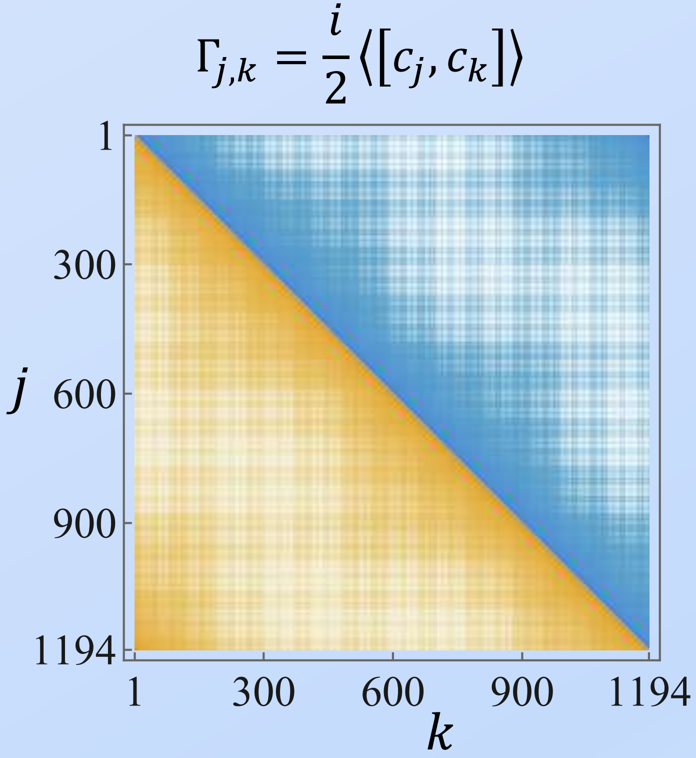
Critical Ising model: $H_{\text{Ising}} = -J\sum_k (X_k X_{k+1} + Z_k) \leftrightarrow J\sum_k \left(f_k f_k^\dagger - f_k^\dagger f_k + (f_k^\dagger - f_k)(f_{k+1}^\dagger + f_{k+1})\right)$





Holographic tensor network states

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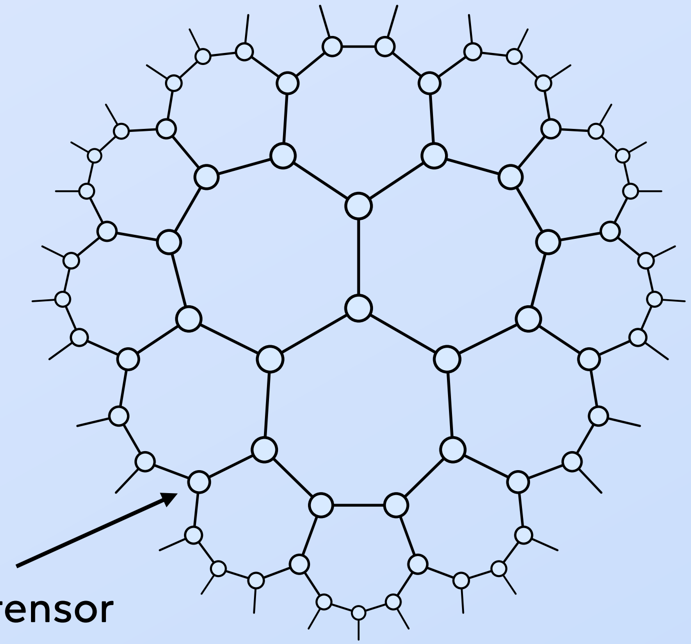
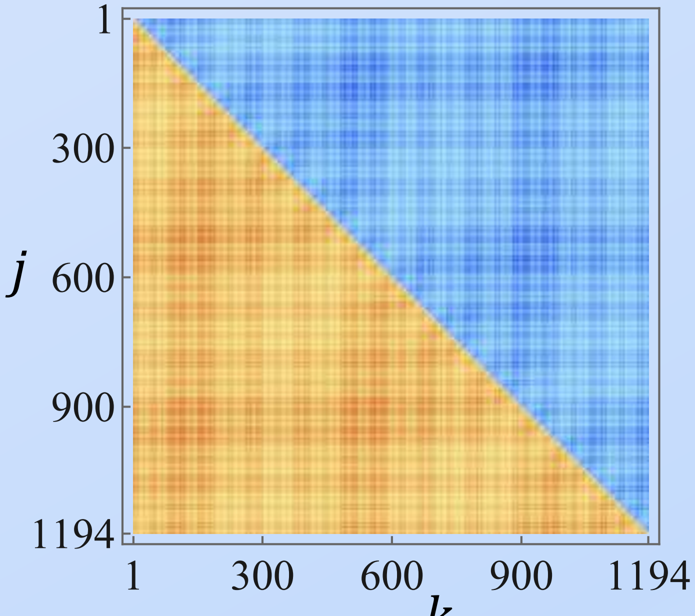


each tensor
fixed by same $\boldsymbol{A}$

$$\Gamma_{j,k} \cdot |\mathrm{j}-\mathrm{k}| = \frac{i}{2}\langle [c_j, c_k] \rangle \cdot |\mathrm{j}-\mathrm{k}|$$



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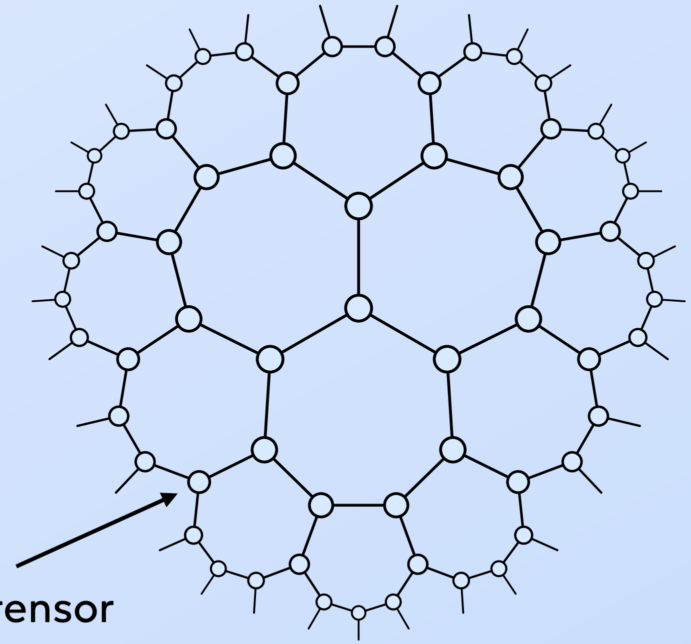

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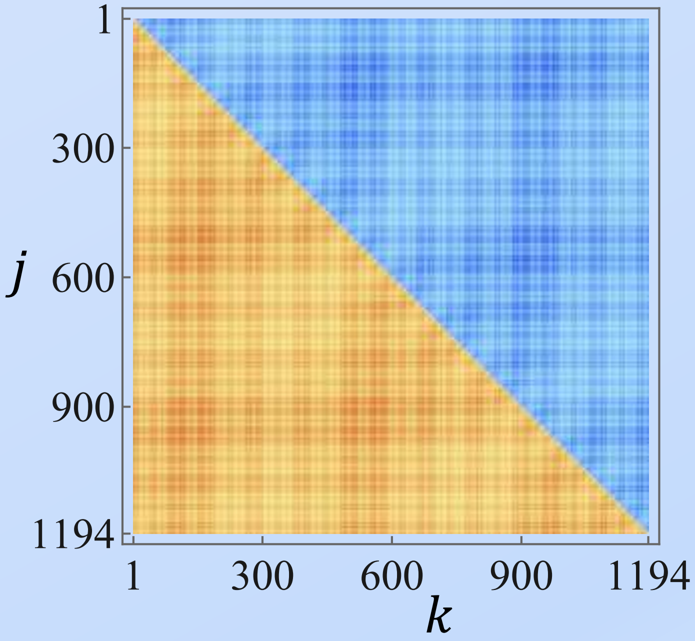
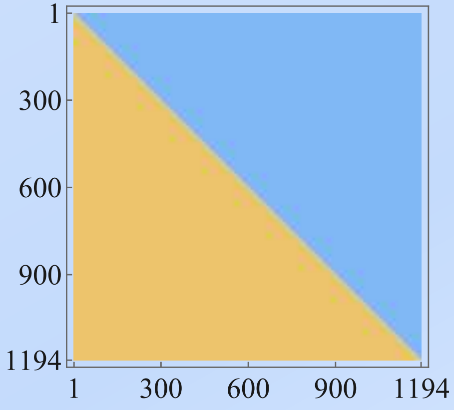
$$\Gamma_{j,k} \cdot |\mathrm{j}-\mathrm{k}| = \frac{i}{2}\langle [c_j, c_k] \rangle \cdot |\mathrm{j}-\mathrm{k}| \approx g_j g_k \underbrace{\Gamma_{j,k}^{\mathrm{Ising}} \cdot |\mathrm{j}-\mathrm{k}|}$$





Holographic tensor network states

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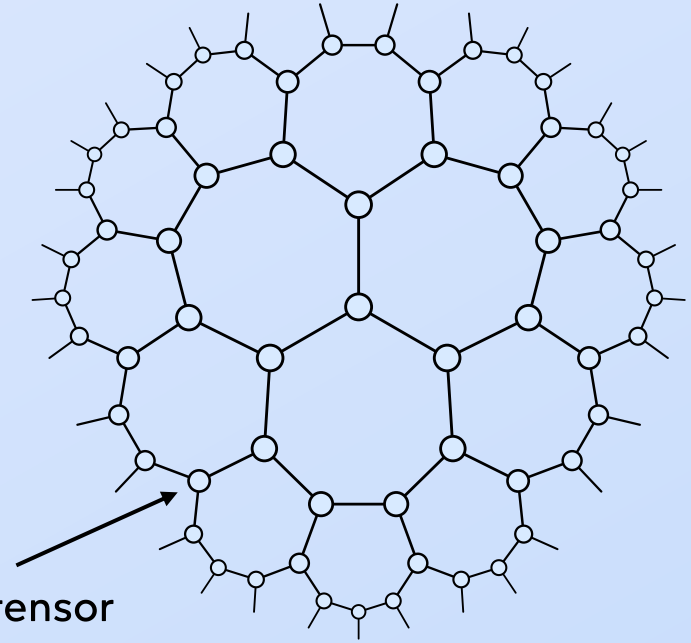

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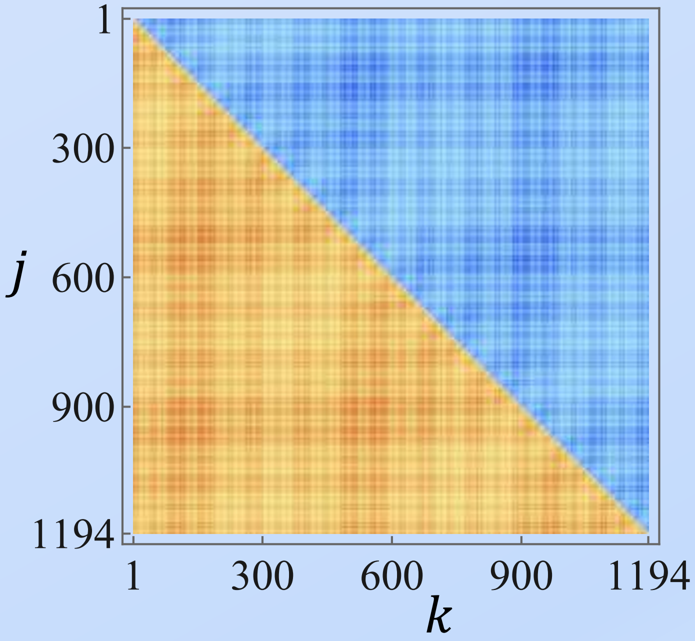

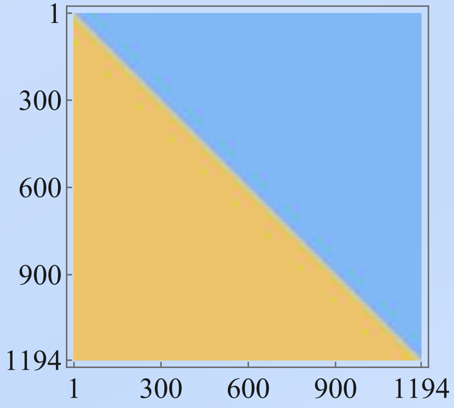

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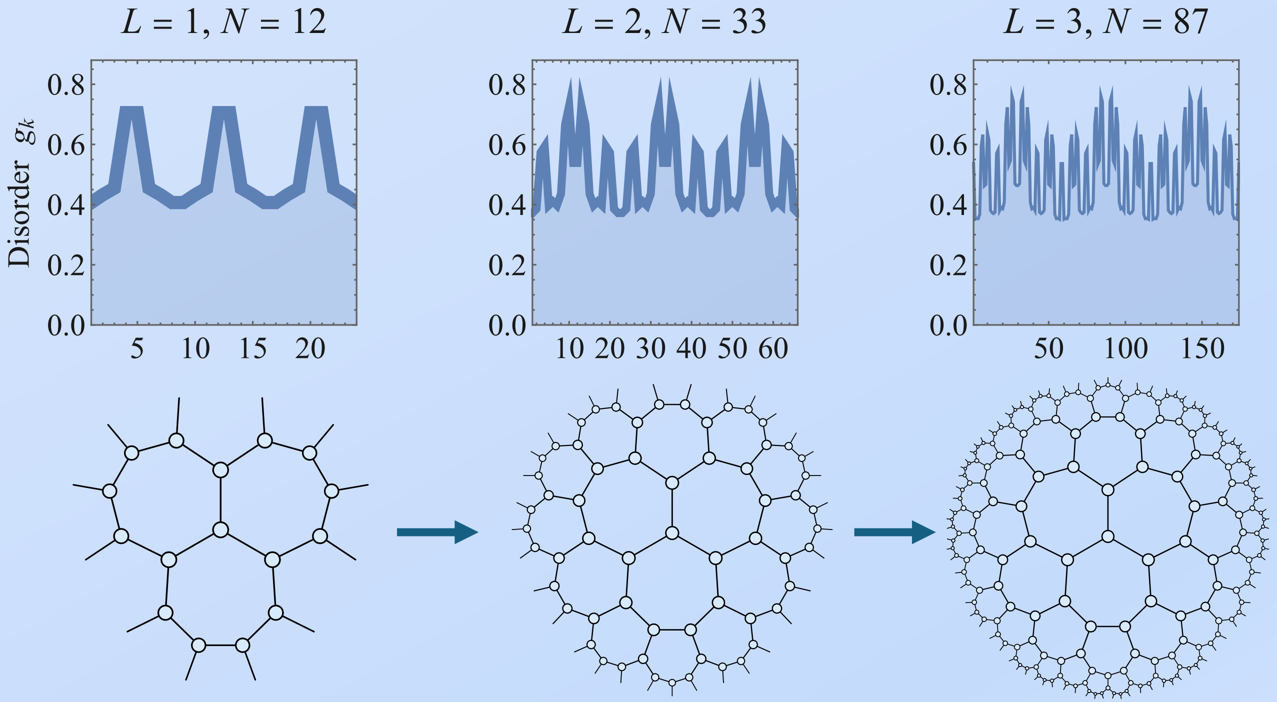
Symmetries:

$\mathbb{Z}_3$ × quasiperiodic



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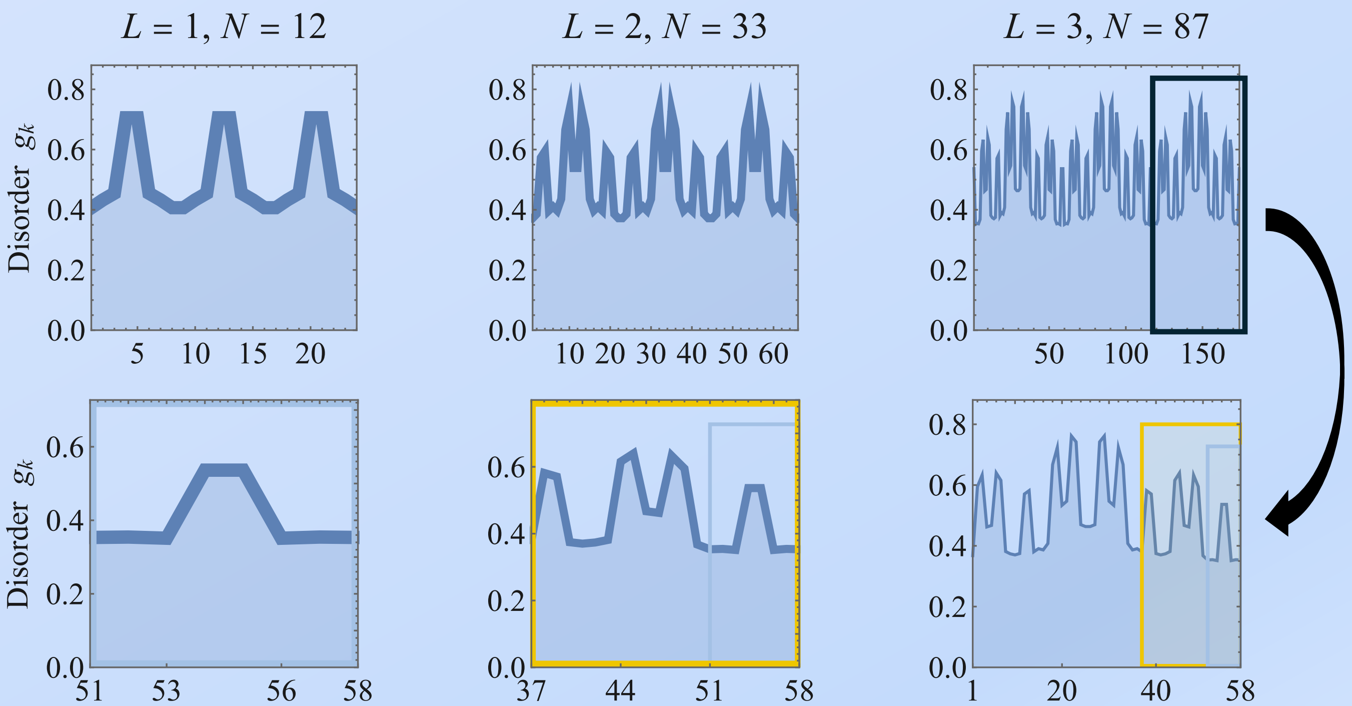
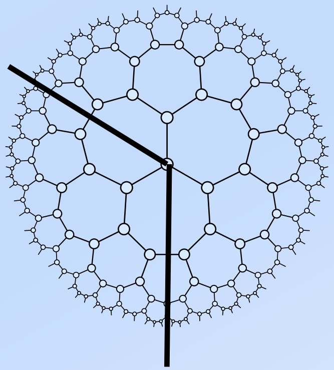
Fractal self-similarity:

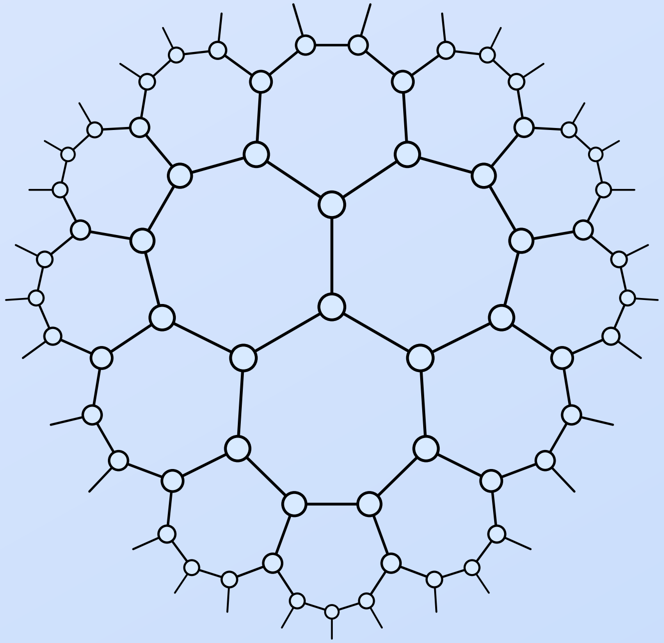




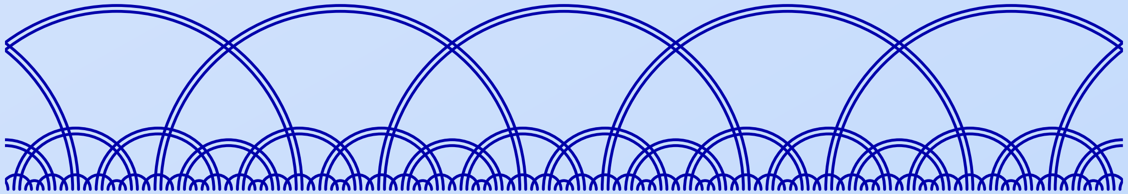
Holographic tensor network states

Matchgate summary:

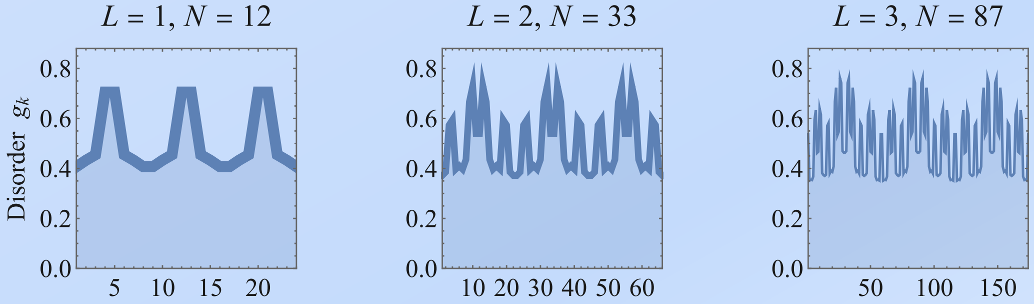
Tensor networks on regular hyperbolic geometries produce boundary states with *quasiperiodic symmetries* enabling *quantum-critical behavior*!



A. Holographic code states



B. Critical ground states



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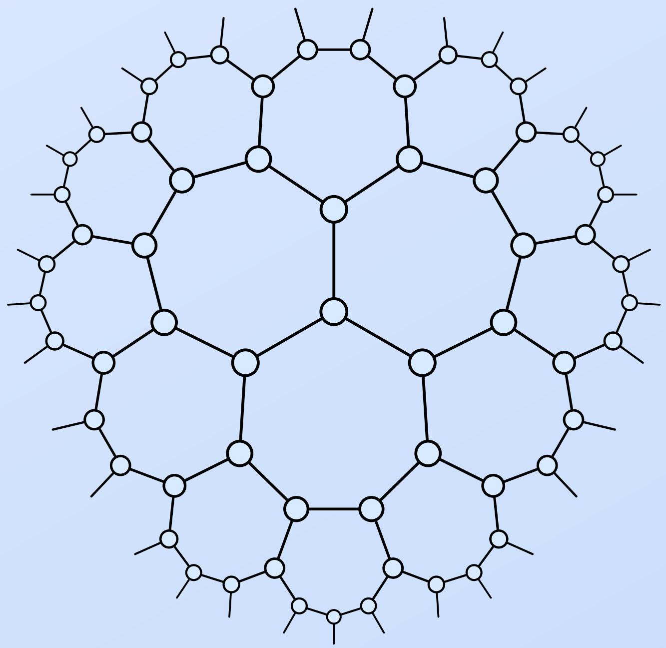

Lessons for ***discrete holography***:

- Bulk *regularity* $\leftrightarrow$
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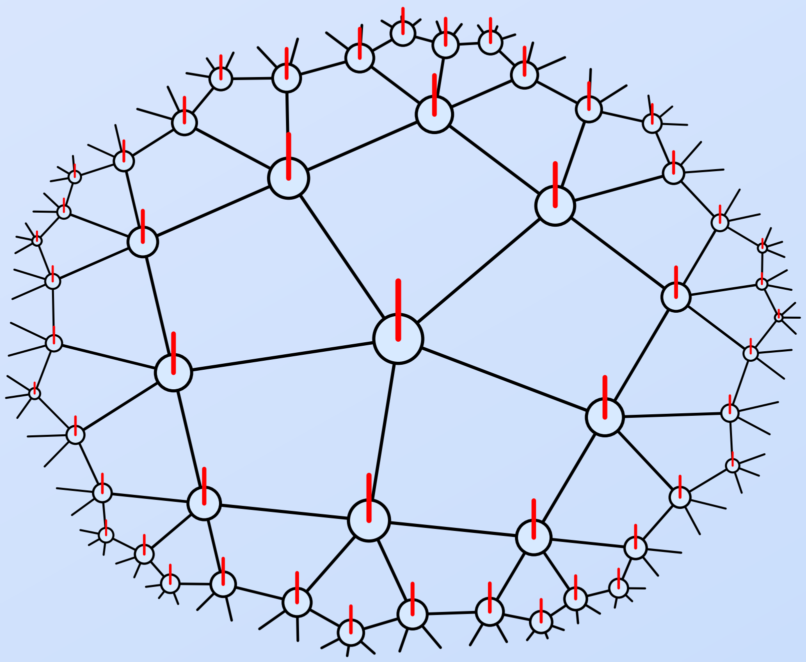

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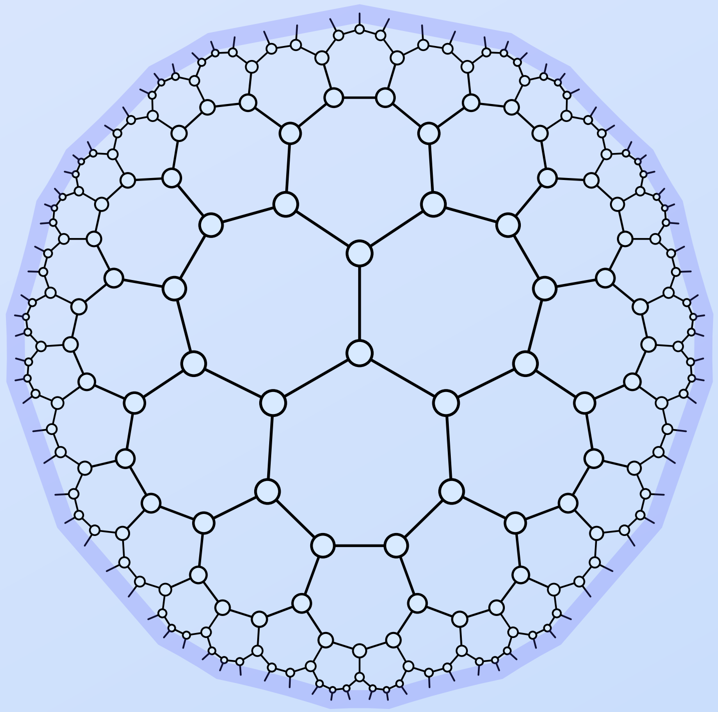

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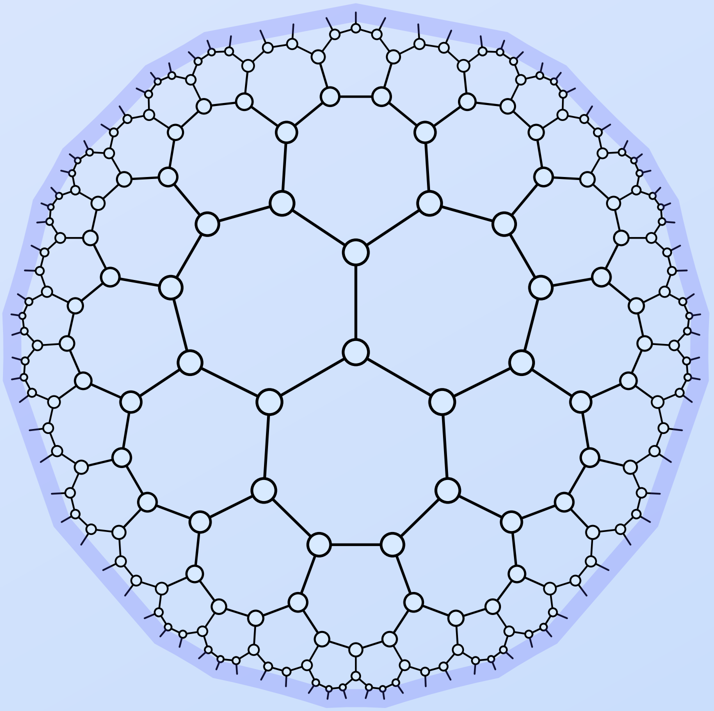

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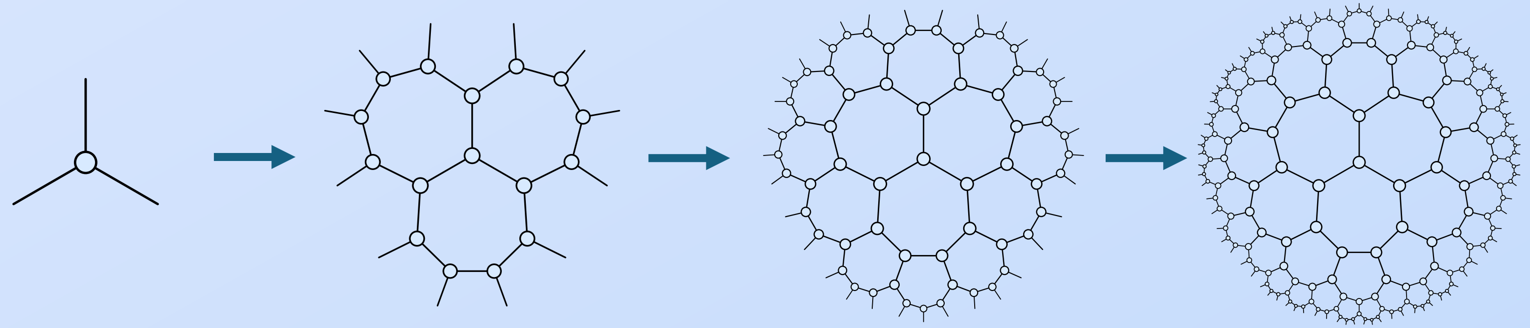
Define *independently* from tensor networks?

Holographic disorder

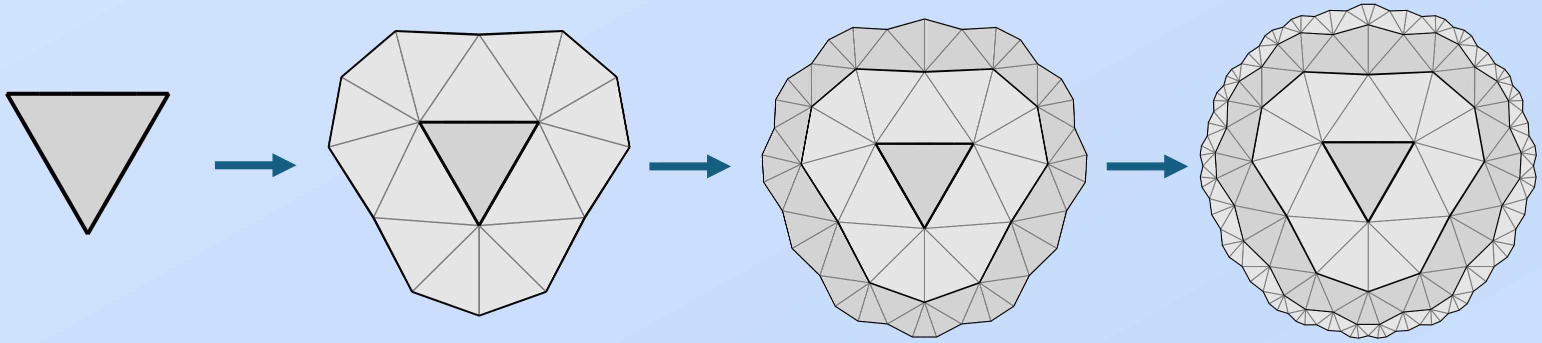
<u>So far:</u> Tensor network ansatz → ground state of disordered H

<u>Now:</u> Quasicrystal symmetries → critical H ?

Quasiperiodicity appearing at *every layer*:



Dual:



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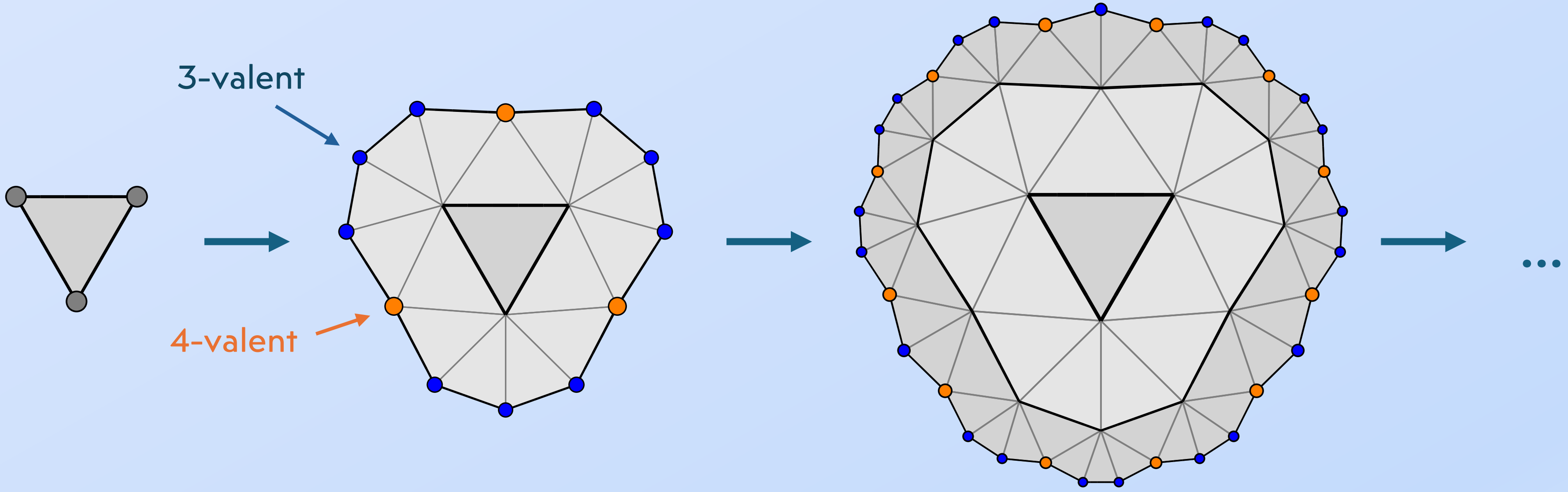
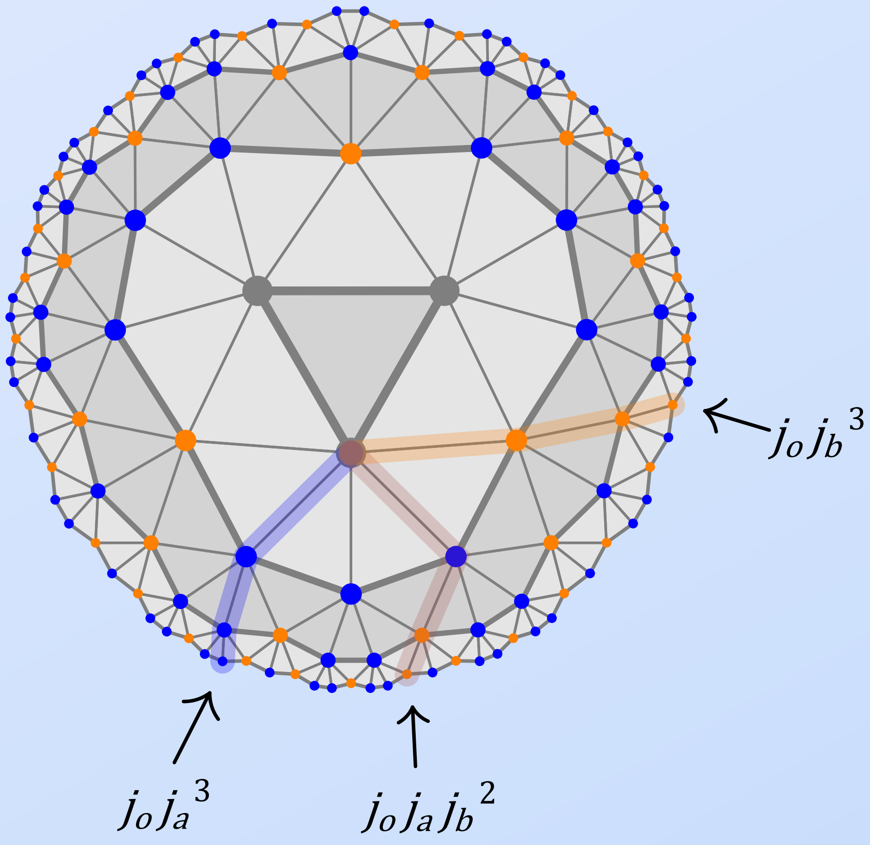


ooo → aaabaaabaaab → aabaabaabaaab... (× 3) → ...

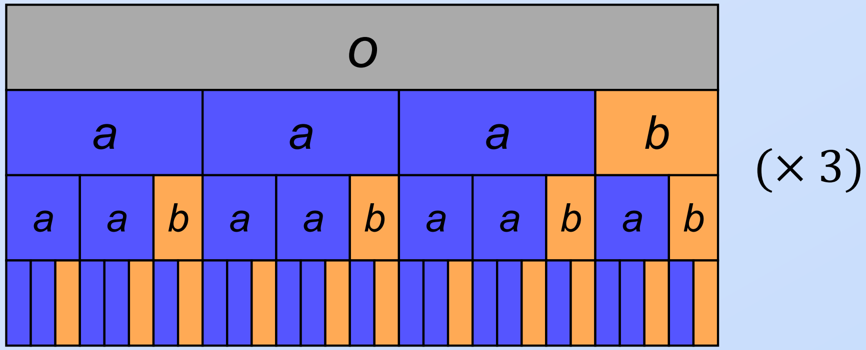
Inflation rule: a→aaab, b→aab

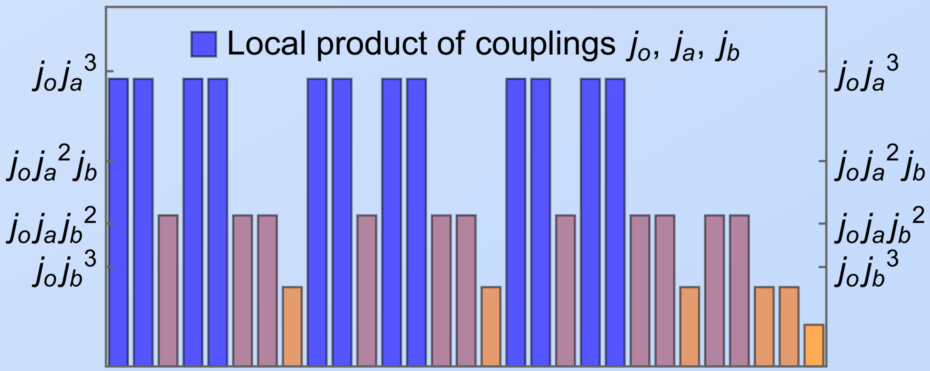
Holographic disorder

Local boundary symmetry = Overlap of each layer's quasiperiodicity



Each vertex = 1 coupling j_i





Multiscale quasicrystal ansatz (MQA)

[AJ/Gluza/Verhoeven/Singh/Eisert 2020]

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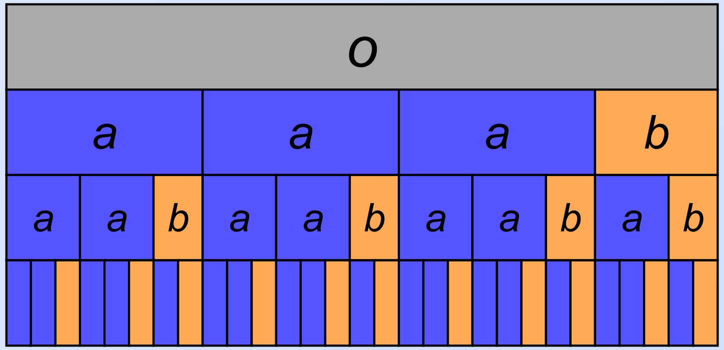
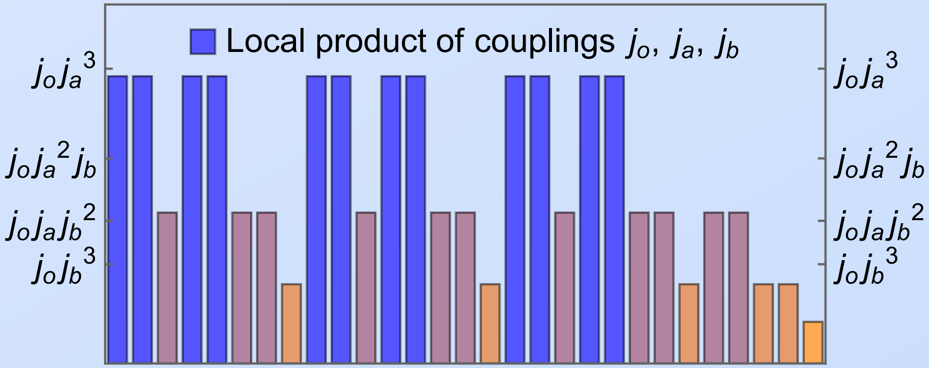


$(\times 3)$



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Pipeline:

1. Quasiperiodic inflation rule: a→aaab, b→aab
2. Assign each letter a coupling: j_a, j_b
3. Construct physical couplings J_k from MQA

$$\rightarrow H = \sum_k J_k \, (\ldots)$$

Q: If H is critical for $J_k = \mathrm{const}$, does the MQA with quasiperiodic J_k preserve criticality?

A: Depends on the inflation rule!

Holographic disorder

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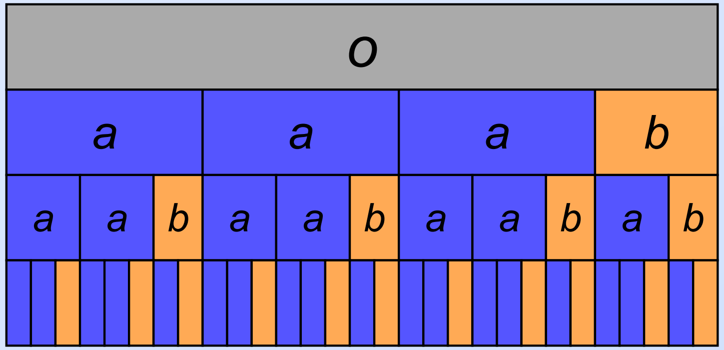

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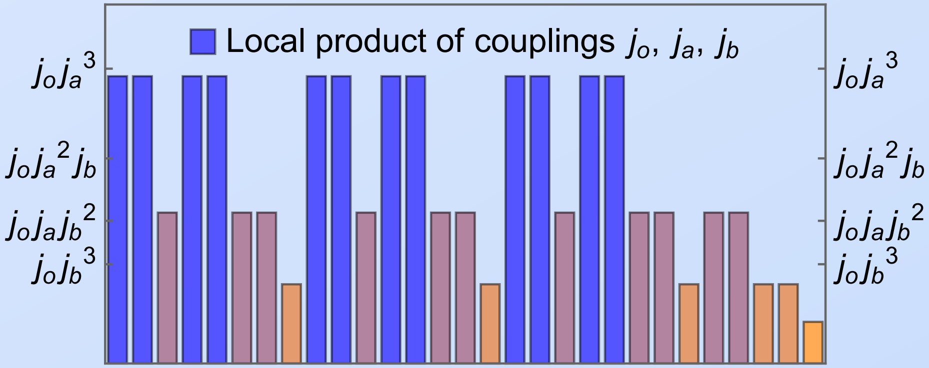

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Test with *critical XXZ model.*

Holographic disorder

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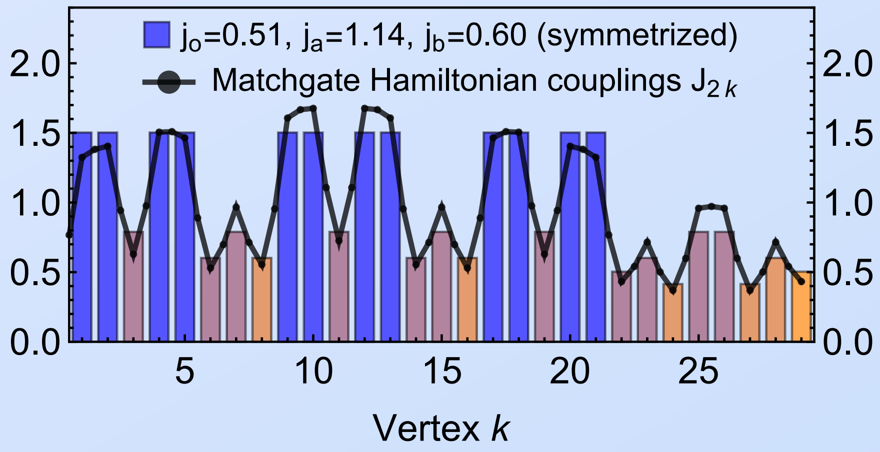
Example 1: {3,7} tiling



MQA reproduces matchgate result.

Pipeline:

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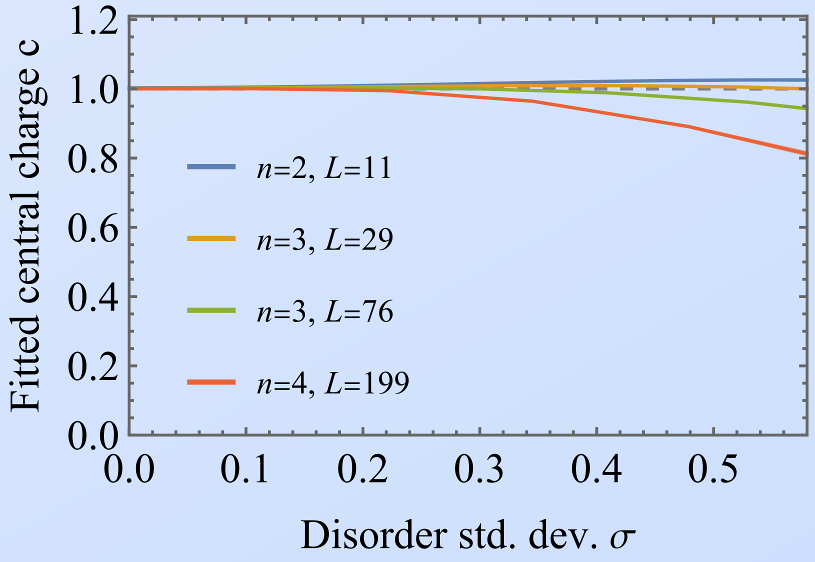

Critical behavior **stable** until very large disorder.

[Saraidaris/AJ 2024]

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Holographic disorder

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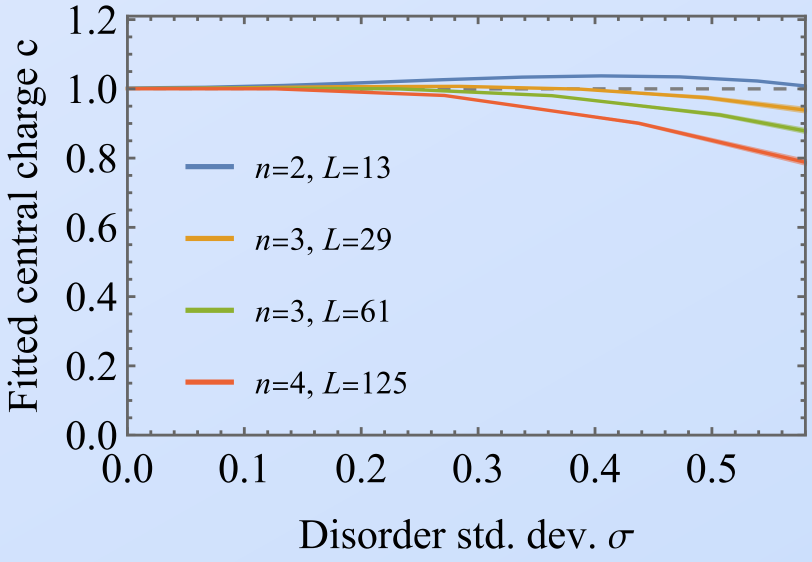
Example 1: {4,5} tiling



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Holographic disorder

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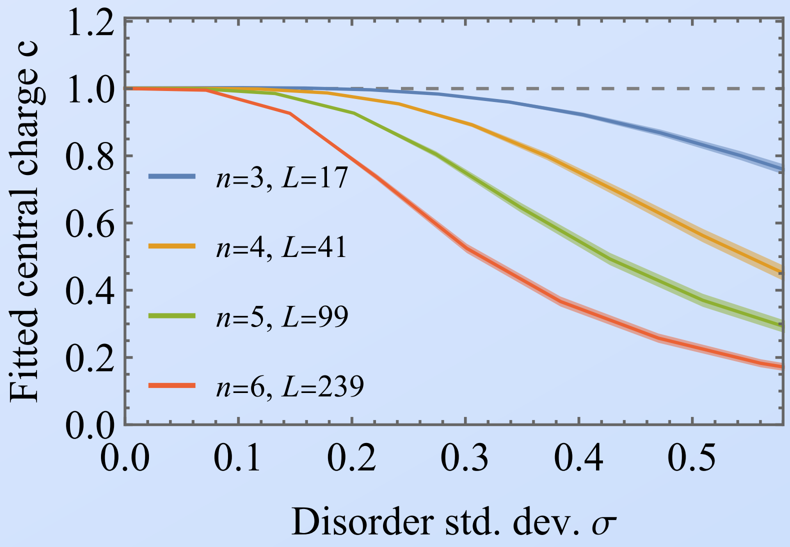
Example 1: Silver-mean sequence



Critical behavior **unstable** even at small disorder.

[Saraidaris/AJ 2024]

Pipeline:

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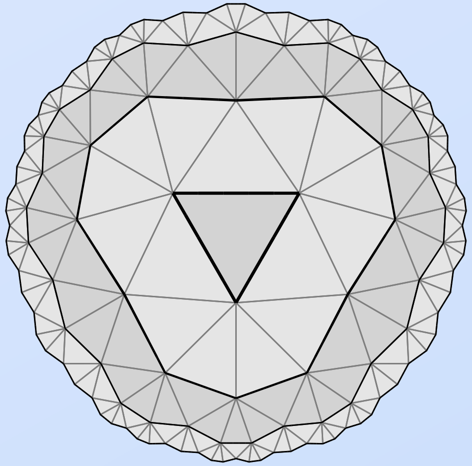
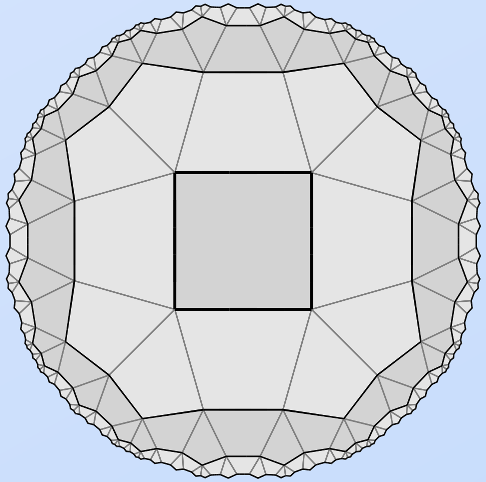
Holographic disorder

Our conclusion: Multi-scale quasiperiodicity does not *by itself* ensure quantum criticality.

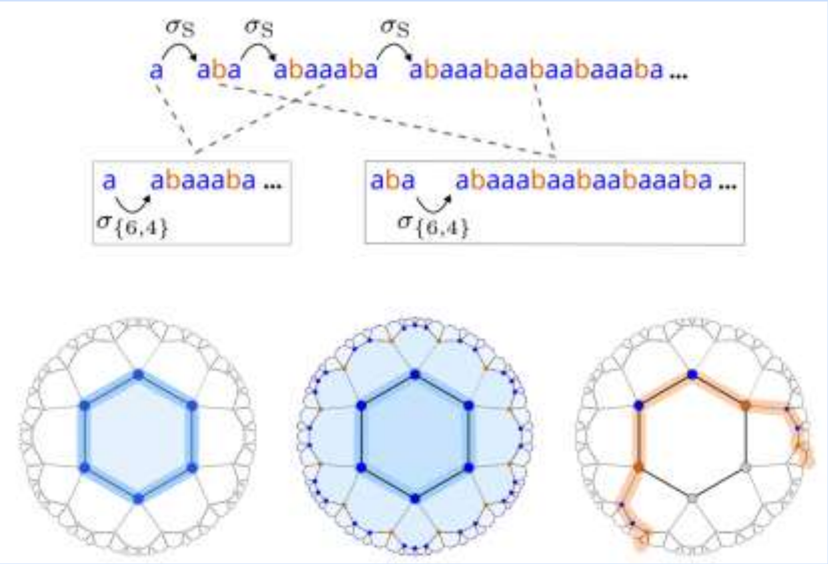
It only works when it encodes the symmetries of a “dual” bulk geometry! [Saraidaris/AJ 2024]



a→aaab, b→aab



a→ababa, b→aba

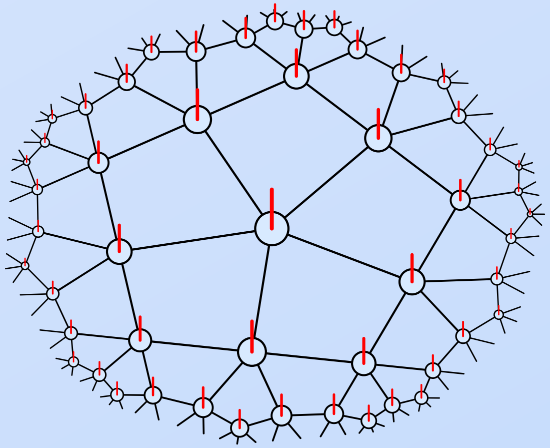


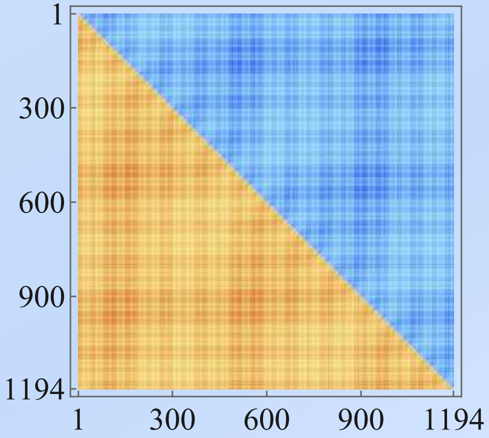
a→aba, b→a

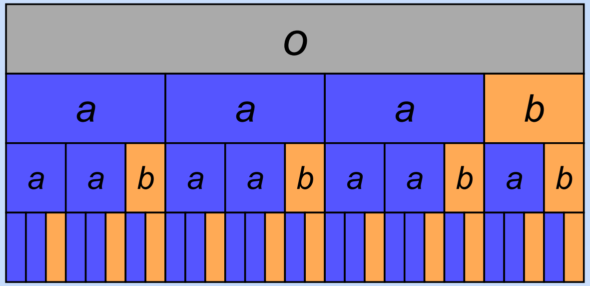
Summary

We explored the symmetries of *discrete-holographic models.*

- Tensor networks as discrete bulk/boundary maps
- Bulk regularity ⇒ boundary quasiperiodicity
- Hyperbolic tensor network states: Critical correlations + entanglement
- Boundary theories defined with *multi-scale quasiperiodic* disorder
- Beyond tensor networks: New types of disordered critical many-body phases







Discussion

What have we learned?

- Formulating holographic dualities beyond the continuum will require understanding discrete symmetries
- Tensor networks can be both a useful *numerical* ansatz and a *conceptual* tool to understand novel quantum phases

Future work:

- *Dynamical* formulation of discrete holography?
- Understanding continuum limits and renormalization techniques
- Realizing quasiperiodic phases on quantum simulators

Thank you for your attention!

This talk was based on:

- AJ, M. Gluza, F. Pastawski, J. Eisert, *“Holography and criticality in matchgate tensor networks,”* arXiv:1711.03109
- AJ, M. Gluza, F. Pastawski, J. Eisert, *“Majorana dimers and holographic quantum error-correcting codes,”* arXiv:1905.03268
- AJ, Z. Zimborás, J. Eisert, *“Tensor network models of AdS/qCFT,”* arXiv:2004.04173
- AJ, M. Gluza, C. Verhoeven, S. Singh, J. Eisert, *“Boundary theories of critical matchgate tensor networks,”* arXiv:2110.02972
- D. Saraidaris, AJ, *“Critical spin models from holographic disorder,”* arXiv:2409.17235

E-Mail: ajahn.quantum@gmail.com

I’m hiring two postdocs at APCTP this Fall!