# Aperiodic quantum chains for discrete holography on hyperbolic tilings

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Oskar Klein Center, Stockholm University

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# Main motivation: holographic dualities

Quantum gravity theory in a (d+1)-dimensional spacetime $\mathcal{M}_{d+1}$

$\Longleftrightarrow$

Quantum theory without gravity on $\partial\mathcal{M}_{d+1}$

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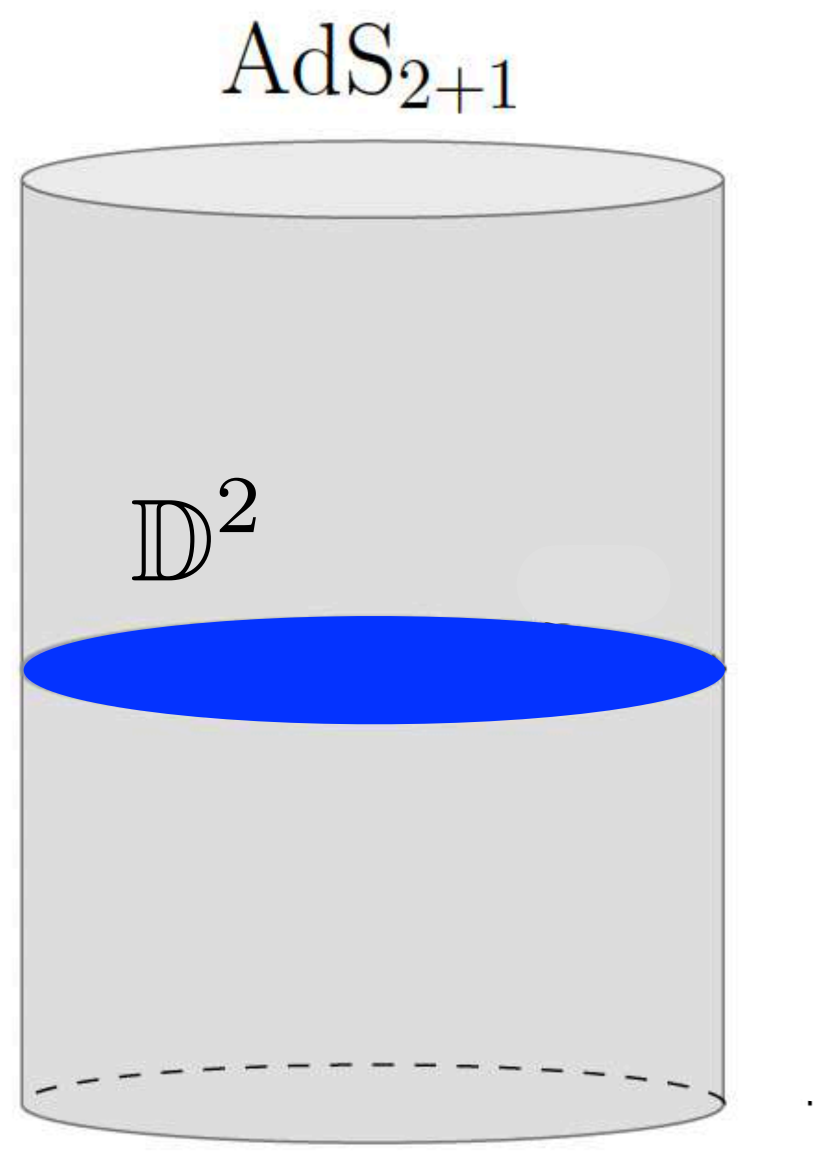
Proposals in the literature: p-adic AdS/CFT [Gubser, Knaute, Parikh, Samberg, Witaszczyk '16; Heydeman, Marcolli, Saberi, Stoica '16], modular discretization of two-dimensional AdS [Axenides, Floratos, Nicolis '14 and '19]

# Regular hyperbolic tilings of the Poincaré disk

Consider $\mathrm{AdS}_{2+1}$ and fix a constant time slice: Poincaré disk

The discrete bulk is obtained by discretizing the Poincaré disk as follows:



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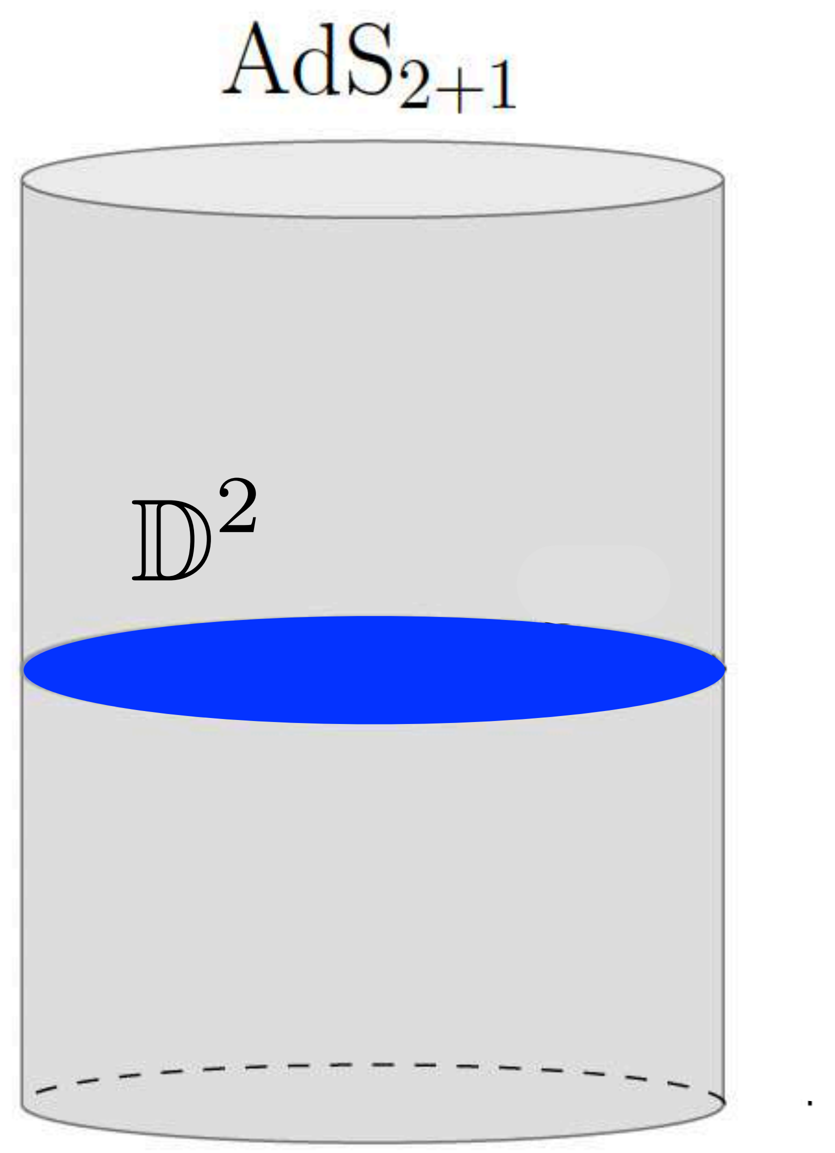
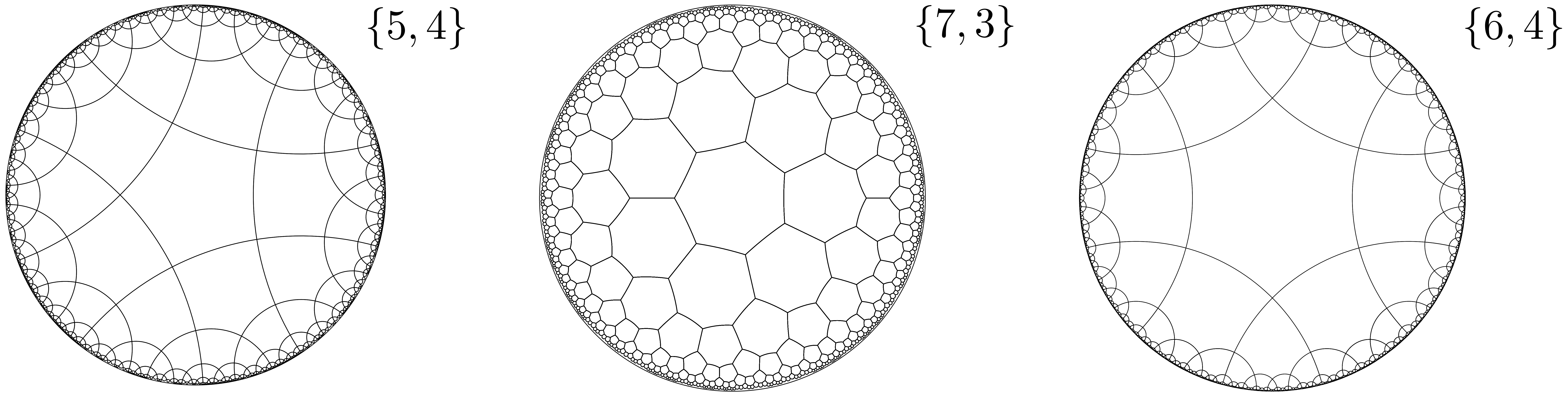



**Regular hyperbolic tilings** $\{p,q\}$: $q$ regular $p$-gons meet at each vertex



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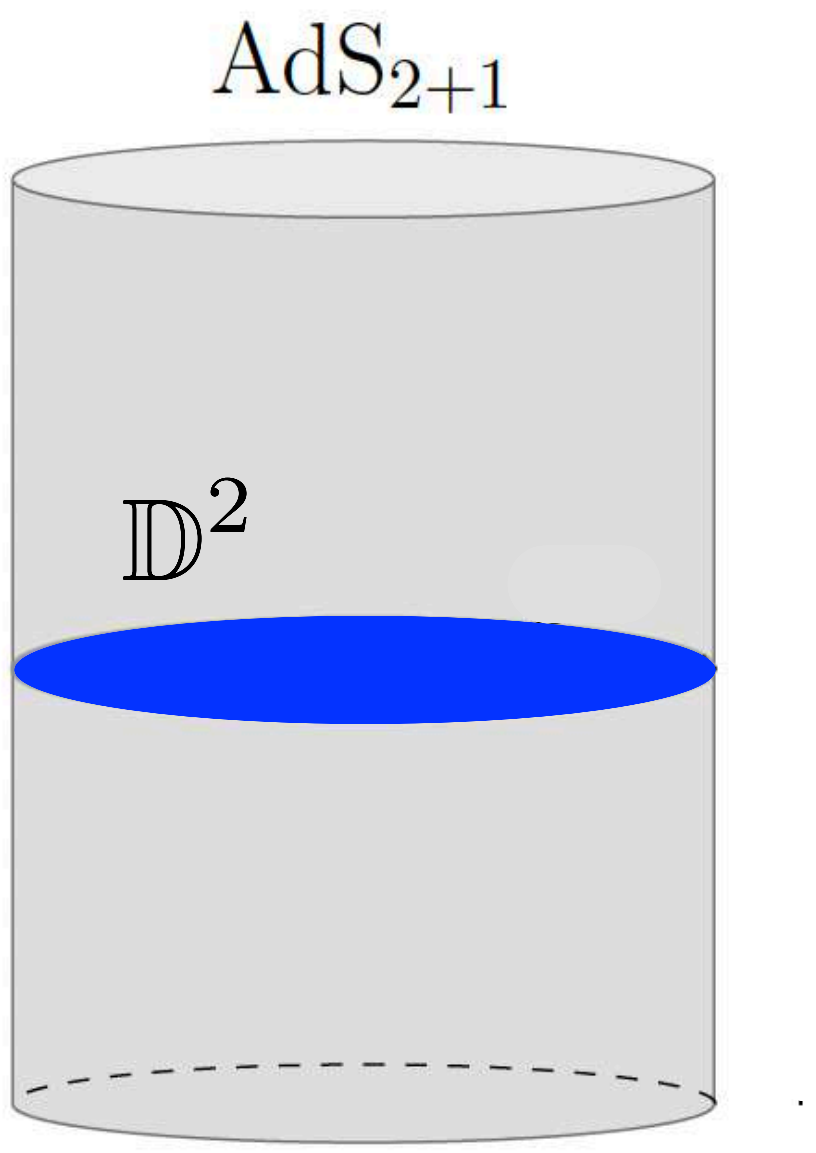
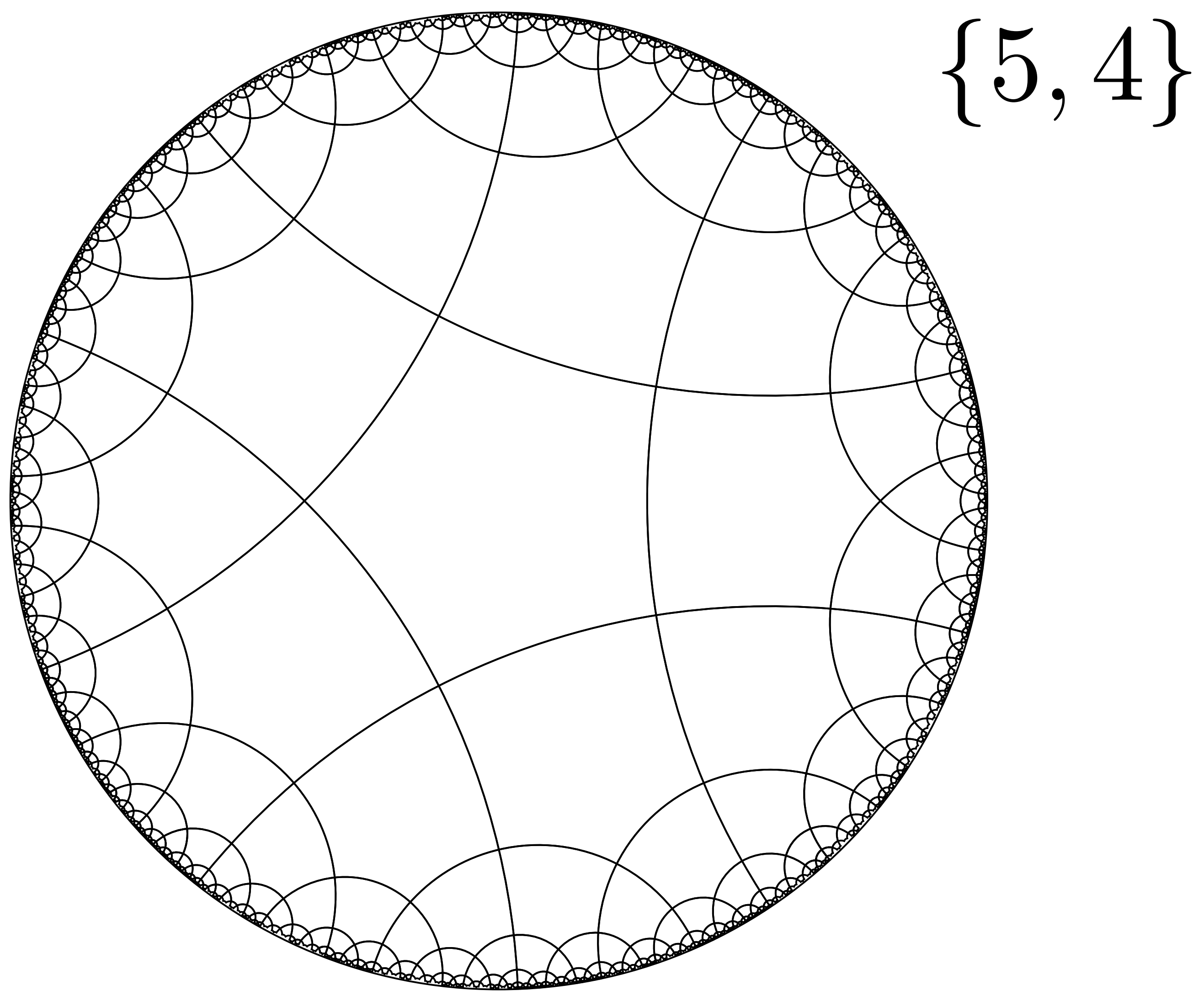
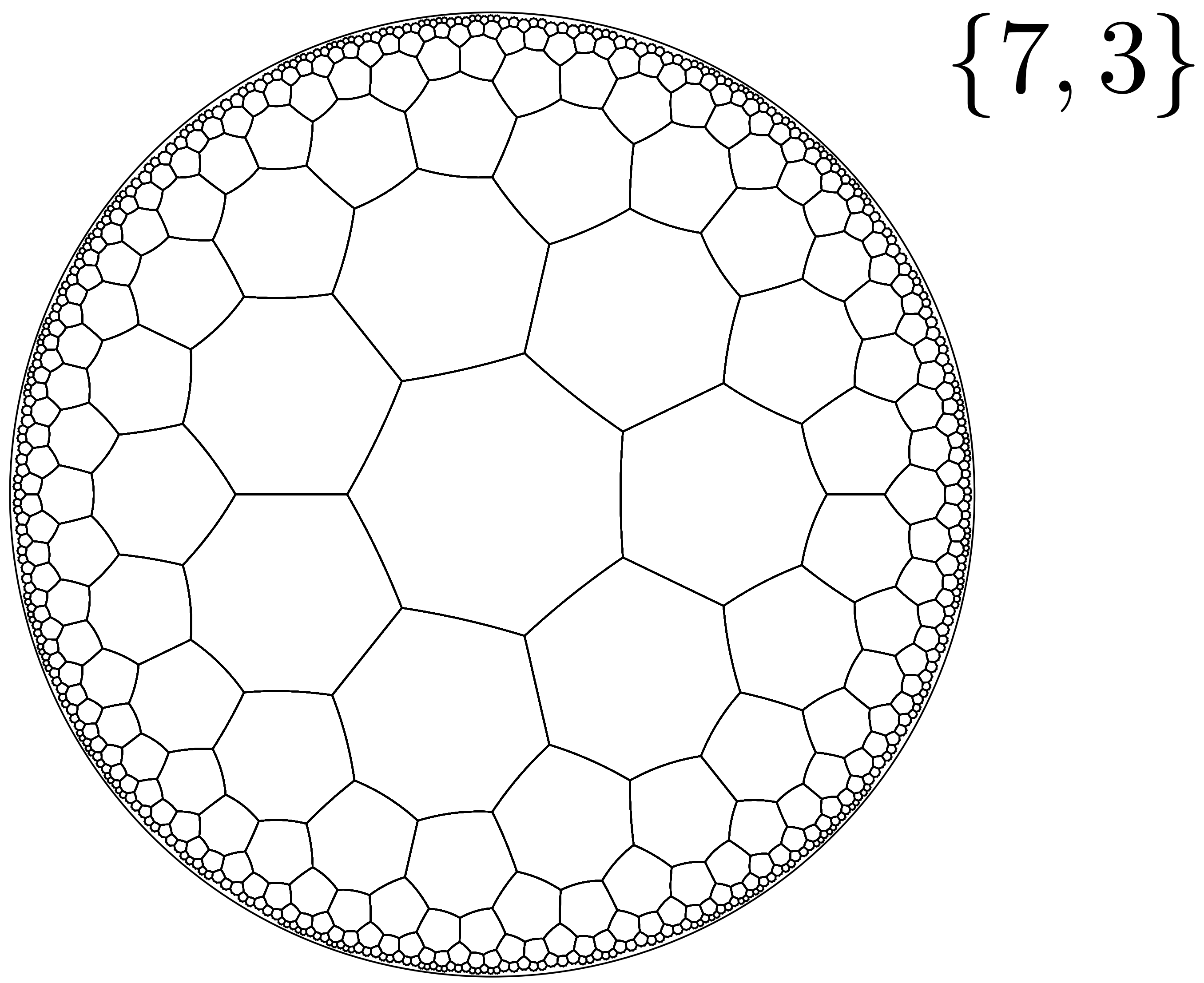
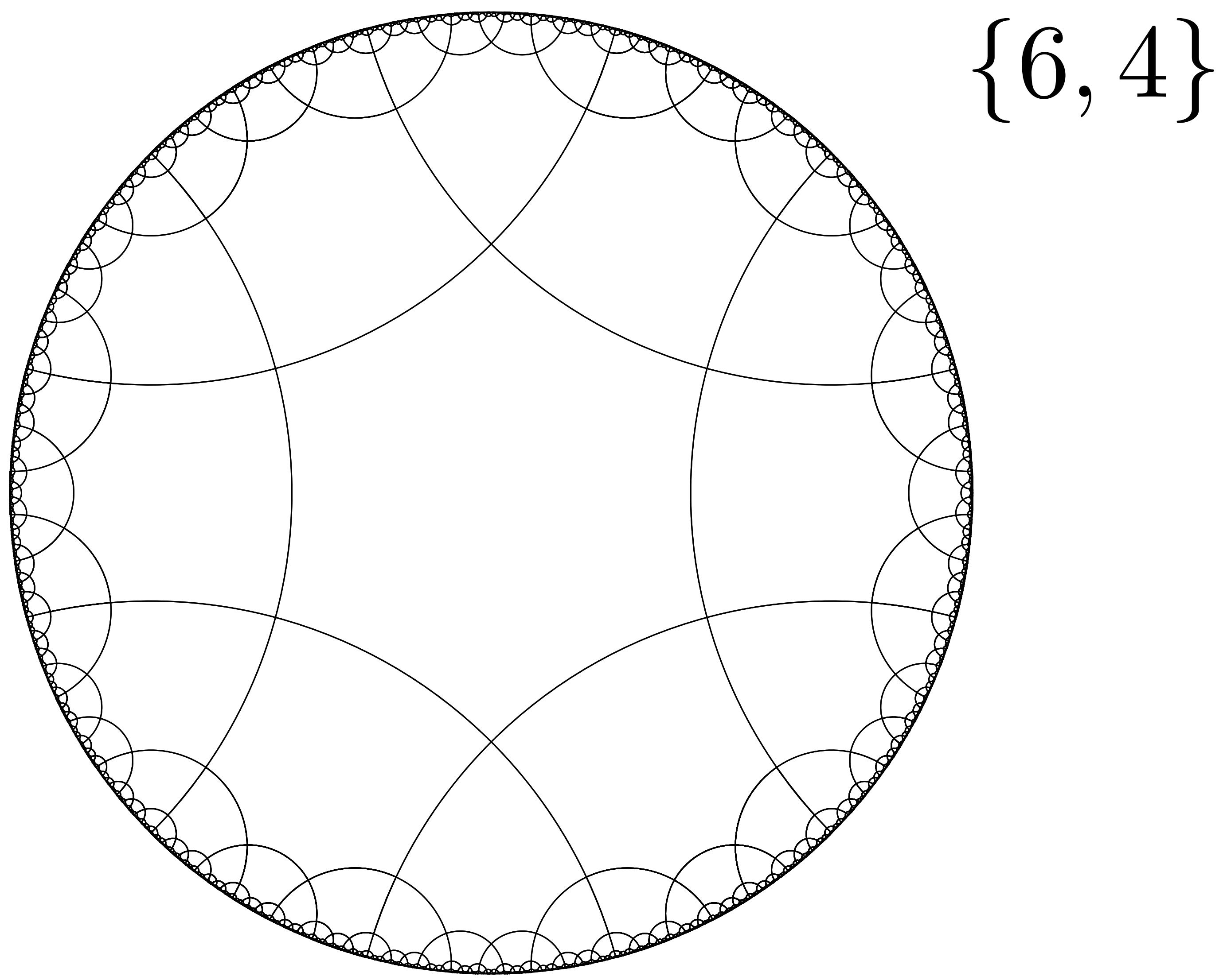
Tiling of hyperbolic space if $(p-2)(q-2)>4$



# Hyperbolic spaces in a lab

Experimental access to physics on hyperbolic spaces

- Circuit electrodynamics

  [Kollár, Fitzpatrick, Houck, ’19]
  [Bienias, Boettcher, Belyansky, Kollár, Gorshkov, ’22]

- Classical electric circuits

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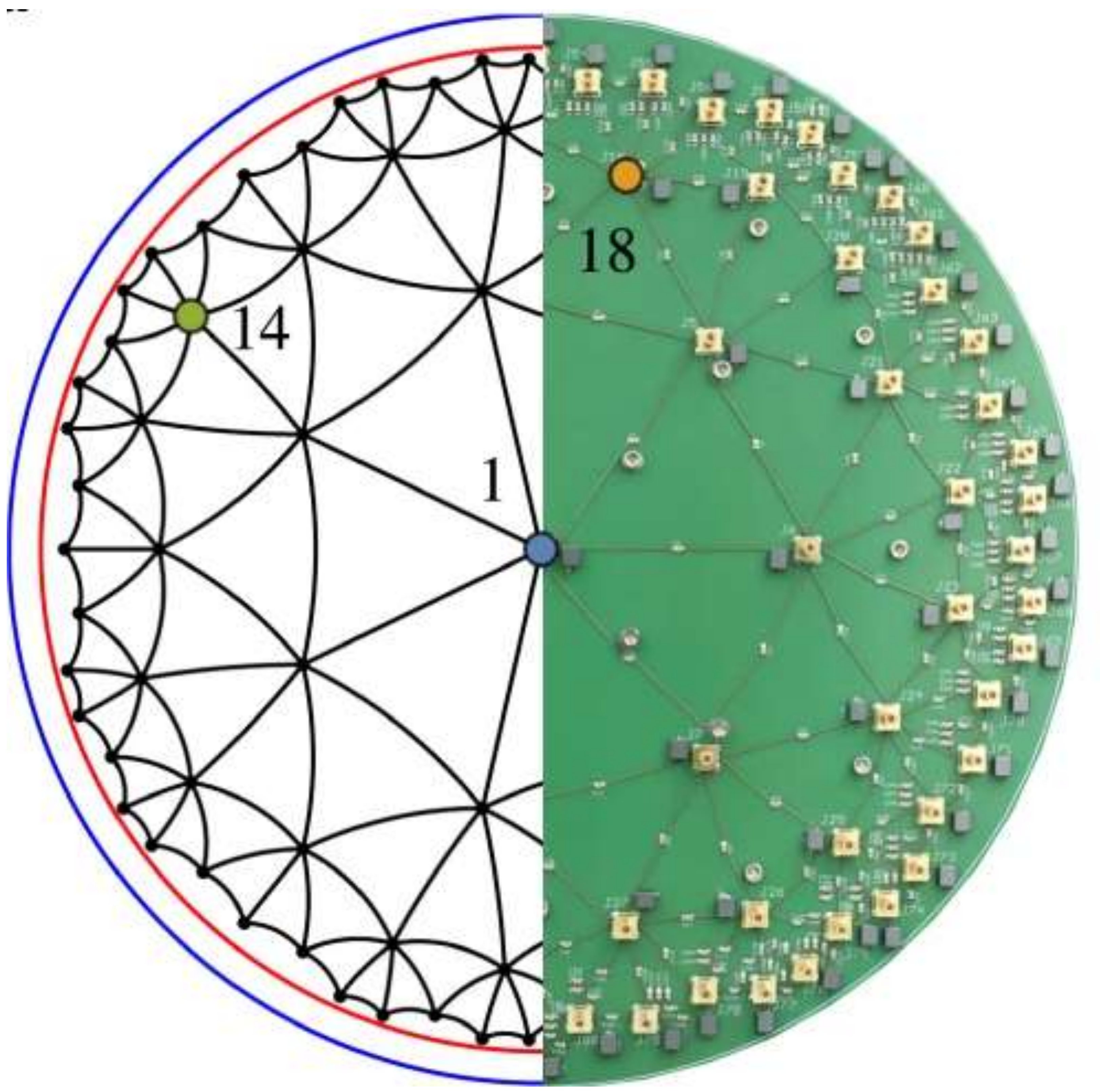

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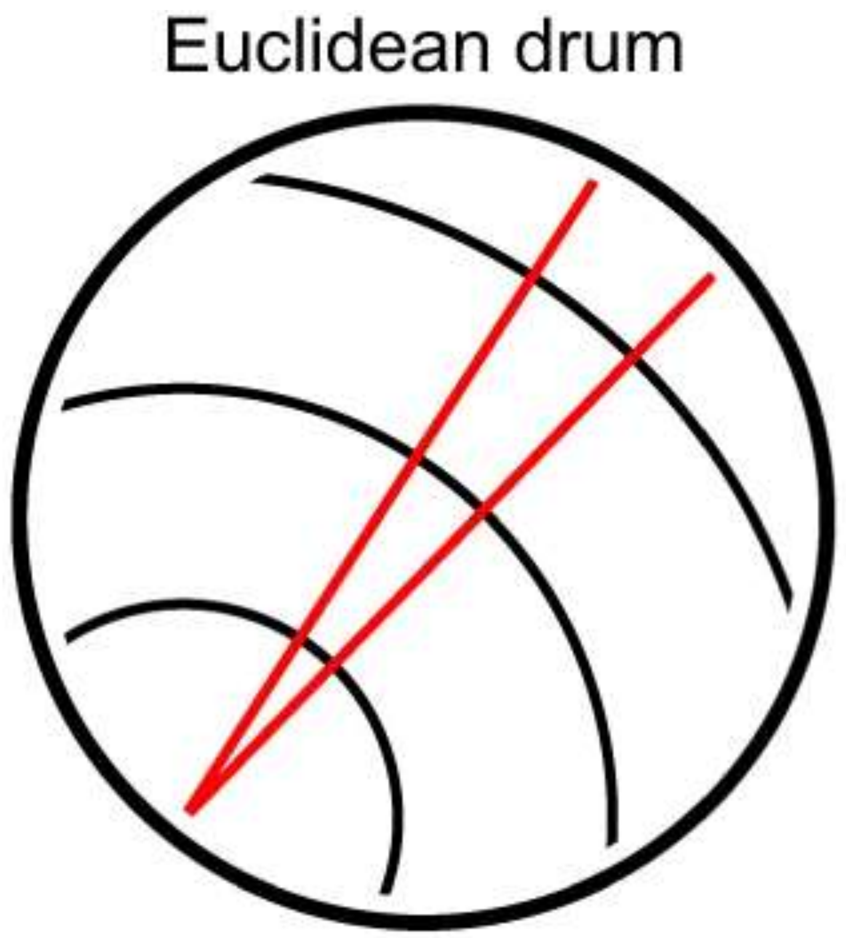
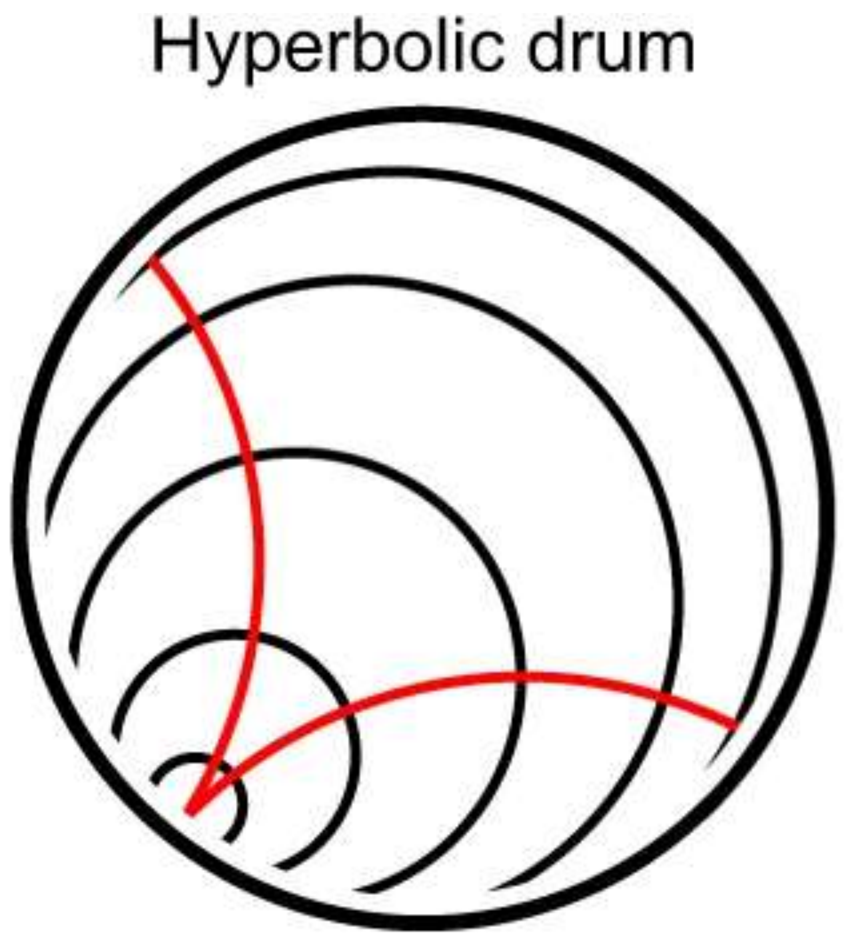
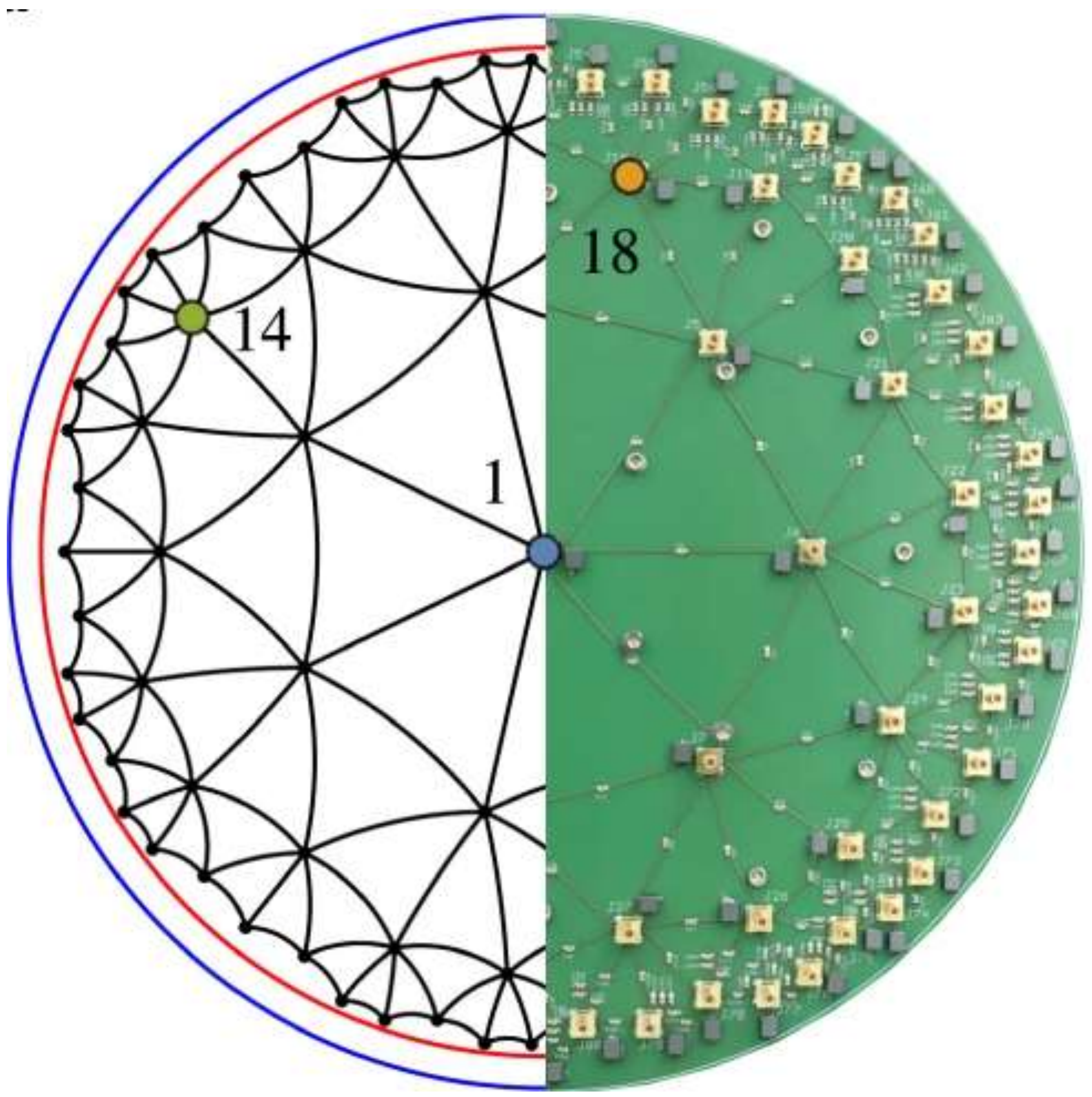



Figures from [Lenggenhager *et al*, ’22]



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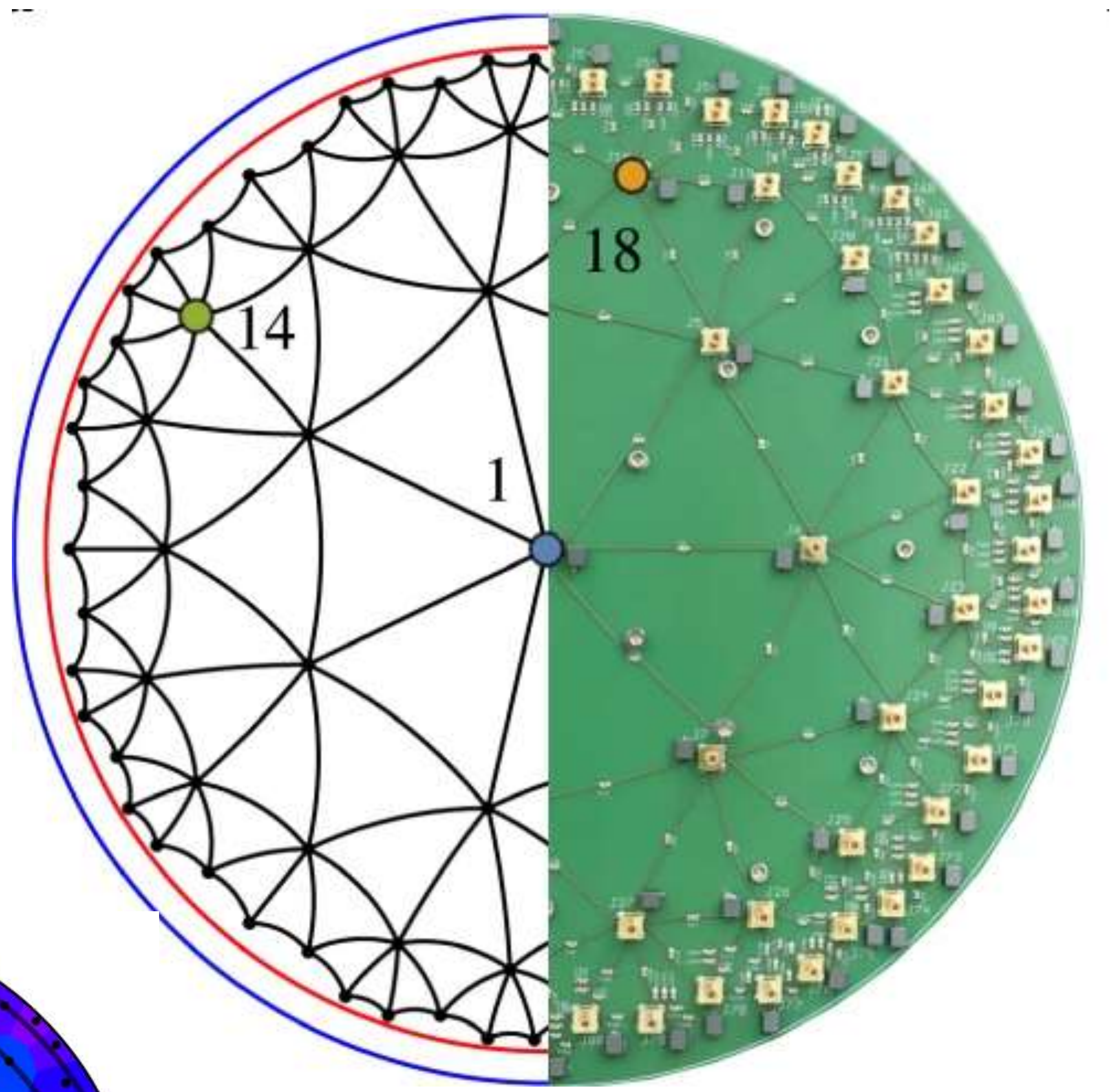


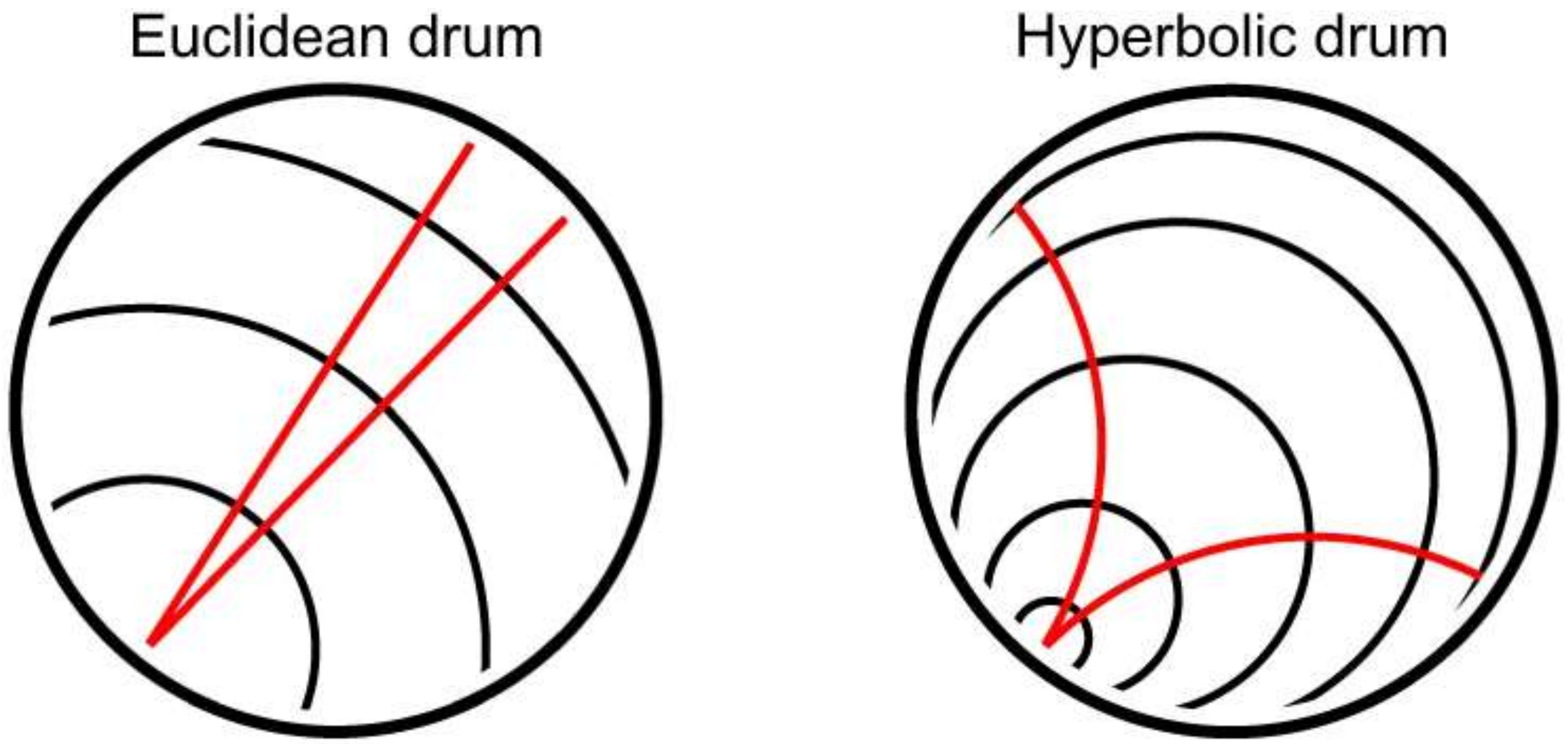


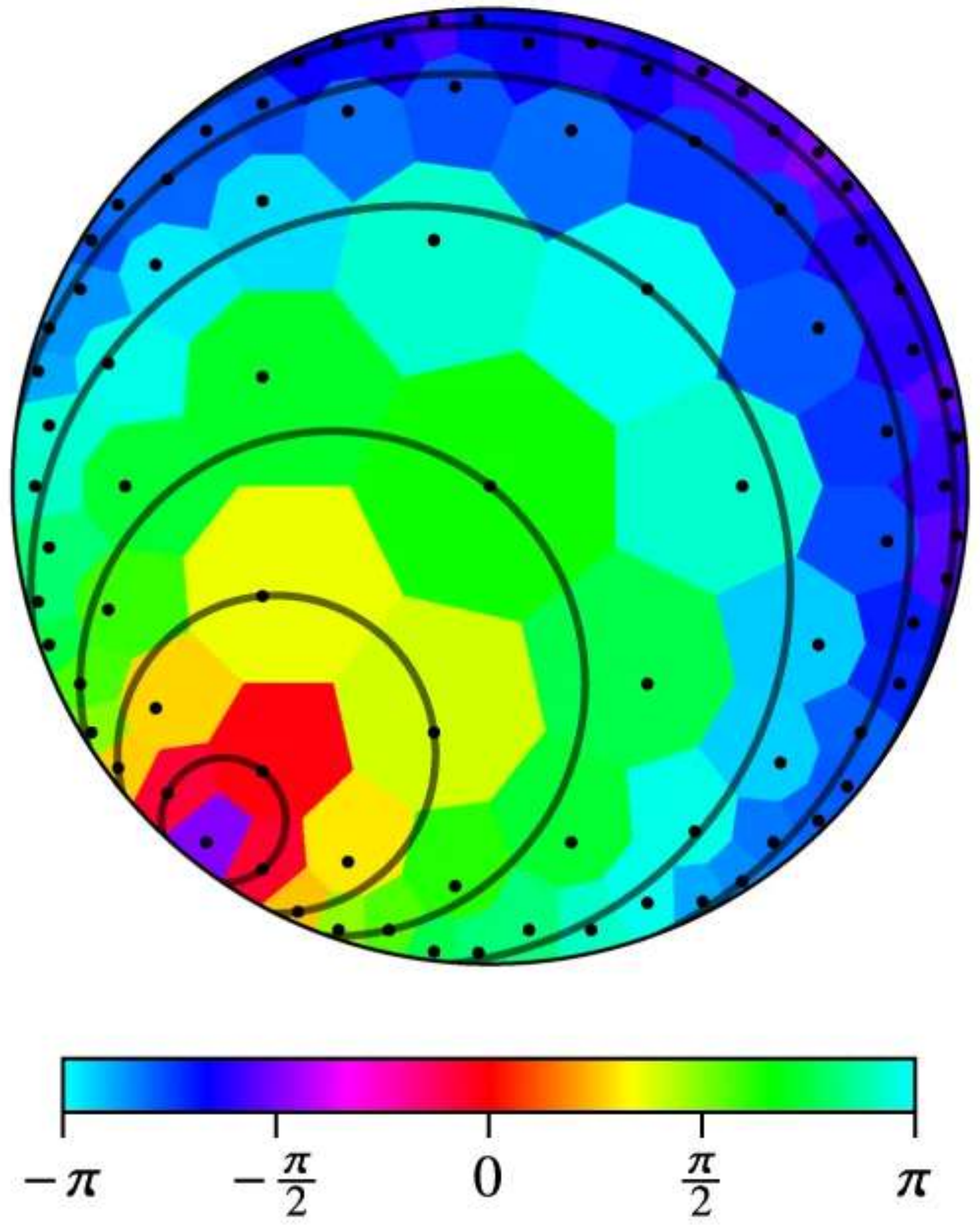


**Realization of effects peculiar to hyperbolic geometries in a lab**

# Hyperbolic tilings: a multidisciplinary playground

## Experimental realizations

Experimental access to physics on hyperbolic spaces

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## Condensed matter theory

- Lattice models on hyperbolic tilings

  [Chen *et al*, '22] [Breuckmann, Placke, Roy, '22]

- Bloch theory in hyperbolic lattices

  [Boettcher, Gorshkov, Kollár, Maciejko, Rayan, Thomale, '22] [Maciejko, Rayan, '21, '22]

## Tensor network models

[Jahn, Gluza, Pastawski, Eisert, 2019]
[Jahn, Zimborás, Eisert, '20]

## Mathematics

- Fuchsian groups and representation theory

  [Katok, '92]

- Gromov-hyperbolic spaces and hyperbolic buildings

  [Gesteau, Marcolli, Parikh, '22]

## Tests of AdS/CFT on hyperbolic tilings

- Correlation functions [Asaduzzaman, Catterall, Hubisz, Nelson, Unmuth-Yockey, '20] [Brower, Cogburn, Owen, '22] [Dey, Basteiro, *et al.* '24]
- Breitenlohner-Freedmann bound

  [Asaduzzaman, Catterall, Hubisz, Nelson, Unmuth-Yockey, '20] [Basteiro, Dusel, Erdmenger, Herdt, Hinrichsen, Meyer, Schaut, '22]

# In this talk...

If an explicit discrete holographic duality on hyperbolic tilings holds, we expect

Theory on the discrete geometry of the tiling

dynamically dual

?

Quantum chain at the boundary of the tiling

# In this talk…

If an explicit discrete holographic duality on hyperbolic tilings holds, we expect

**Fixed** discrete
geometry on the tiling

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**Ground state** of the
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First step in this program

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*SciPost Phys.* **13** (2022) 103

*SciPost Phys.* **15** (2023) 218

*JHEP* **02** (2025) 142

with P. Basteiro, R. N. Das, J. Erdmenger, J. Karl, R. Meyer, Z.-Y.Xian

In this talk, we define a class of boundary theories that reflect the properties of the bulk $\{p, q\}$ tiling

**Goal:** how do different bulk discretizations affect the corresponding boundary theories?

# Outline

- Aperiodic quantum chains at the boundary of hyperbolic tilings
- Ground state and entanglement properties
- Tensor networks as a bulk description from aperiodic chains
- Towards adding a metric in the bulk
- Conclusions and future perspectives

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- Aperiodic spin chains at the boundary of hyperbolic tilings
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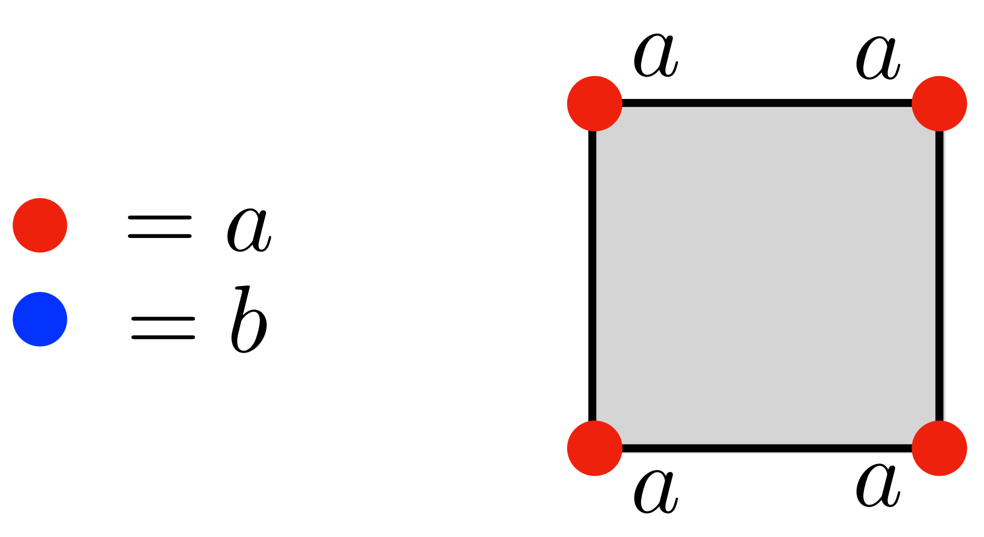
# Hyperbolic tilings through inflation rules [Boyle, Dickens, Flicker, 2020]

Let us construct
a $\{4,5\}$ tiling



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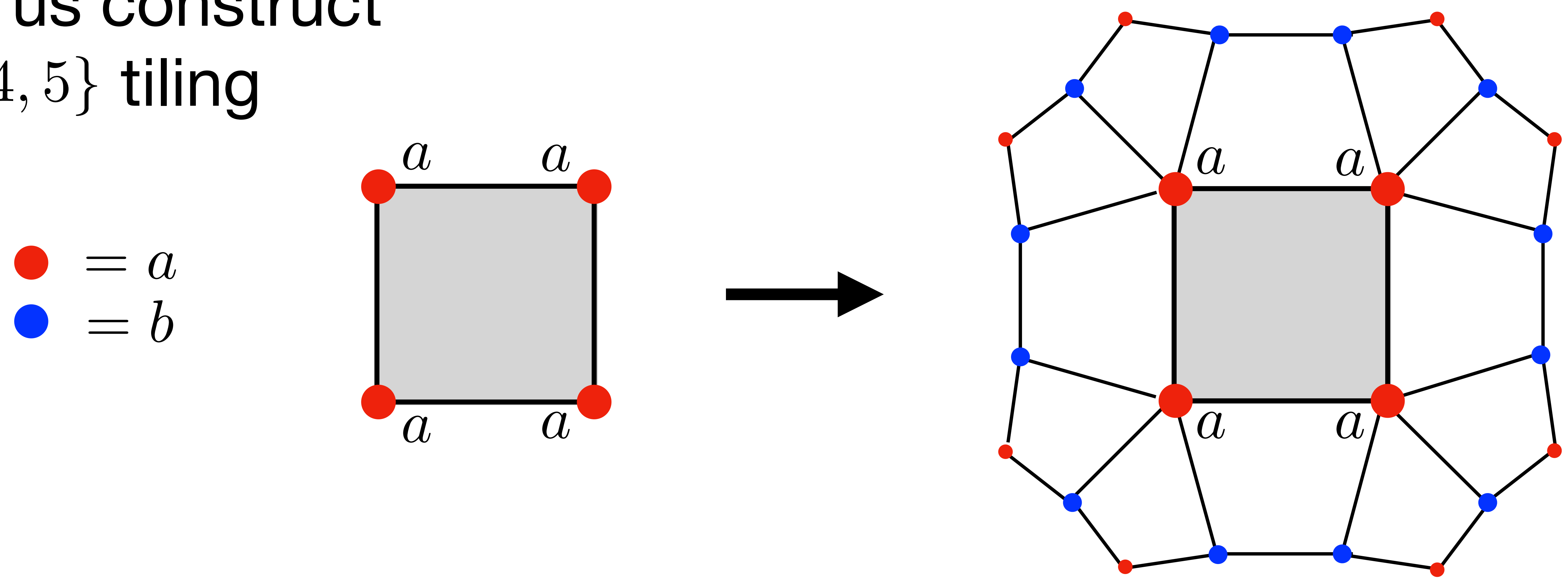

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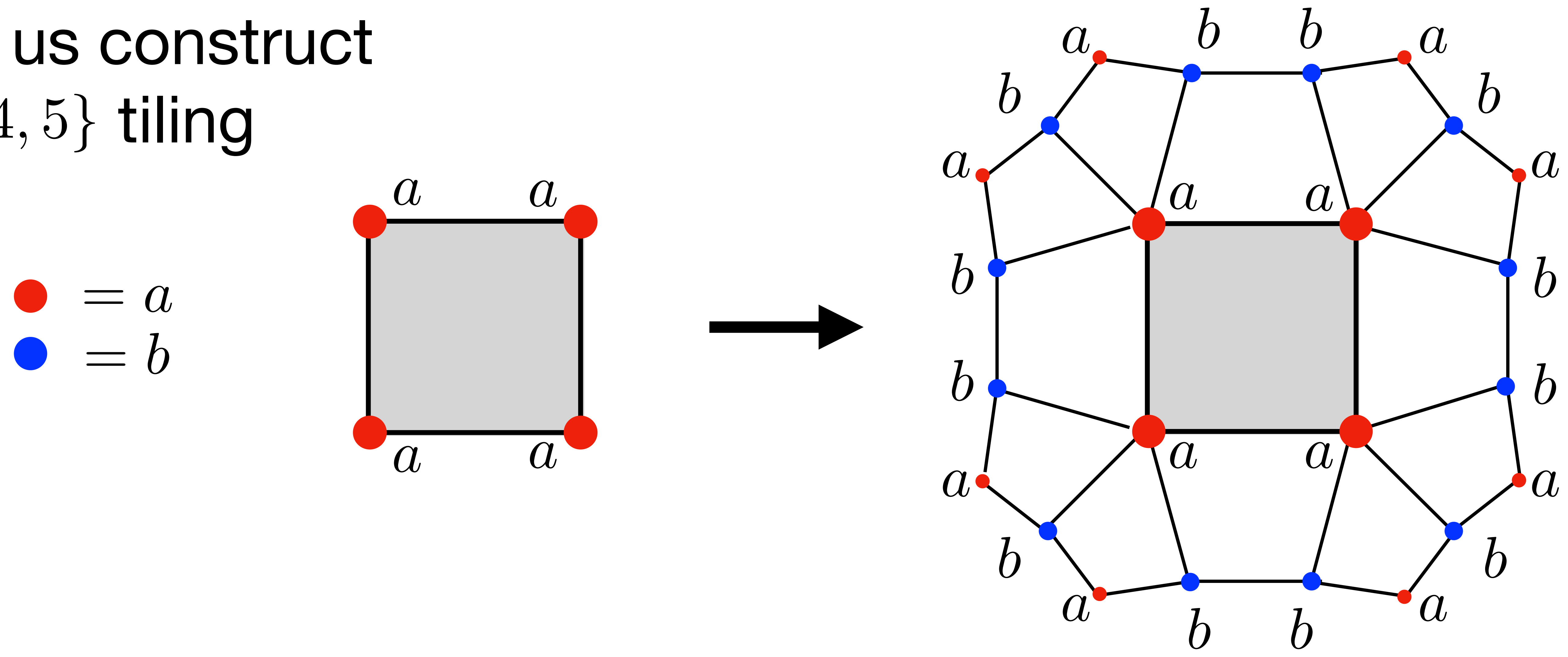


$$a \to babab$$

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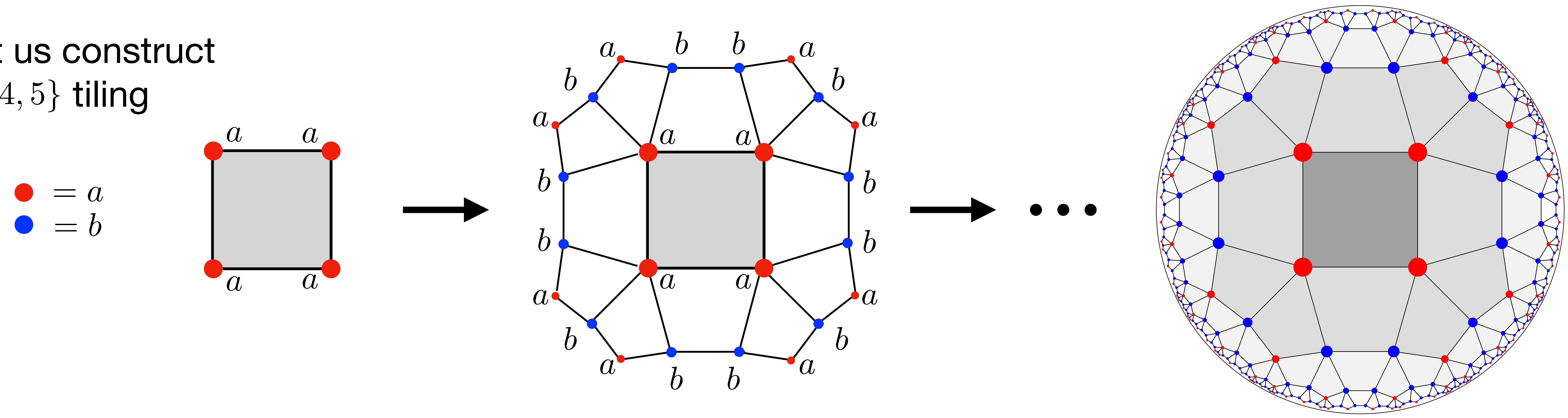


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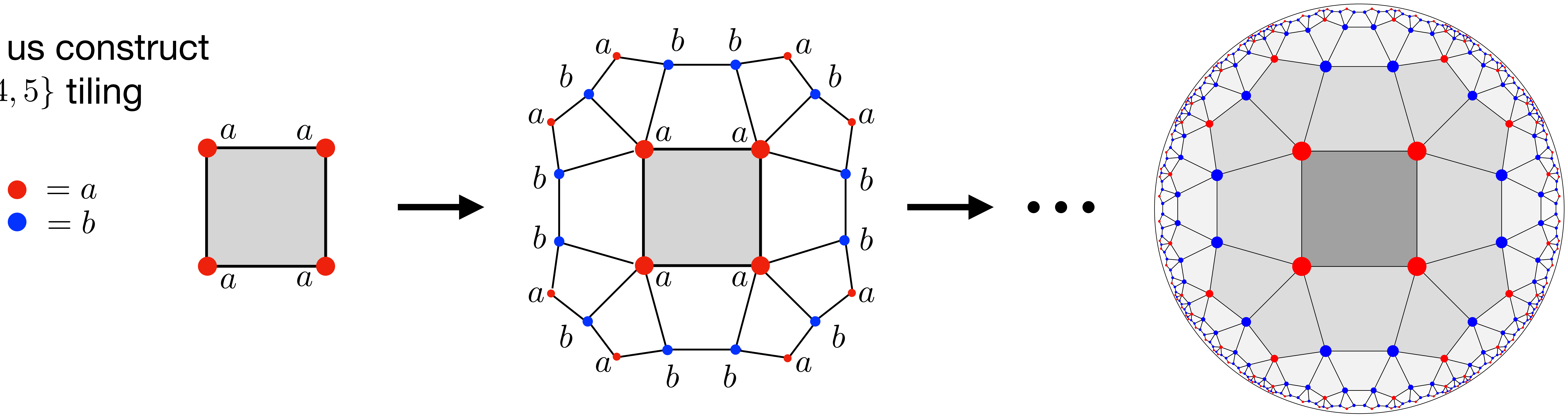


$$\Sigma_{\{4,5\}} : \begin{cases} a \to babab \\ b \to bab \end{cases}$$

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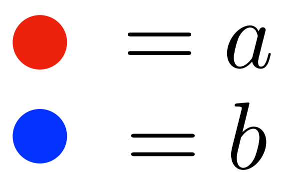


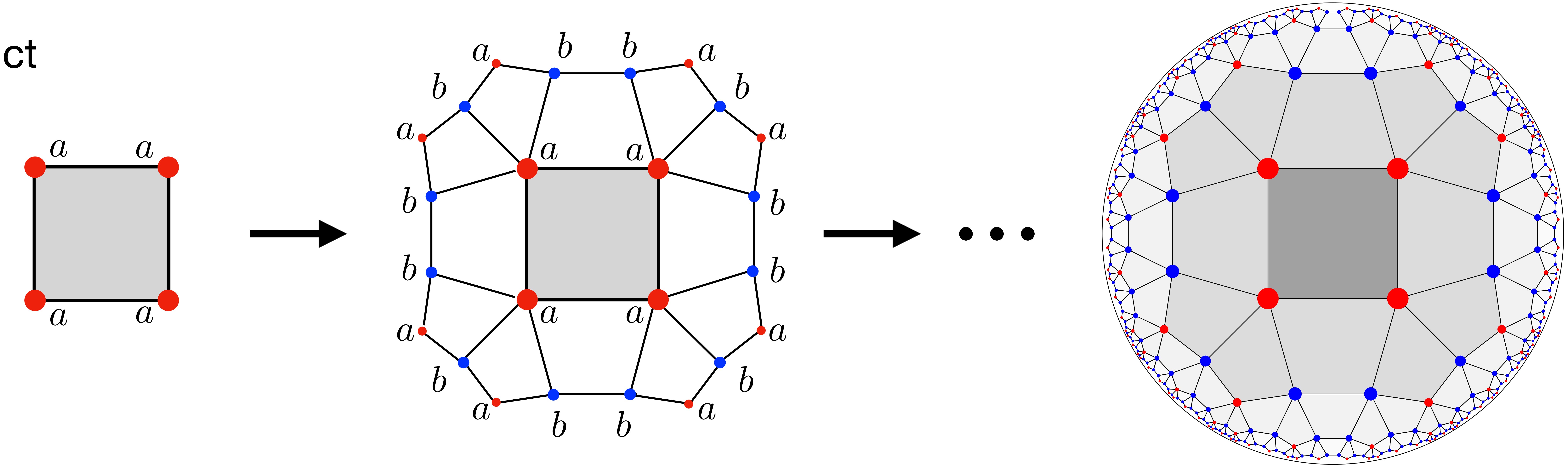


**Inflation rule**

$$\Sigma_{\{p,q\}} : \begin{cases} a \to w_a(a,b) \\ b \to w_b(a,b) \end{cases}$$

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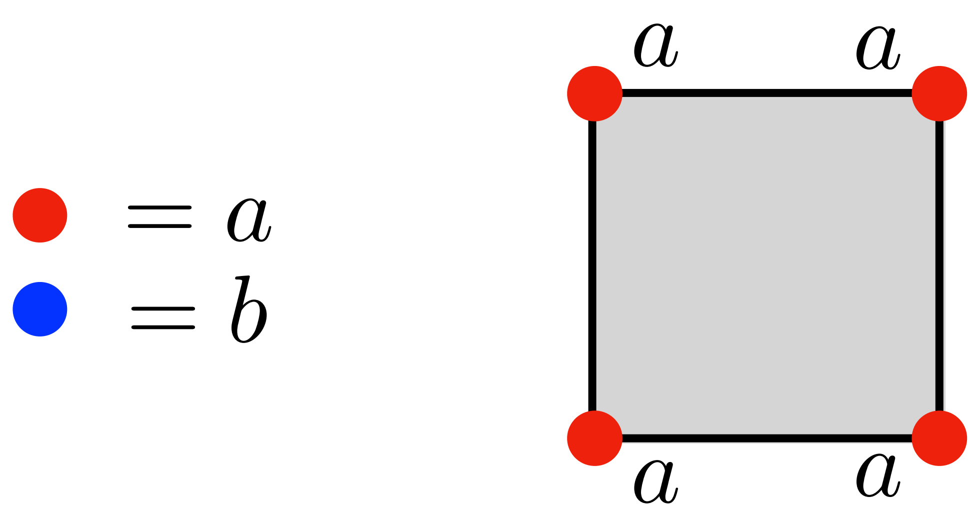


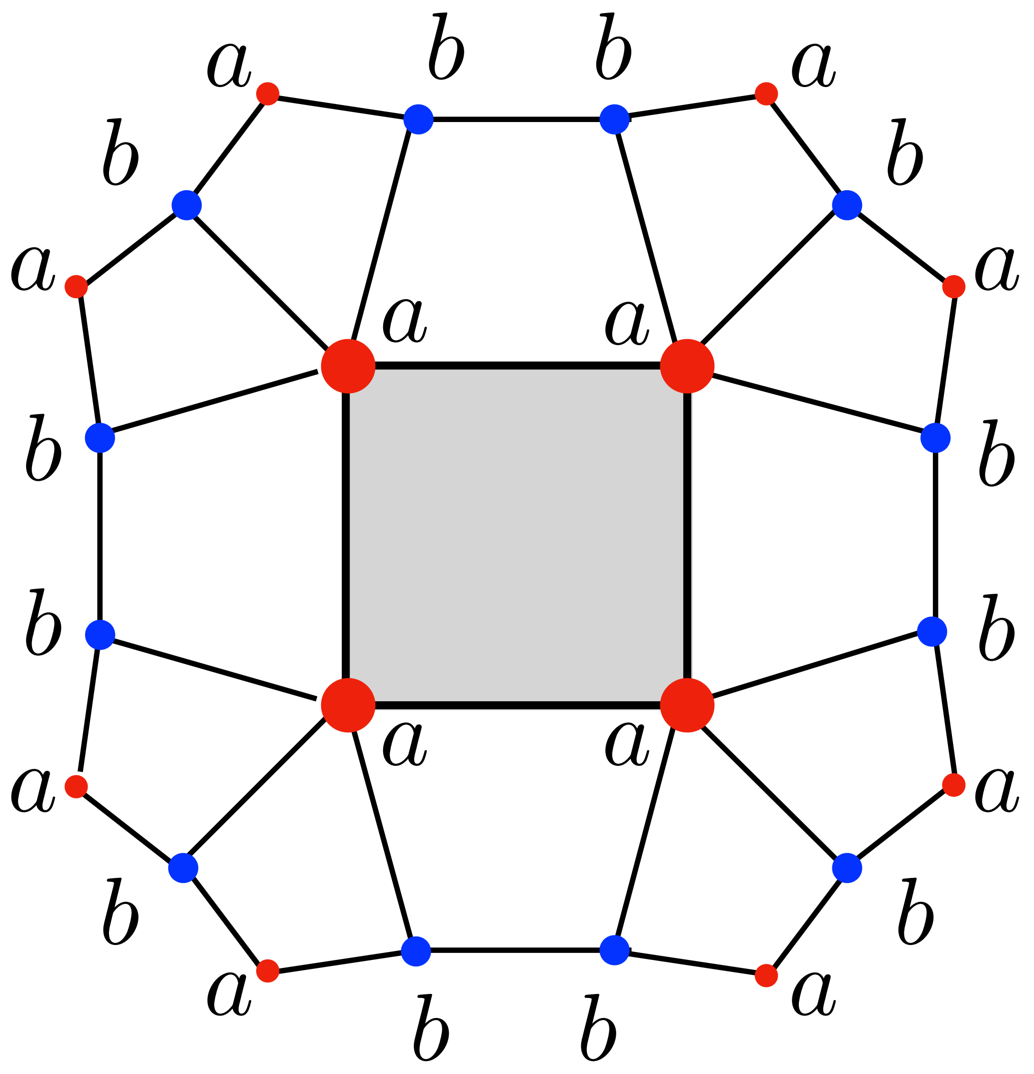


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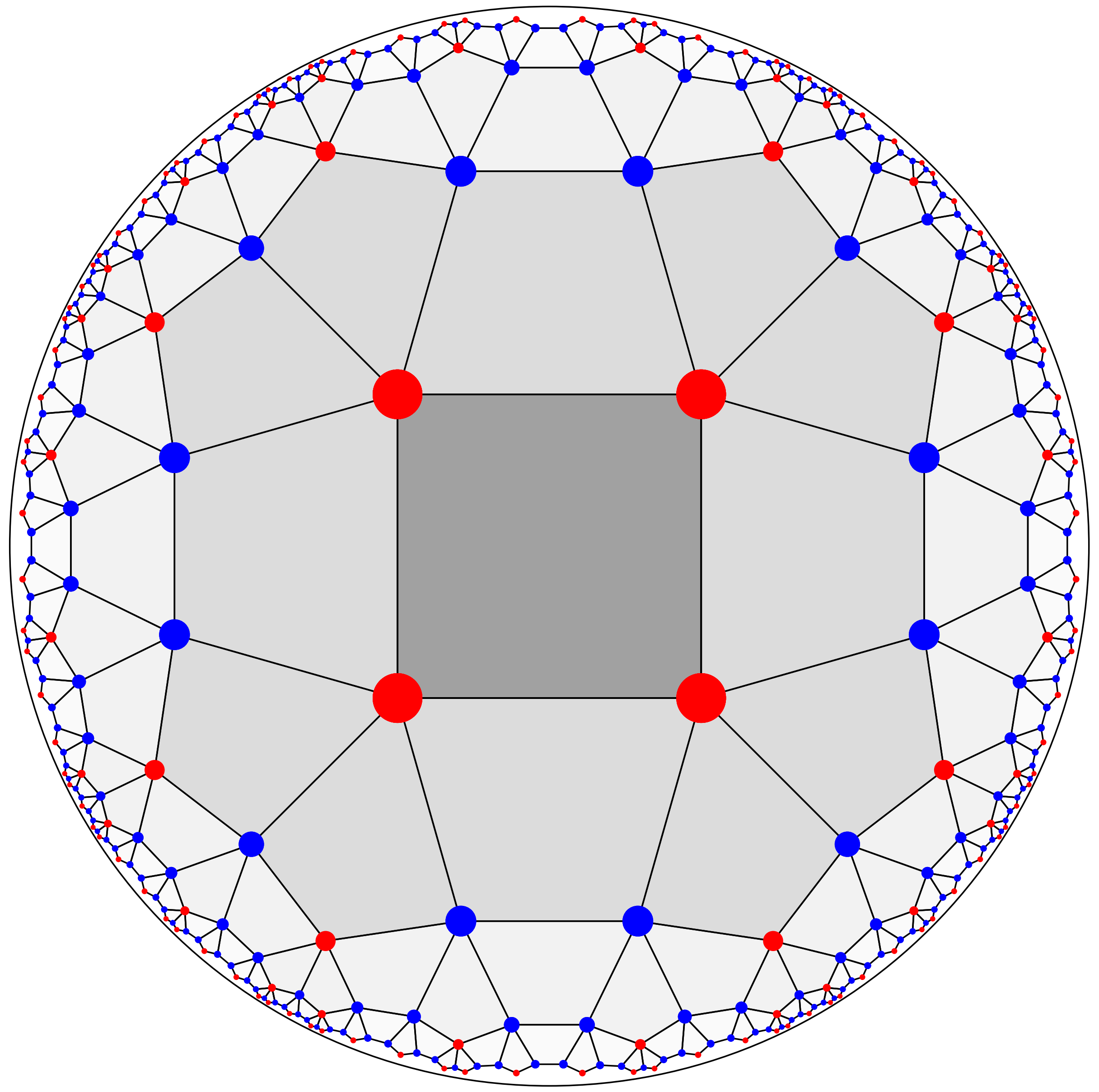

**Inflation rule**

$$\Sigma_{\{p,q\}}: \begin{cases} a \to w_a(a,b) \\ b \to w_b(a,b) \end{cases}$$

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The boundary of the tiling after a large number of inflation steps is characterised by a long **aperiodic sequence**

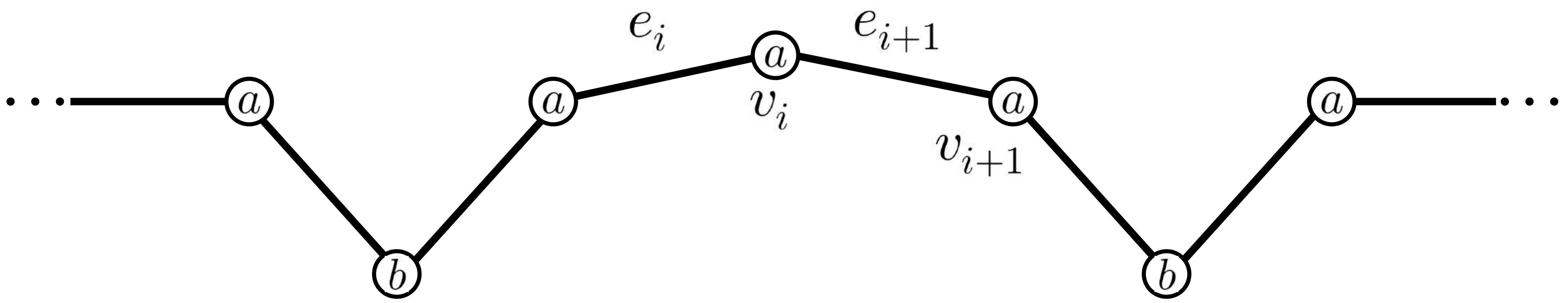
# Construction of the boundary theory: aperiodic spin chains

## Theory at the boundary of the tiling

Edge

Vertex

Letter



# Construction of the boundary theory: aperiodic spin chains

## Theory at the boundary of the tiling

| | | |
|---|---|---|
| Edge | $\longrightarrow$ | Degree(s) of freedom |
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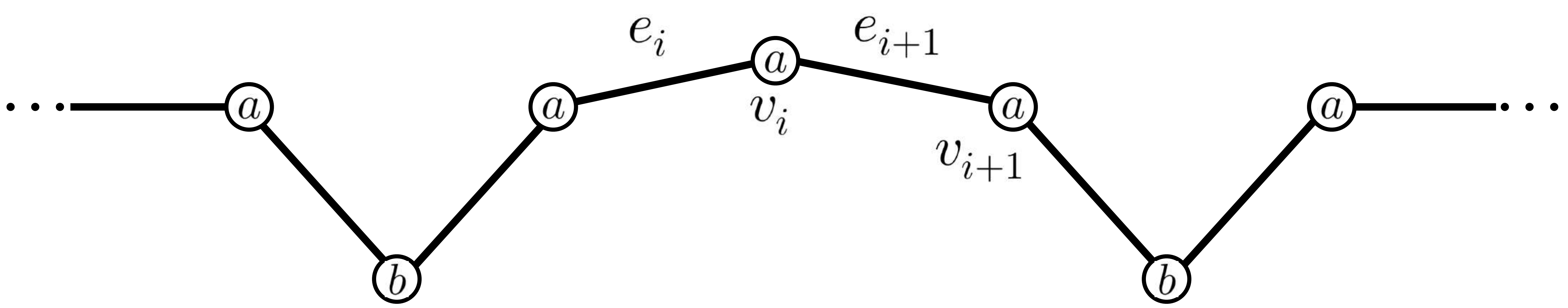

# Construction of the boundary theory: aperiodic spin chains

## Theory at the boundary of the tiling

| | | |
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| Vertex | $\longrightarrow$ | Coupling |
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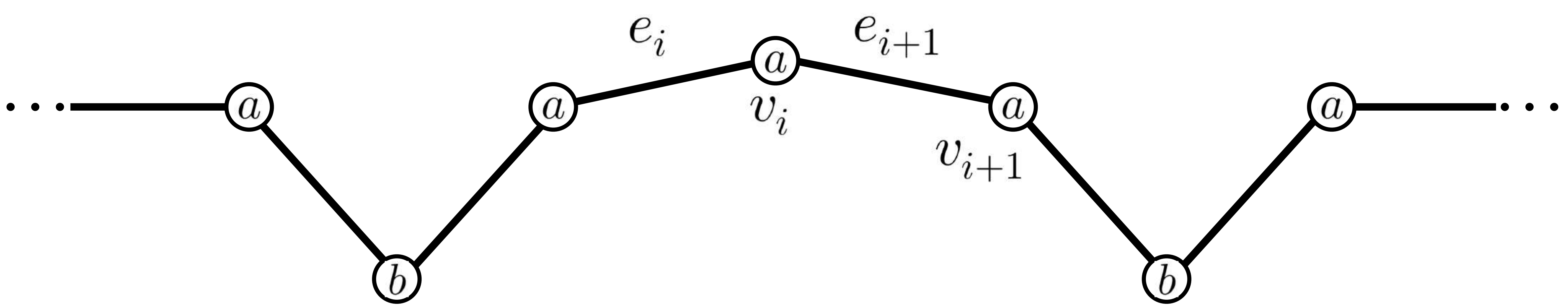

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**Theory at the boundary of the tiling**

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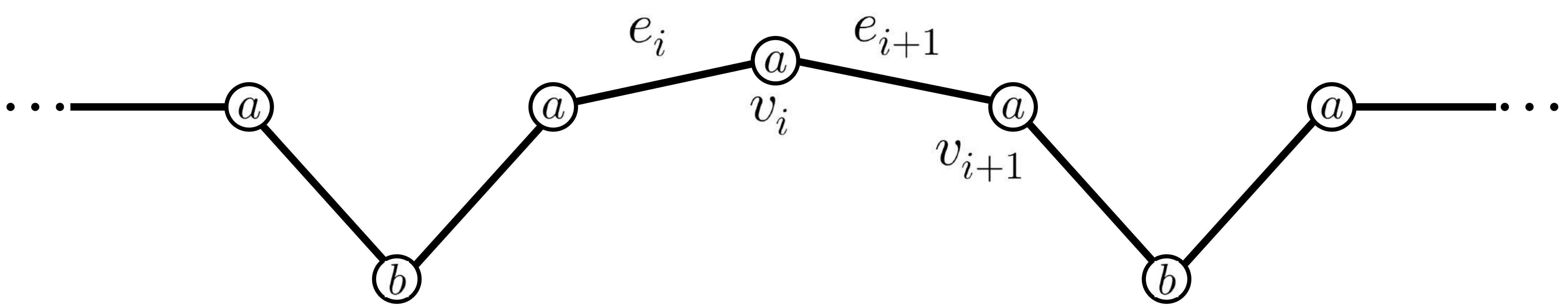

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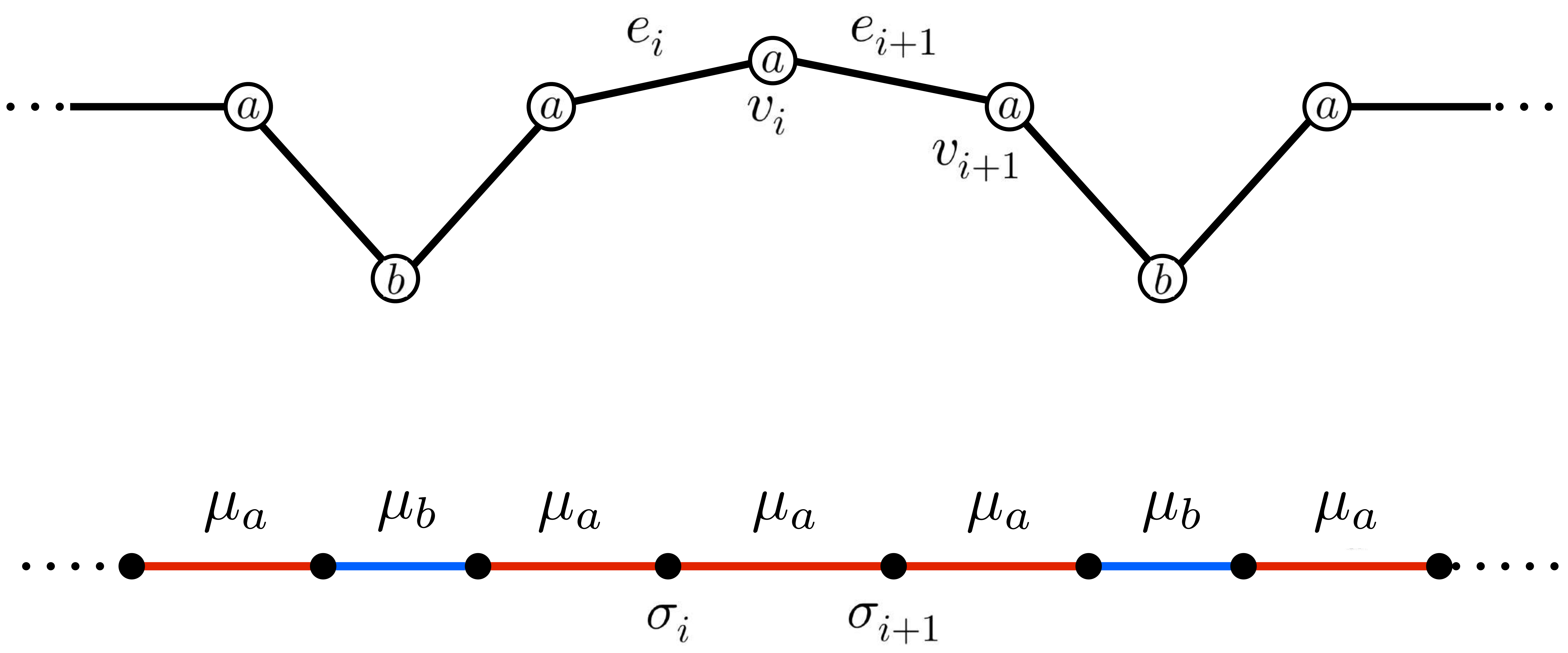

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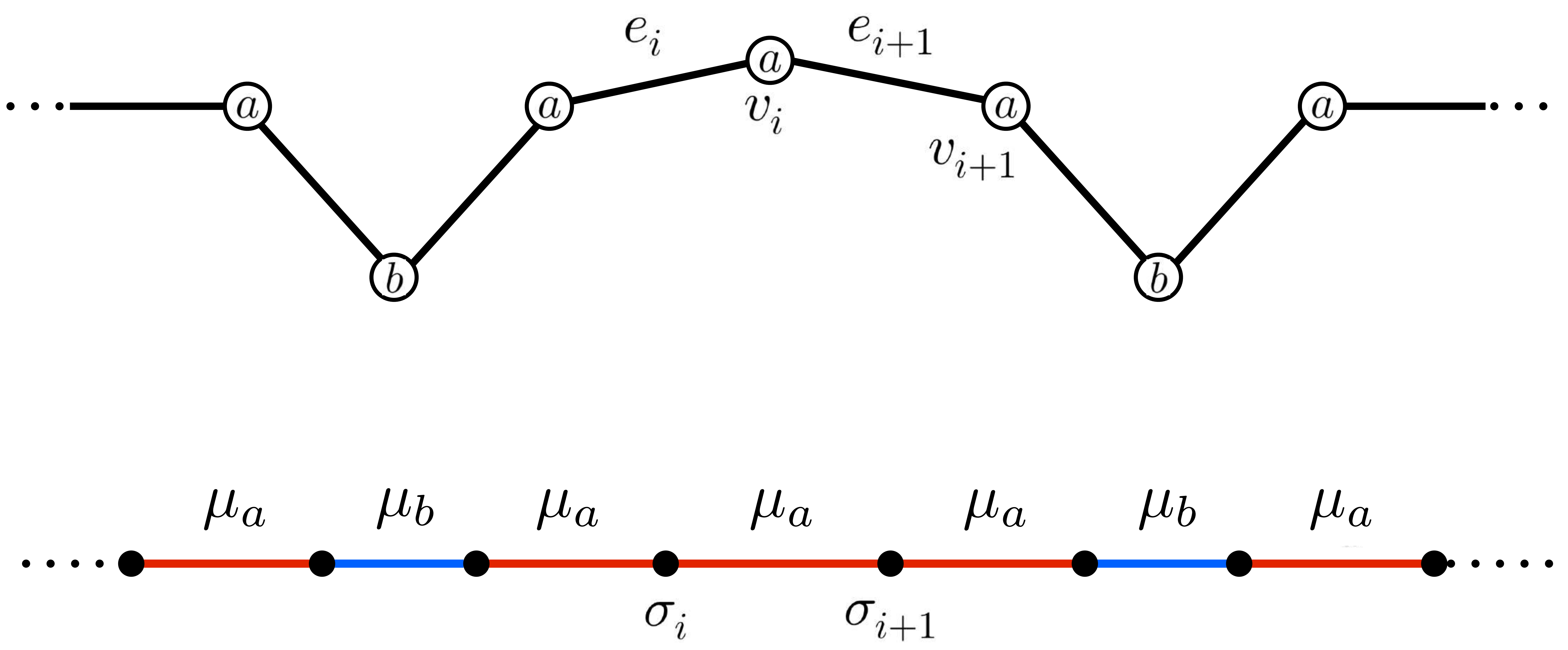


## Aperiodic spin chains [Luck ’93; Hermisson ’97; Vidal, Mouhanna, Giamarchi ‘99]

Spin chains with inhomogeneous couplings: two different couplings are allowed between consecutive spins and their sequence is fixed by a given aperiodic sequence

Boundary of a $\{p,q\}$ tiling of the Poincaré disk $=$ **aperiodic spin chain** with couplings following the aperiodic sequence induced at the boundary by the bulk discretization

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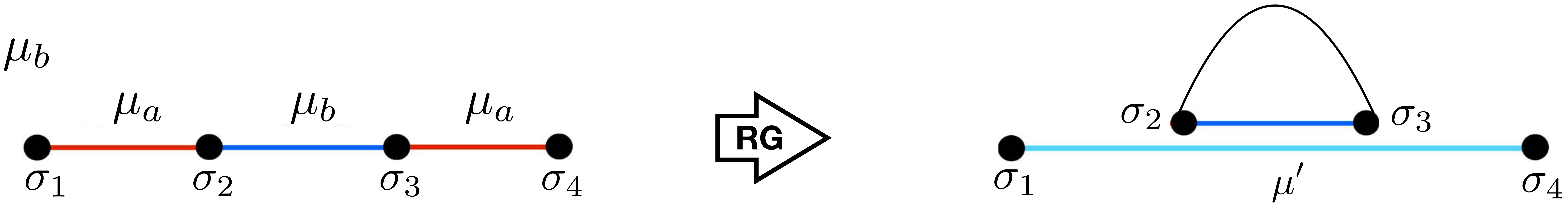
$\mu_a \ll \mu_b$

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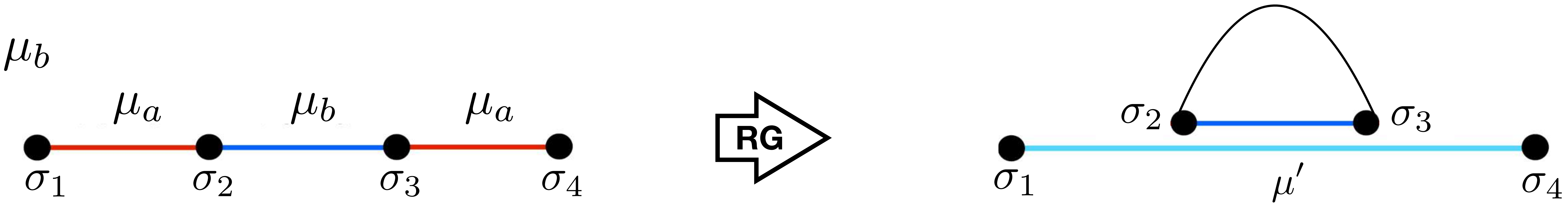


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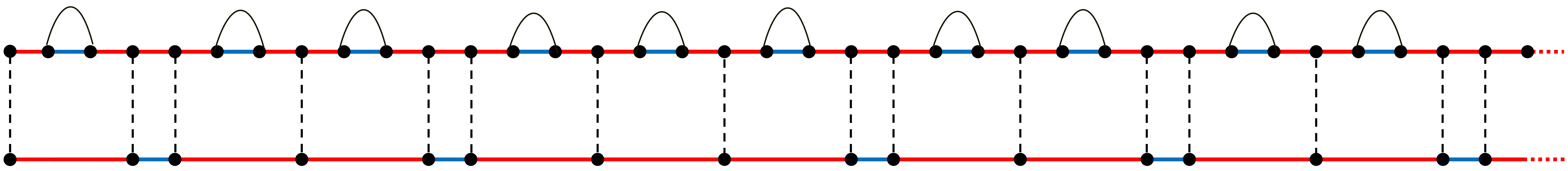

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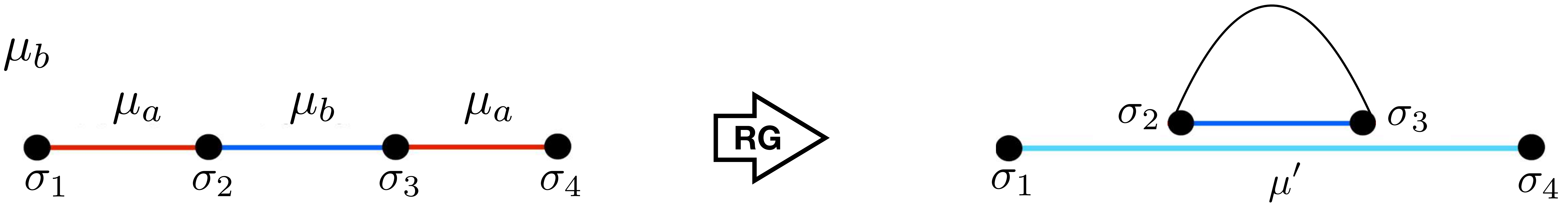


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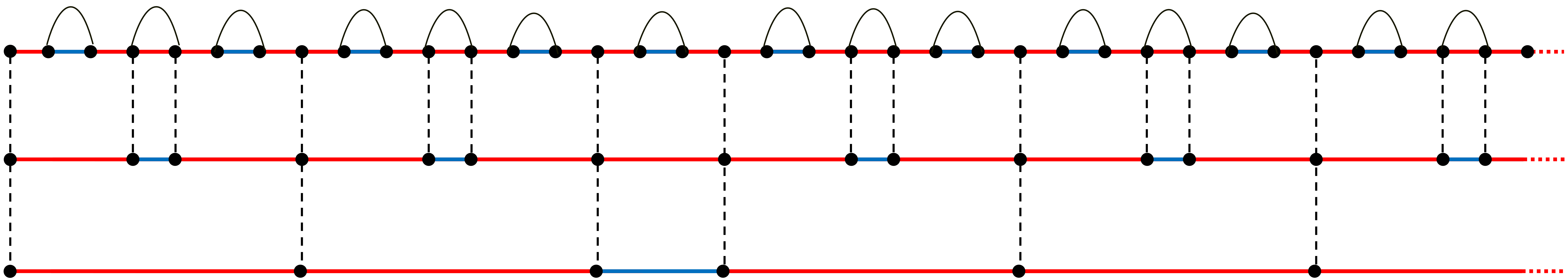

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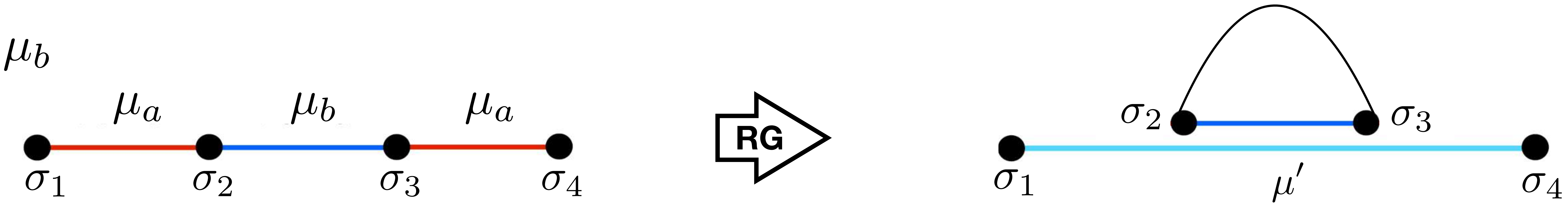


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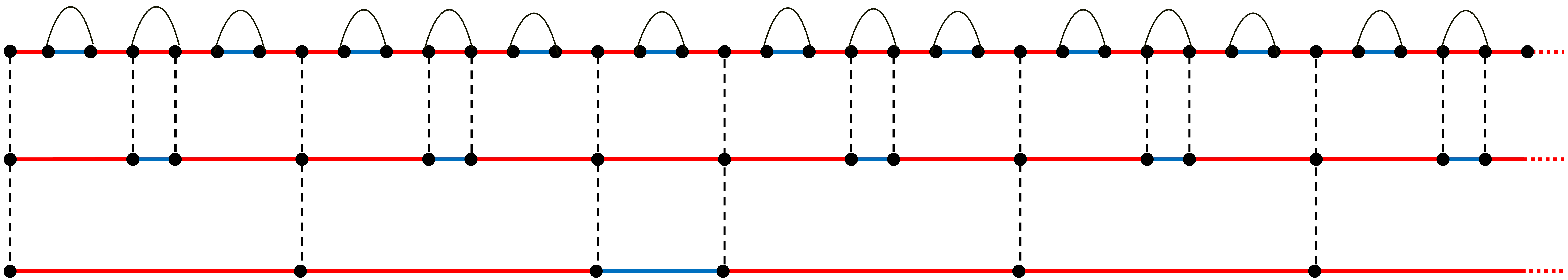
**Check**

$\mu_a \ll \mu_b$ for any renormalized pair of couplings



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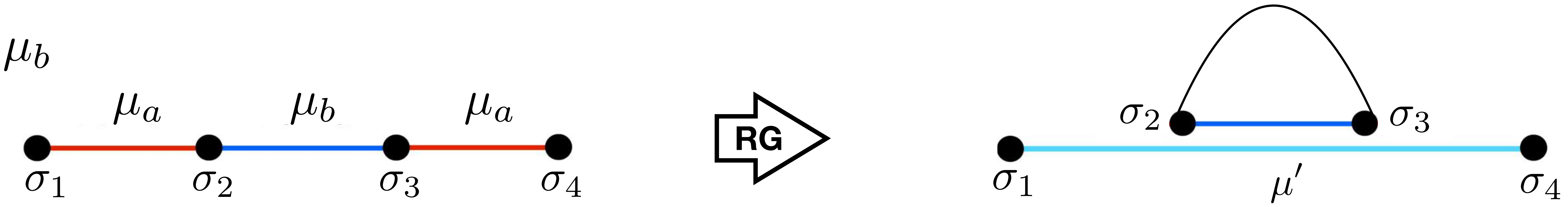


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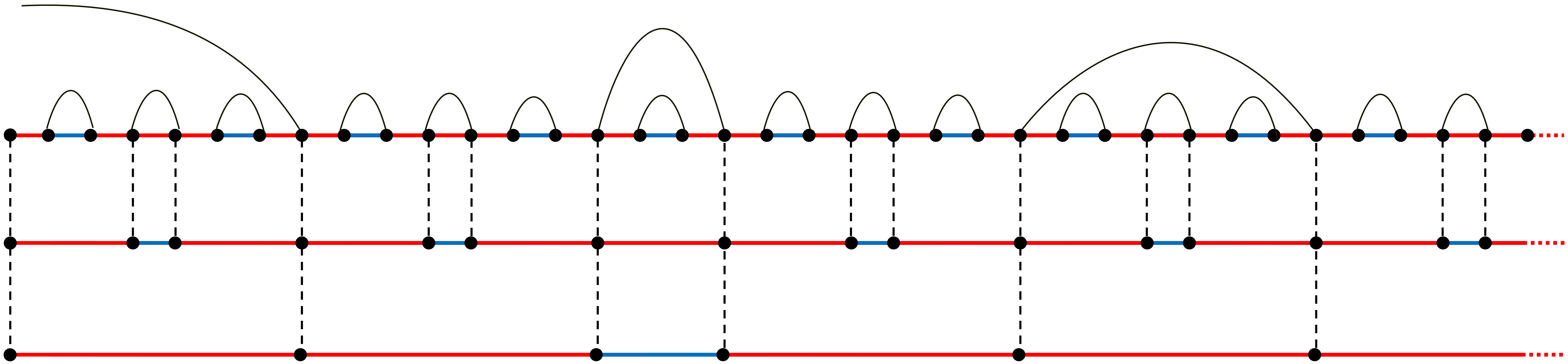


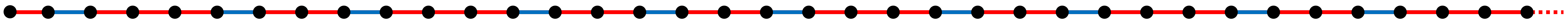
# Aperiodic singlet phase

**Aperiodic XXX chains at the boundary of the following tilings**

$$H_{\Sigma_{\{p,q\}}} = \sum_{i\in\mathbb{Z}} \mu_i \left[\, \sigma_i^{(x)}\sigma_{i+1}^{(x)} + \sigma_i^{(y)}\sigma_{i+1}^{(y)} + \sigma_i^{(z)}\sigma_{i+1}^{(z)} \right] \qquad \mu_i \in \{\mu_a, \mu_b\}$$

$\{6,q\} \quad q \geqslant 4$

$\{3,7\}, \{3,8\}, \{5,4\}, \{5,5\}$



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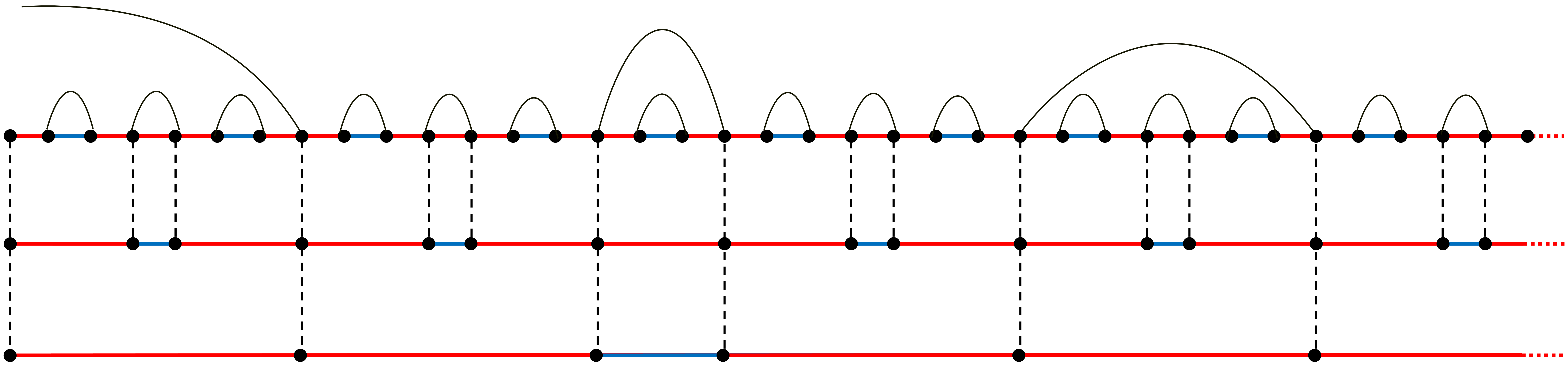
$$\frac{\mu_a}{\mu_b} \left( \xrightarrow{\text{RG}} \right)^n r^*$$



# Aperiodic singlet phase

**Aperiodic XXX chains at the boundary of the following tilings**

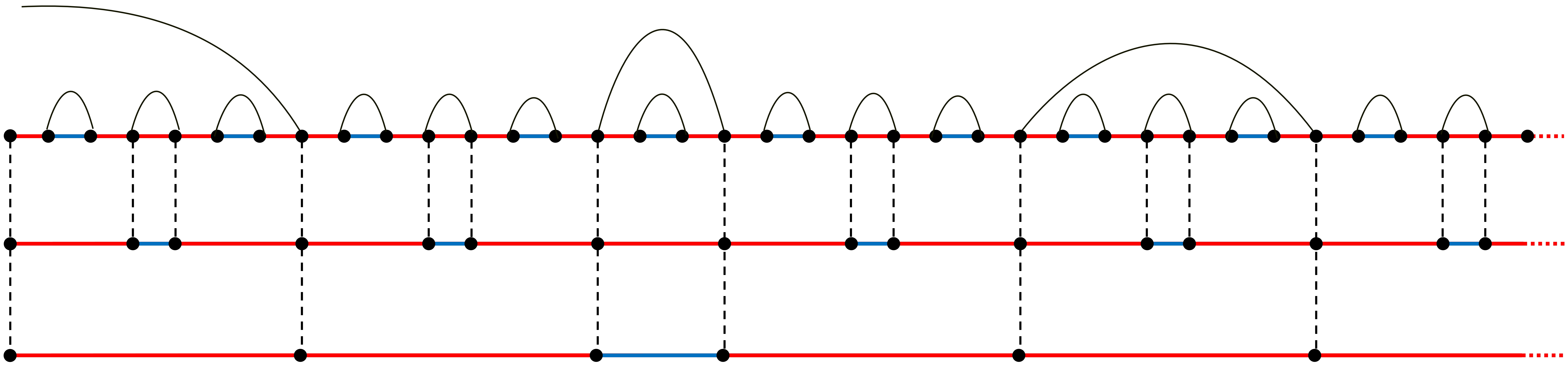
$$\{6, q\} \quad q \geqslant 4$$

$$\{3,7\}, \{3,8\}, \{5,4\}, \{5,5\}$$

$$H_{\Sigma_{\{p,q\}}} = \sum_{i \in \mathbb{Z}} \mu_i \left[ \sigma_i^{(x)} \sigma_{i+1}^{(x)} + \sigma_i^{(y)} \sigma_{i+1}^{(y)} + \sigma_i^{(z)} \sigma_{i+1}^{(z)} \right] \qquad \mu_i \in \{\mu_a, \mu_b\}$$

$$\frac{\mu_a}{\mu_b} \left( \xrightarrow{\text{RG}} \right)^n r^* = 0$$

*Strong-disorder fixed point*



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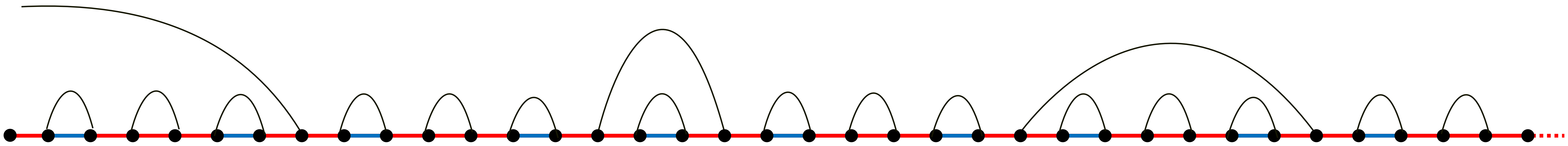
*Strong-disorder fixed point*

The ground state is described by an **aperiodic singlet phase**



$$\rho_{12}^{(\text{EPR})} = |\text{EPR}\rangle_{12\ 12}\langle\text{EPR}| \qquad |\text{EPR}\rangle_{12} = \frac{|0\rangle_1 \otimes |1\rangle_2 - |1\rangle_1 \otimes |0\rangle_2}{\sqrt{2}} \qquad \rho = \left( \bigotimes_{\text{singlets}} \rho_{s_1 s_2}^{(\text{EPR})} \right)$$

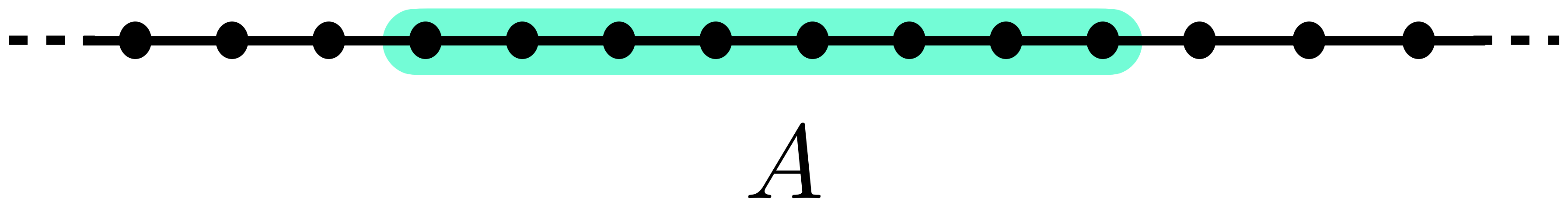
Singlet distribution from the inflation rule

# Entanglement entropy in the aperiodic singlet phase

$\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$

$\rho_A = \mathrm{Tr}_{\mathcal{H}_B}\rho$

$\boxed{S_A = -\mathrm{Tr}(\rho_A \ln \rho_A)}$ **entanglement entropy**



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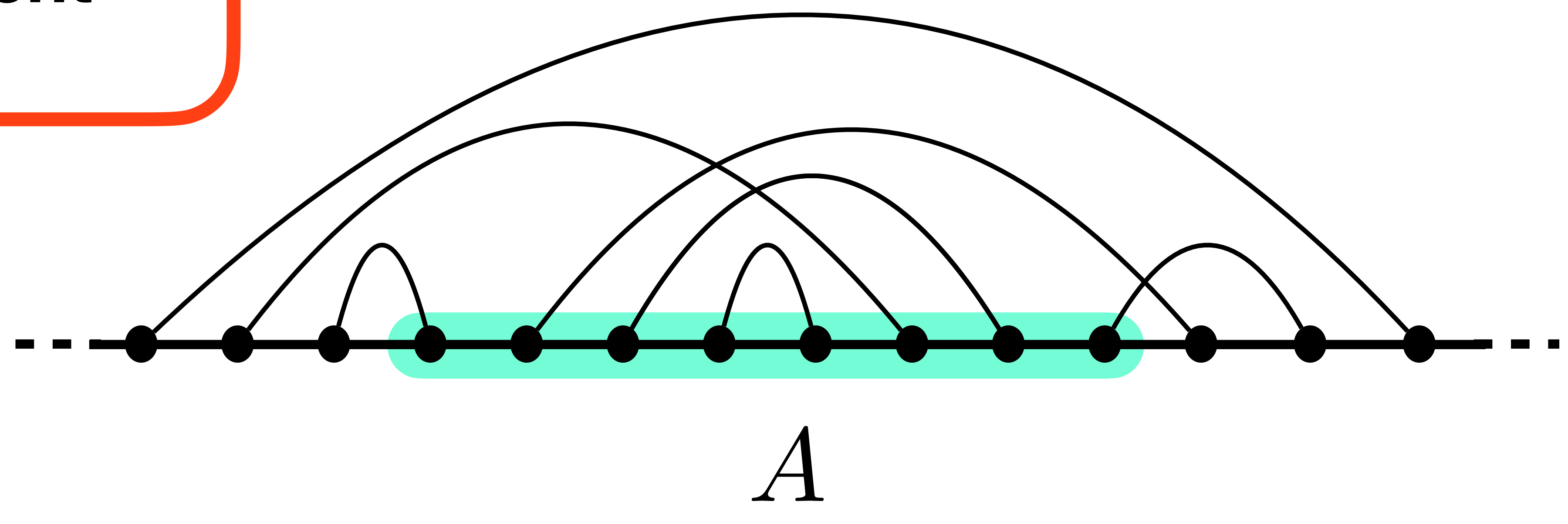

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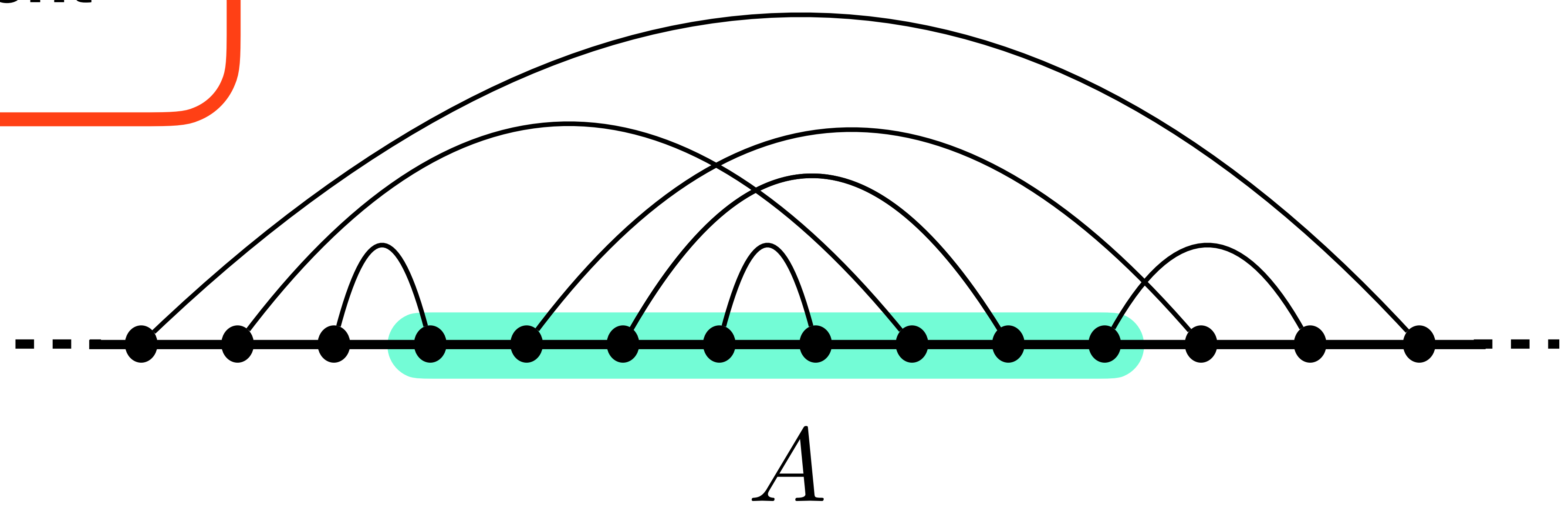


$S_A = \bar{n}_A \ln 2$ average number of singlets shared between $A$ and its complement [Iglói, Juhász, Zimborás '07]

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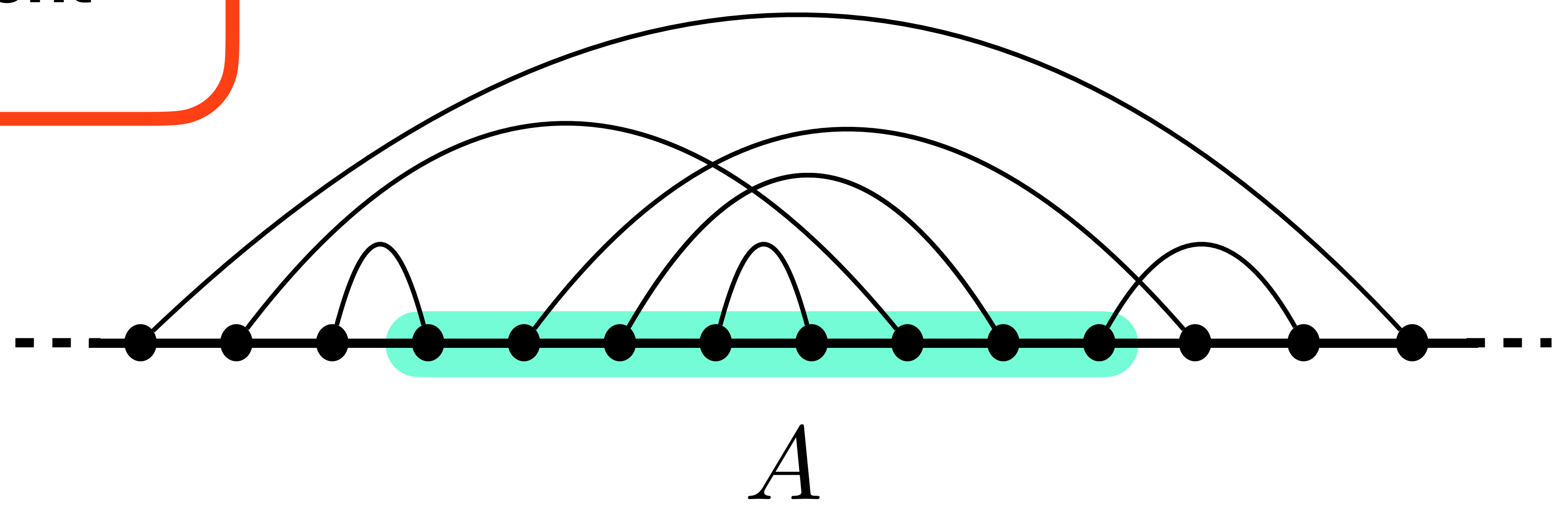


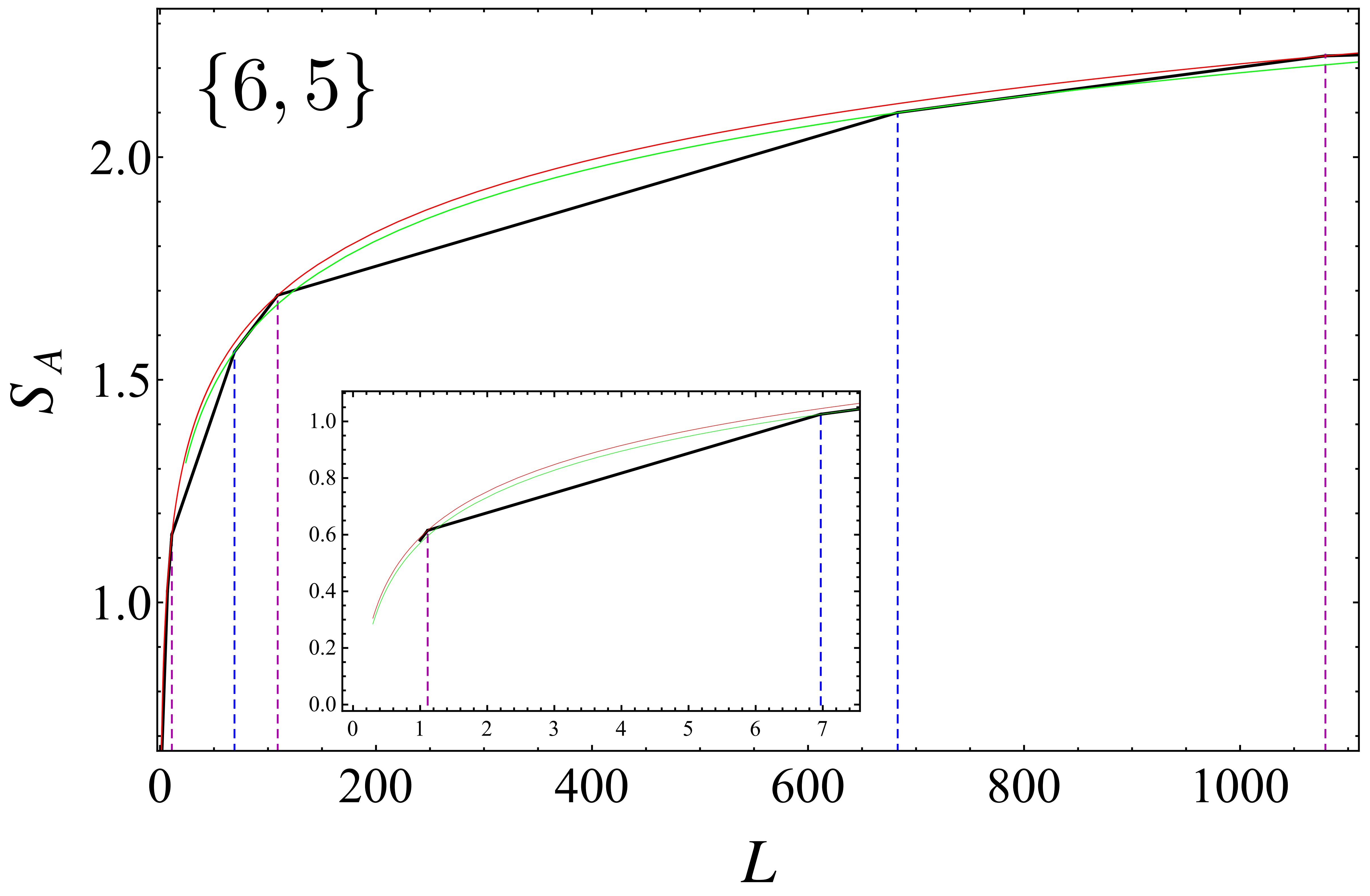

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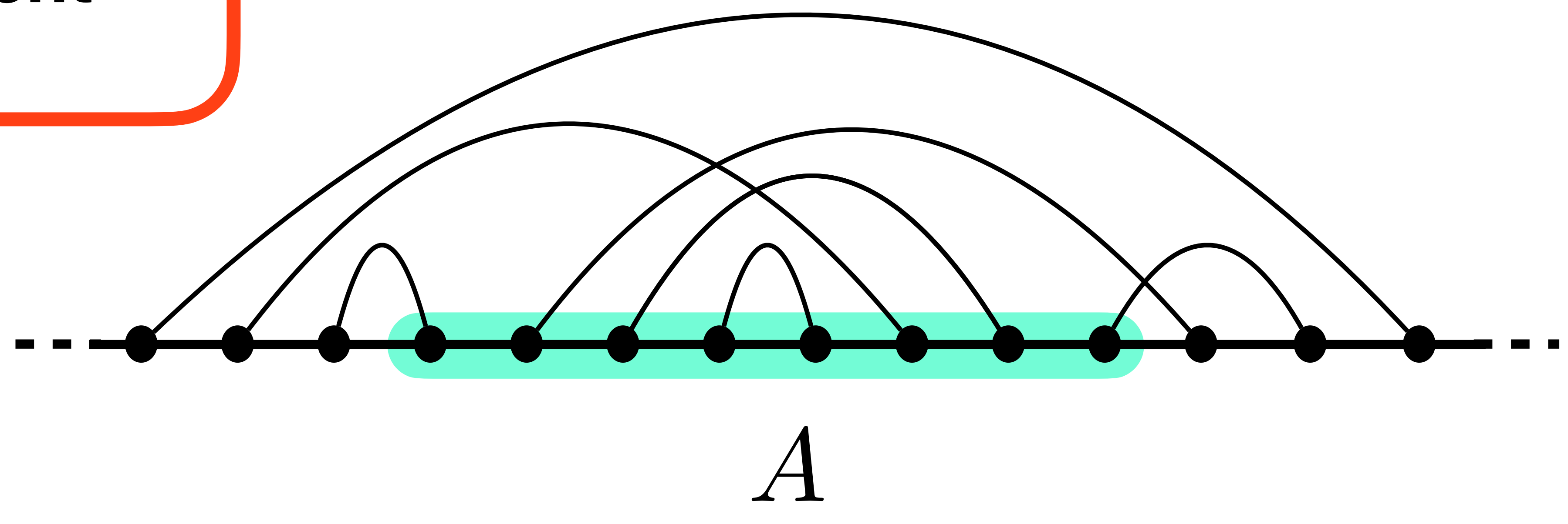


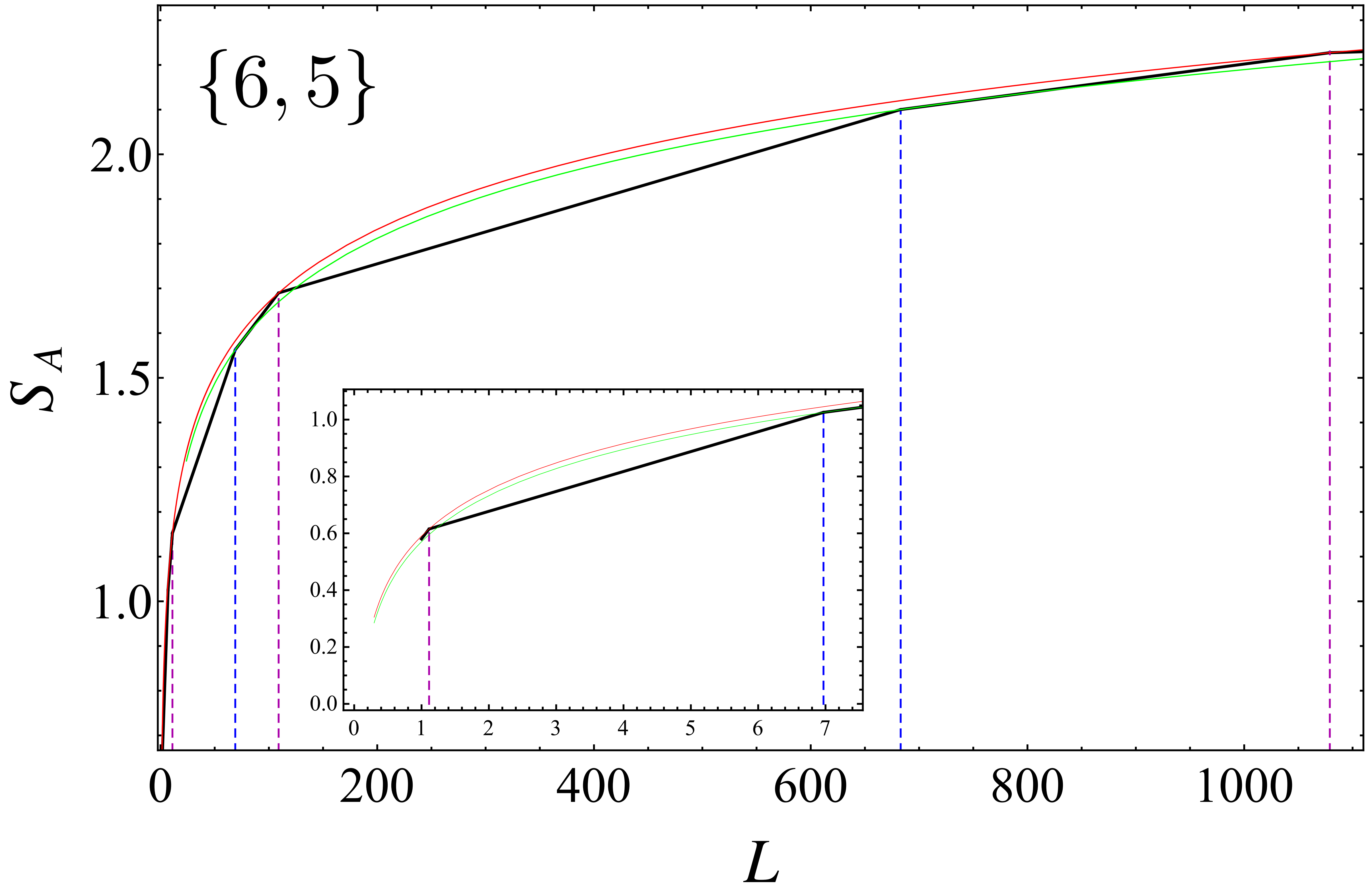


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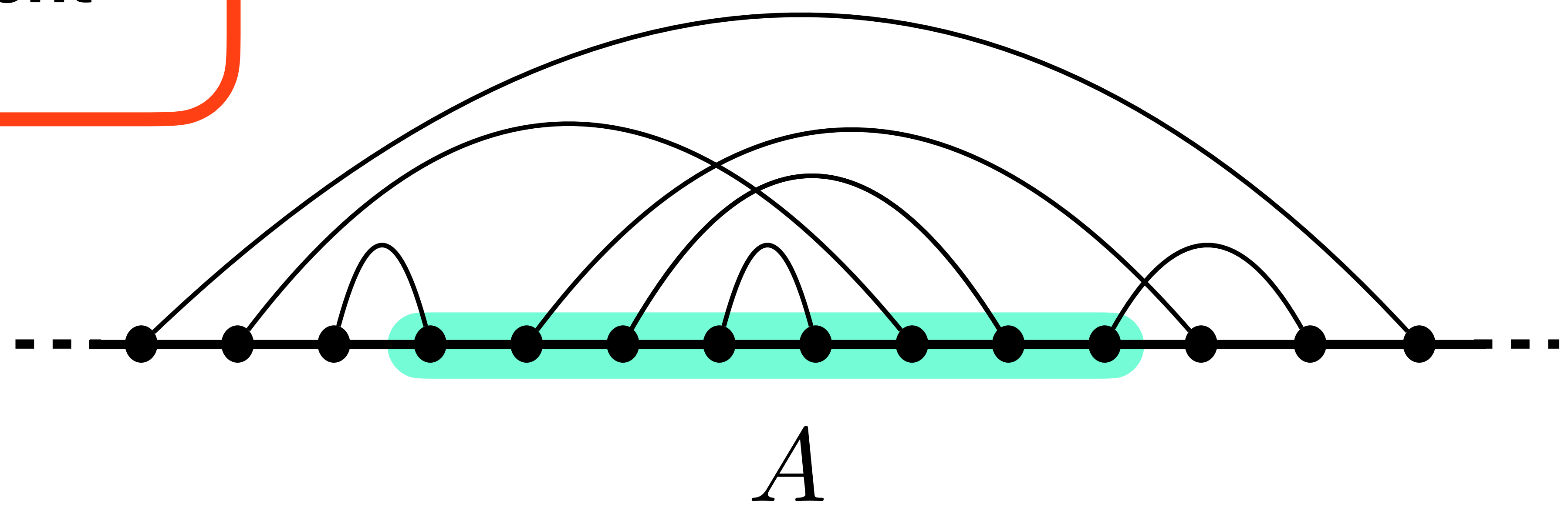


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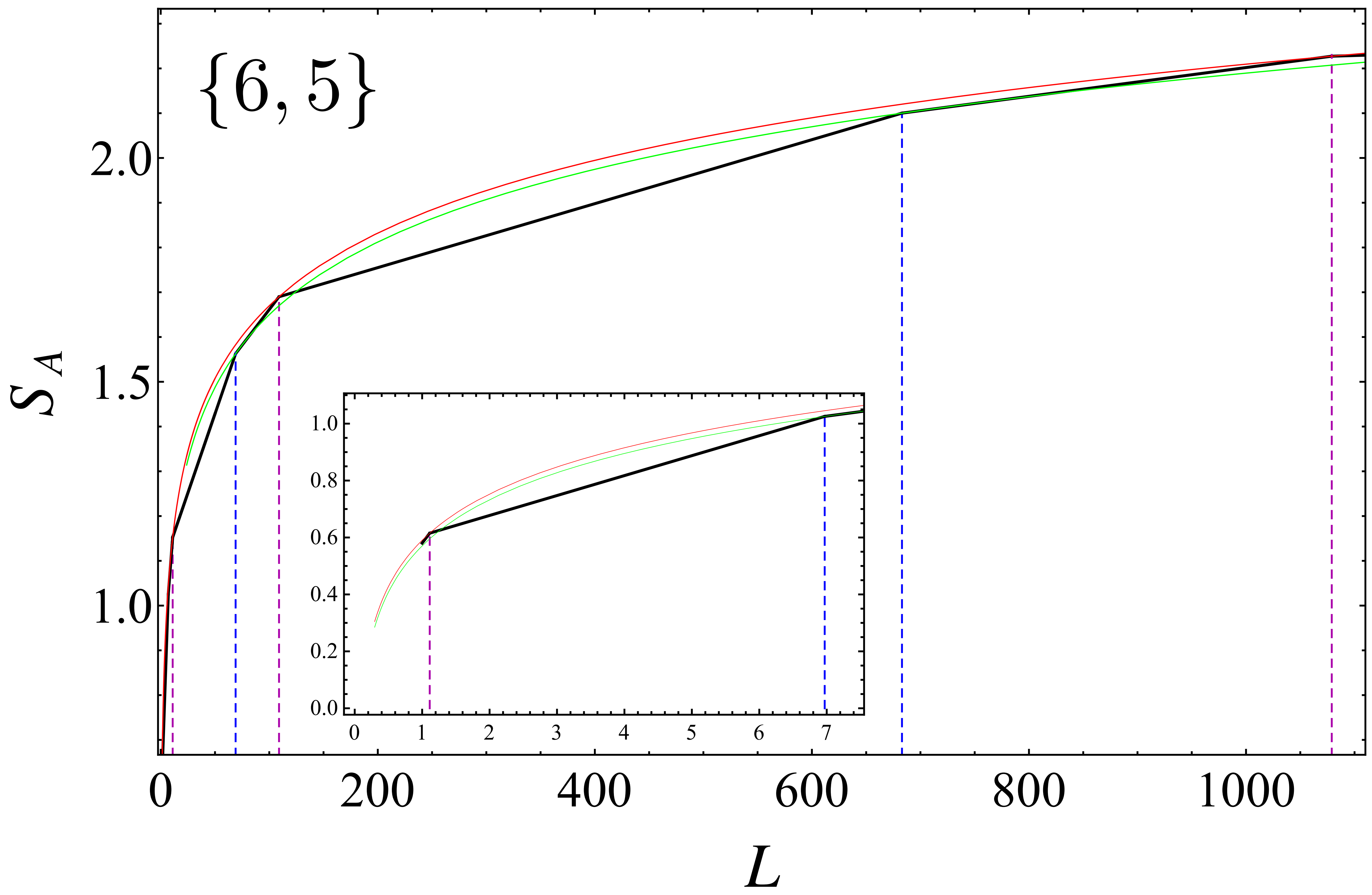


$$S_{A,\mathrm{env}} = \frac{c_{\mathrm{eff}}(p,q)}{3} \ln L + \kappa$$

The coefficient $c_{\mathrm{eff}}$ is dependent on $p$ and $q$

For $\{6,q\},\ q \geq 4:$

$$c_{\mathrm{eff}}(6,q) = \frac{6 - 2q + \sqrt{q^2 - 5q + 6}}{\ln\left(2q - 5 + 2\sqrt{q^2 - 5q + 6}\right)} \frac{6 \ln 2}{6 - 2q}$$

[Basteiro, GDG, Erdmenger, Karl, Meyer, Xian, '22]

# Outline

- Aperiodic spin chains at the boundary of hyperbolic tilings
- Ground state and entanglement properties
- Tensor networks as a bulk description from aperiodic chains
- Towards adding a metric in the bulk
- Conclusions and future perspectives

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We chose the aperiodic chain as boundary theory: how does the bulk encode information about its ground state?

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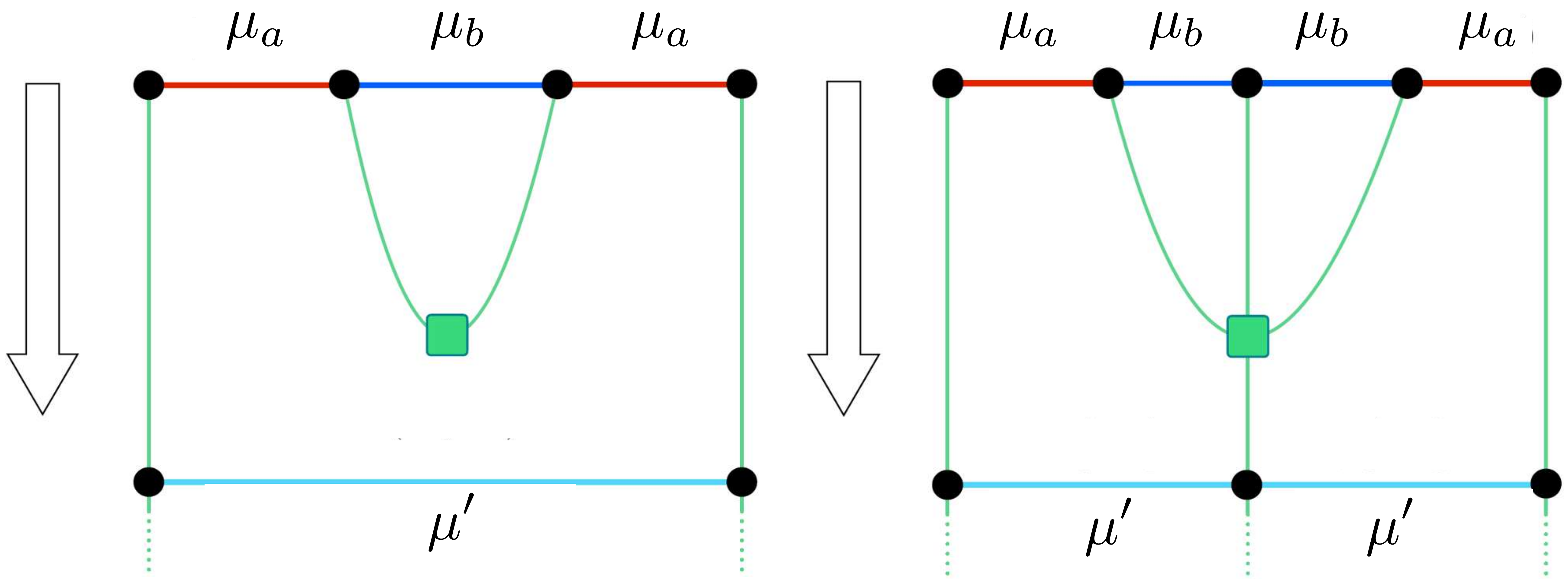


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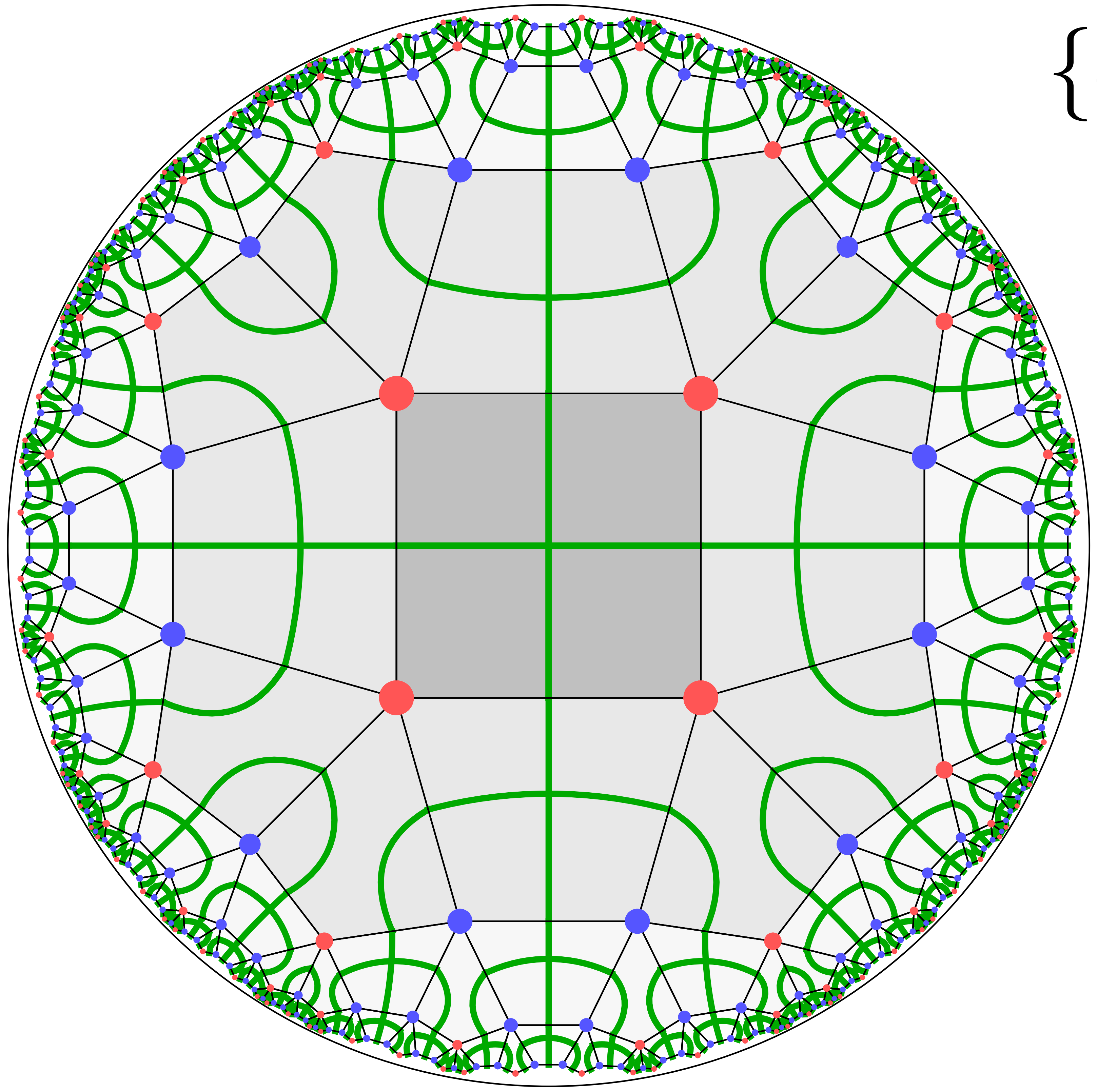
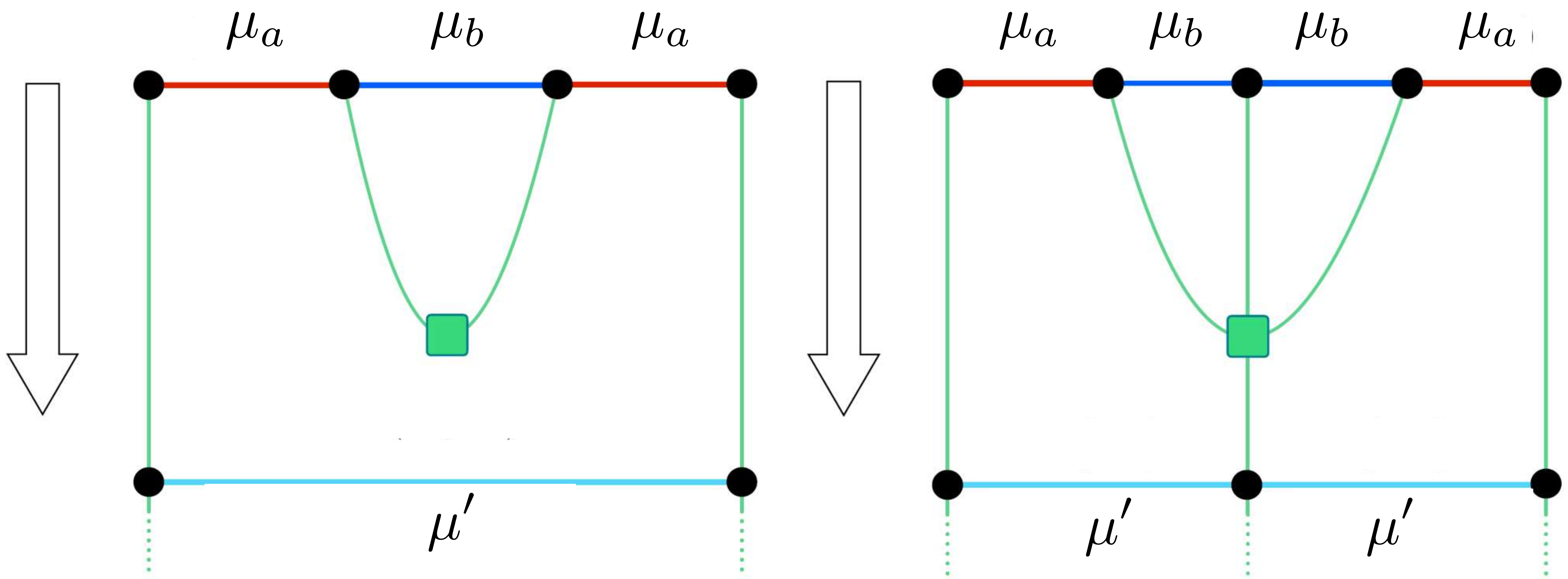


$\{4, 5\}$

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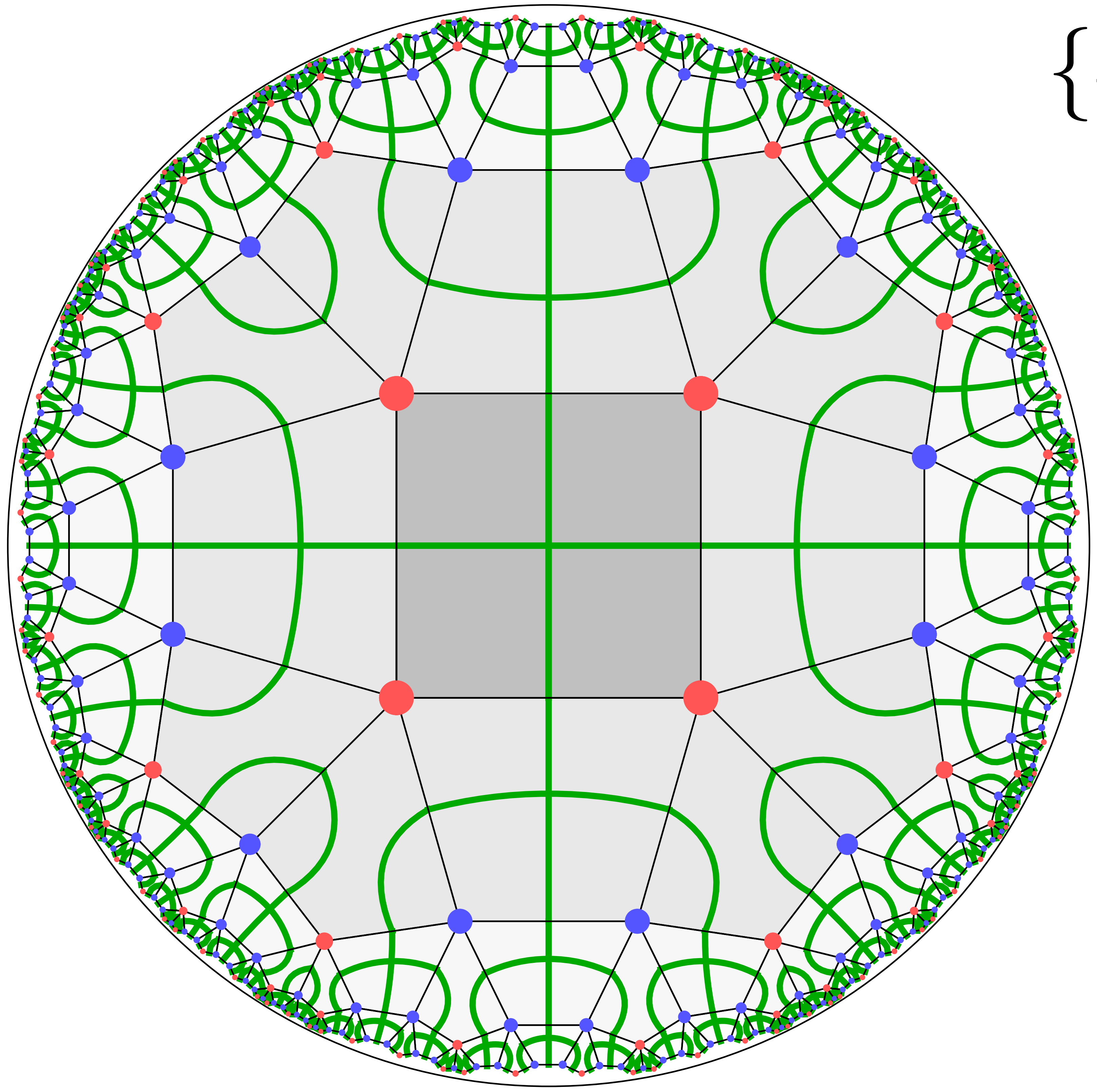

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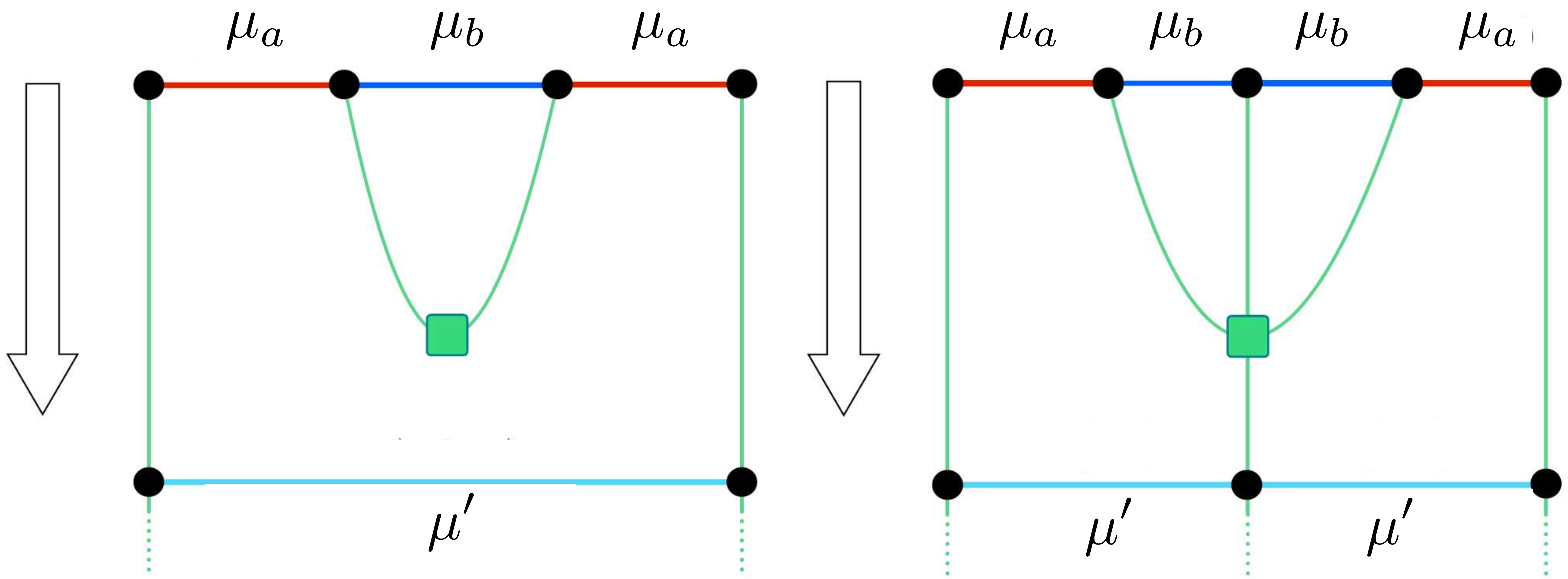


[Basteiro, GDG, Erdmenger, Karl, Meyer, Xian, ’22]

- This TN reproduces exactly the ground state of the boundary aperiodic spin chain

- It encodes the entanglement structure of the ground state of the boundary theory

**For aperiodic singlet phases**
results known through direct computation are retrieved

**No aperiodic singlet phase**
Quantitative results, such as bounds on the entanglement entropy, can be obtained

# Outline

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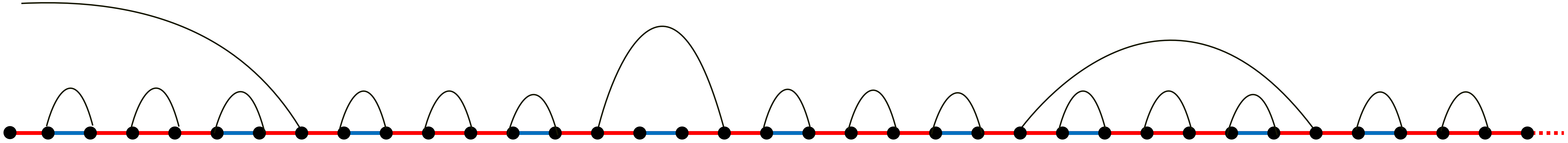
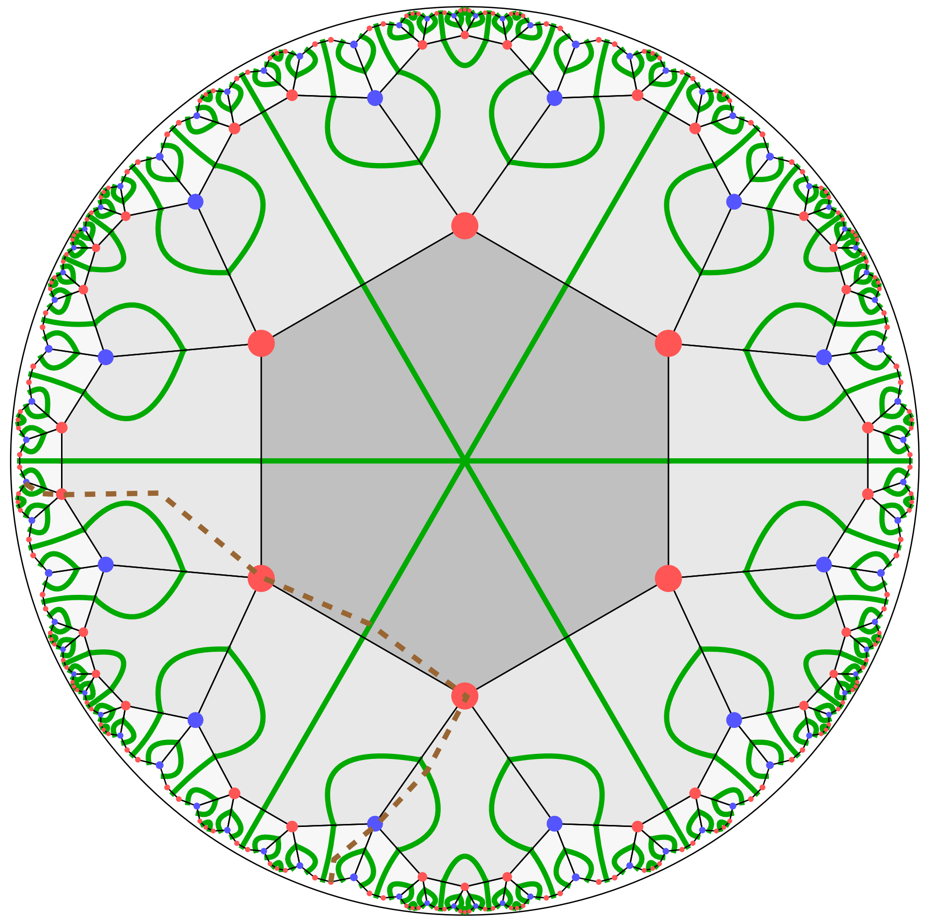
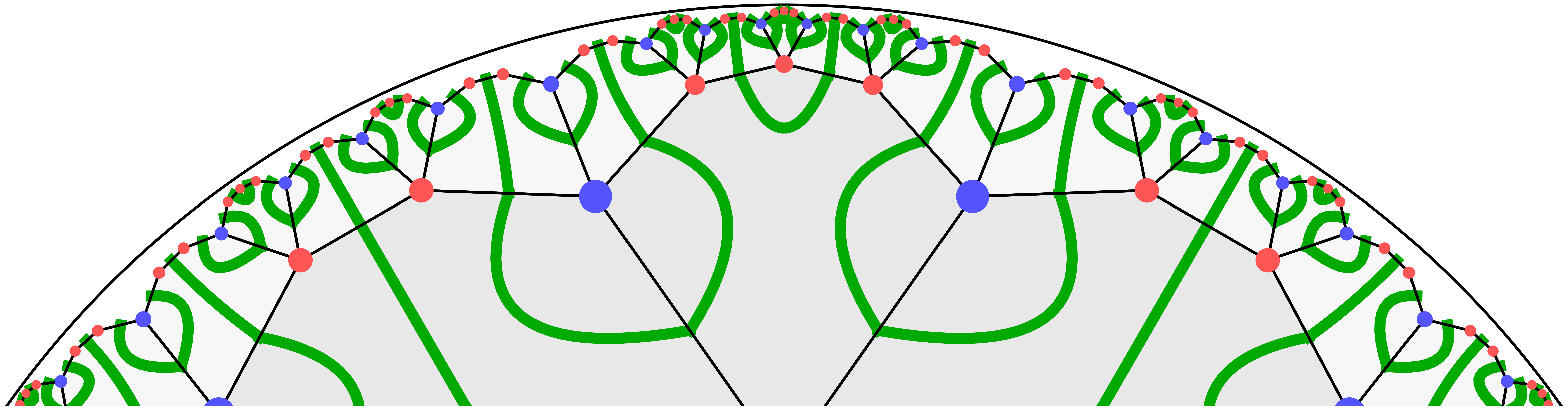
# Sachdev-Ye-Kitaev (SYK) model







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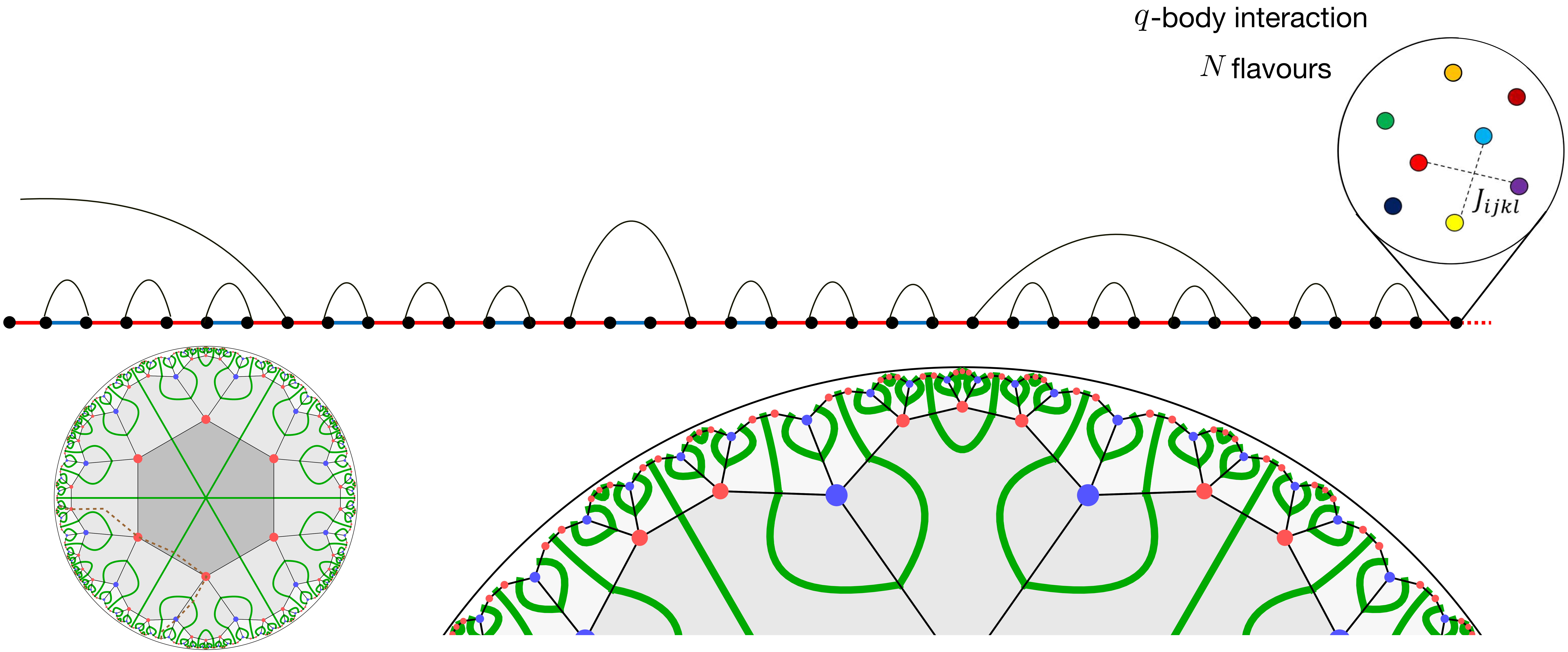


# Sachdev-Ye-Kitaev (SYK) model

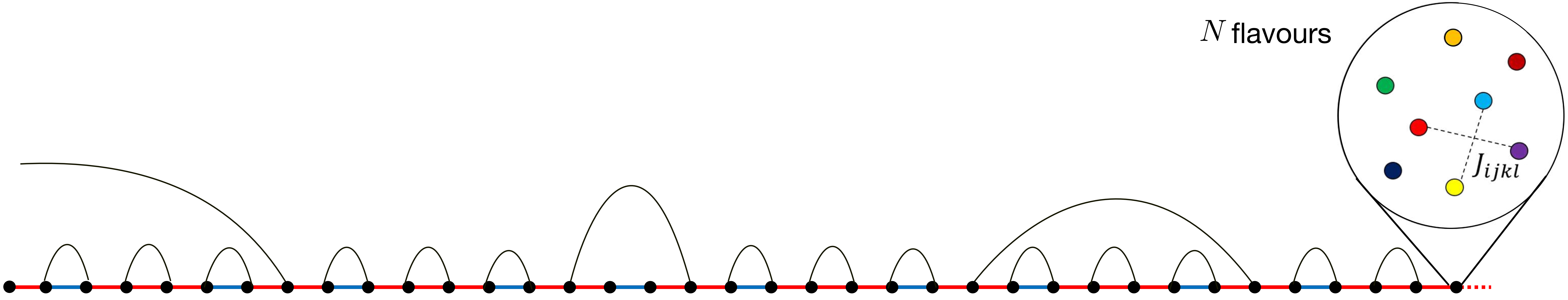
[Sachdev, Ye, '13]
[Kitaev, '15]

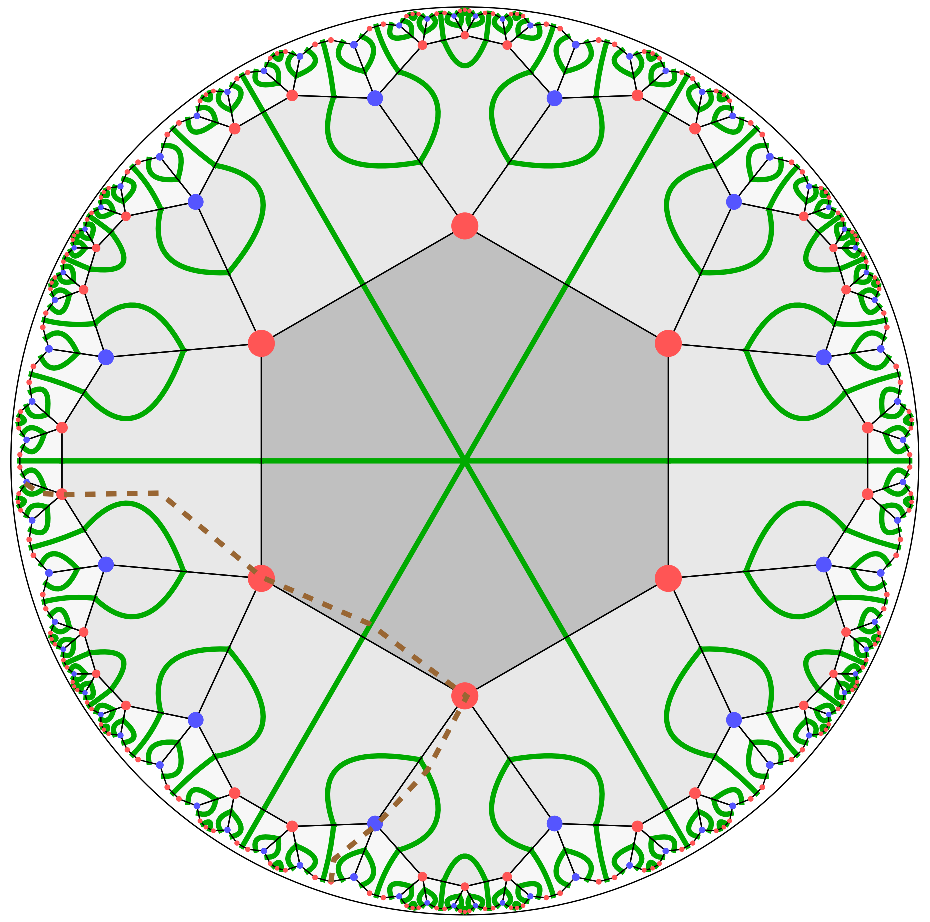
$$H = \mathrm{i}^{q/2} \sum_{1\leq j_1<j_2<\cdots<j_q\leq N} J_{j_1 j_2 \cdots j_q, x} \psi_x^{j_1} \psi_x^{j_2} \cdots \psi_x^{j_q}$$

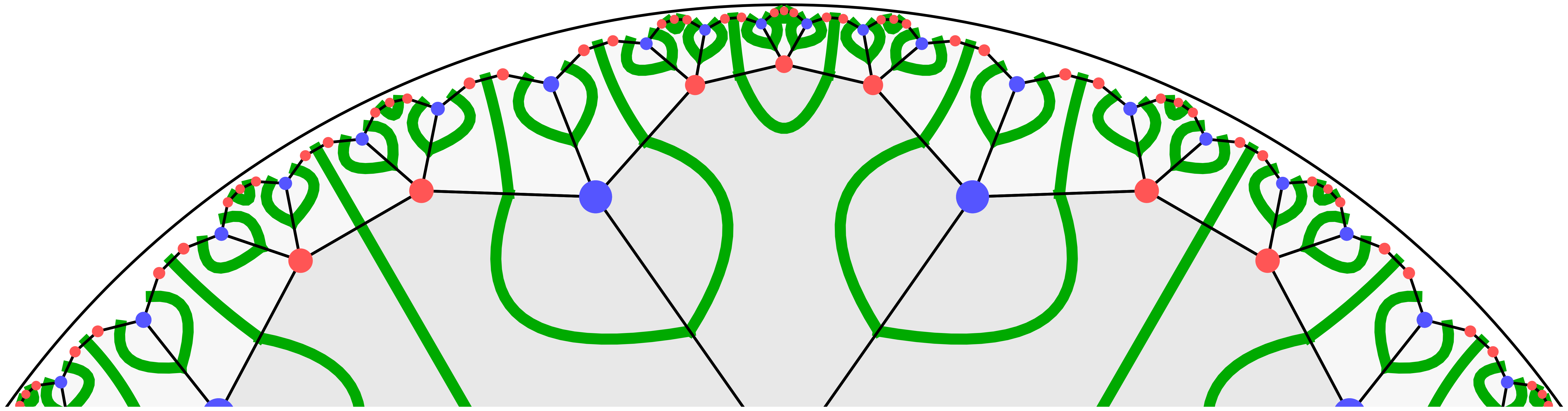
$$\langle J^2_{j_1 j_2 \cdots j_q} \rangle \propto \frac{\mathcal{J}^2}{N^{q-1}}$$

$q$-body interaction

$N$ flavours





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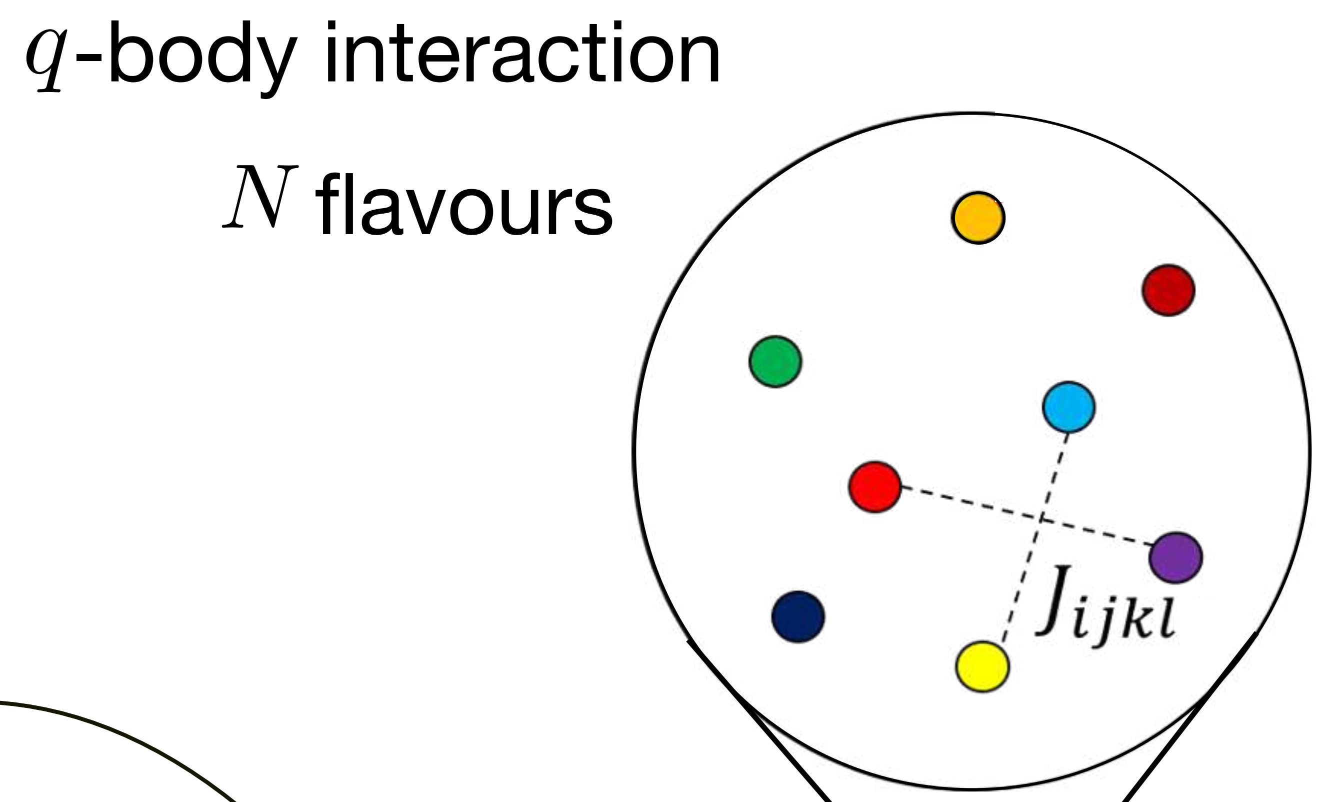
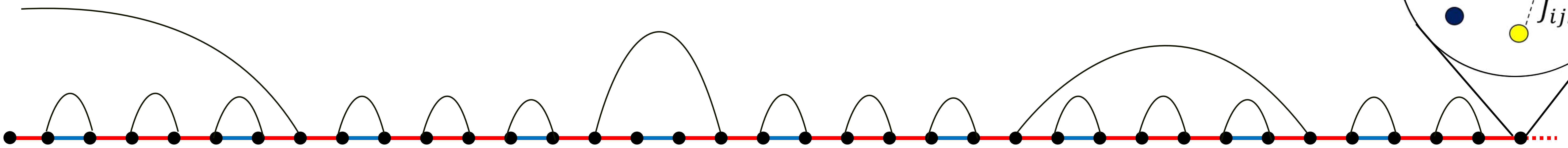
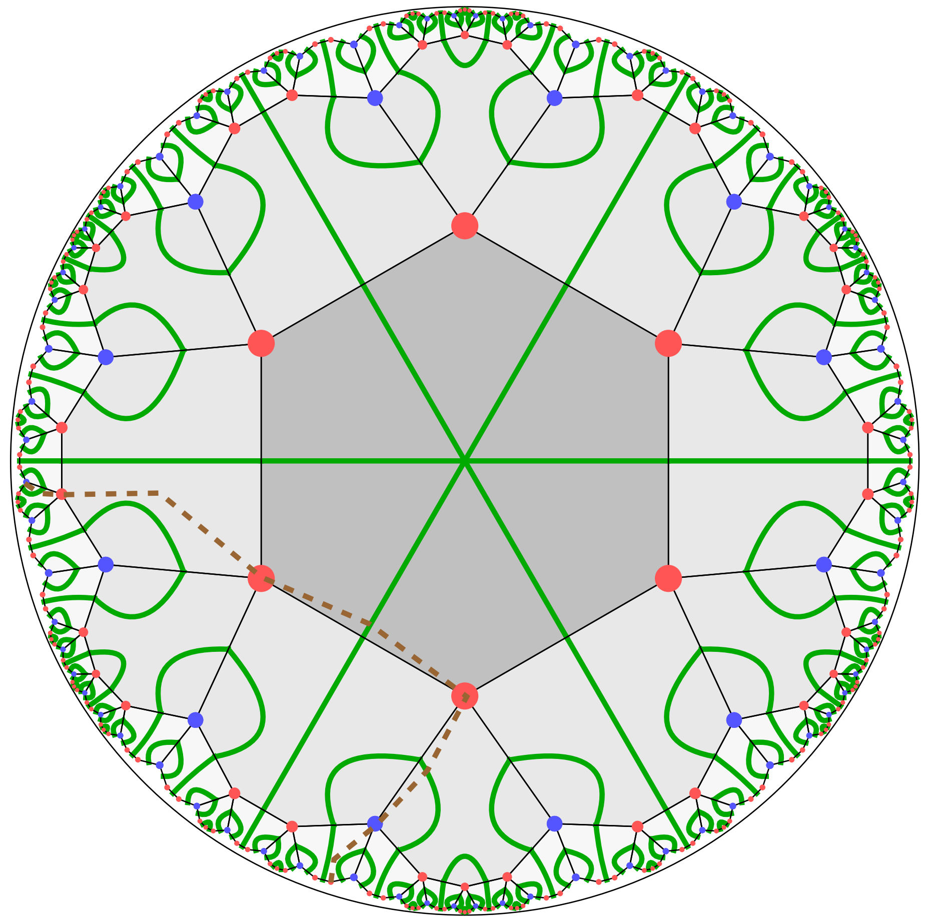
**SYK model**
0+1-dim

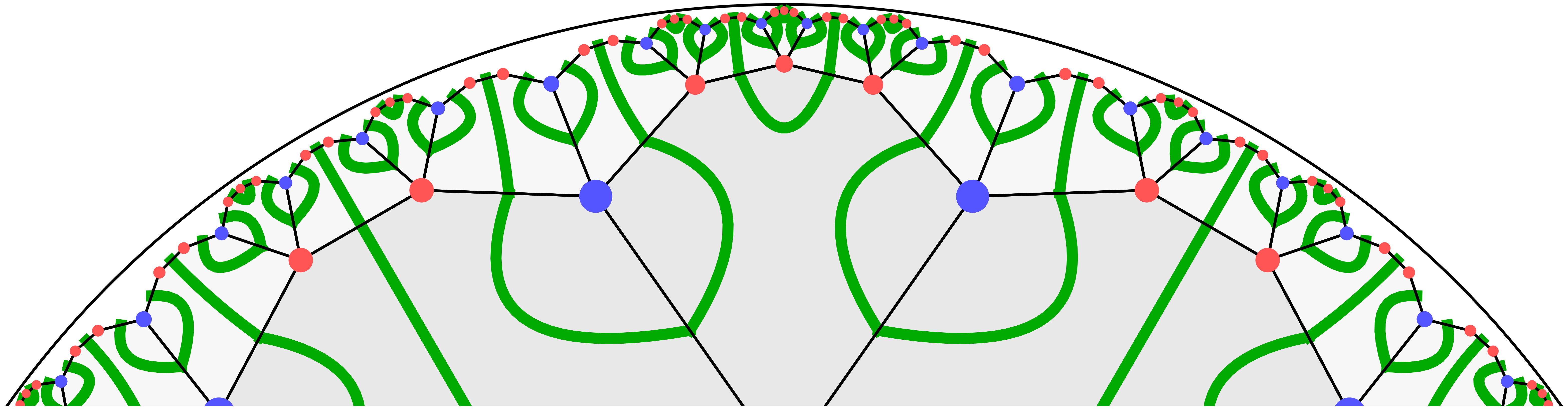
Solvable in the limit $N \gg 1$

Holographically dual to Jackiw-Tetelboim (JT) gravity in nearly AdS2 [Maldacena, Stanford, '16]







# Aperiodic Sachdev-Ye-Kitaev (SYK) chain

[Basteiro, GDG, Erdmenger, Meyer, Xian, ’25]

$$H = \sum_{x=1}^{L} \left( \mathrm{i}^{q/2} \sum_{1 \le j_1 < j_2 < \cdots < j_q \le N} s_x J_{j_1 j_2 \cdots j_q, x} \psi_x^{j_1} \psi_x^{j_2} \cdots \psi_x^{j_q} + \mathrm{i} \sum_{1 \le j \le N} \mu_x \psi_x^j \psi_{x+1}^j \right) \qquad \langle J^2_{j_1 j_2 \cdots j_q} \rangle \propto \frac{\mathcal{J}^2}{N^{q-1}}$$

$\mu_x = \mu_a, \mu_b$ aperiodic modulation of the $\{6,4\}$ tiling

**SYK model**
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Solvable in the limit $N \gg 1$

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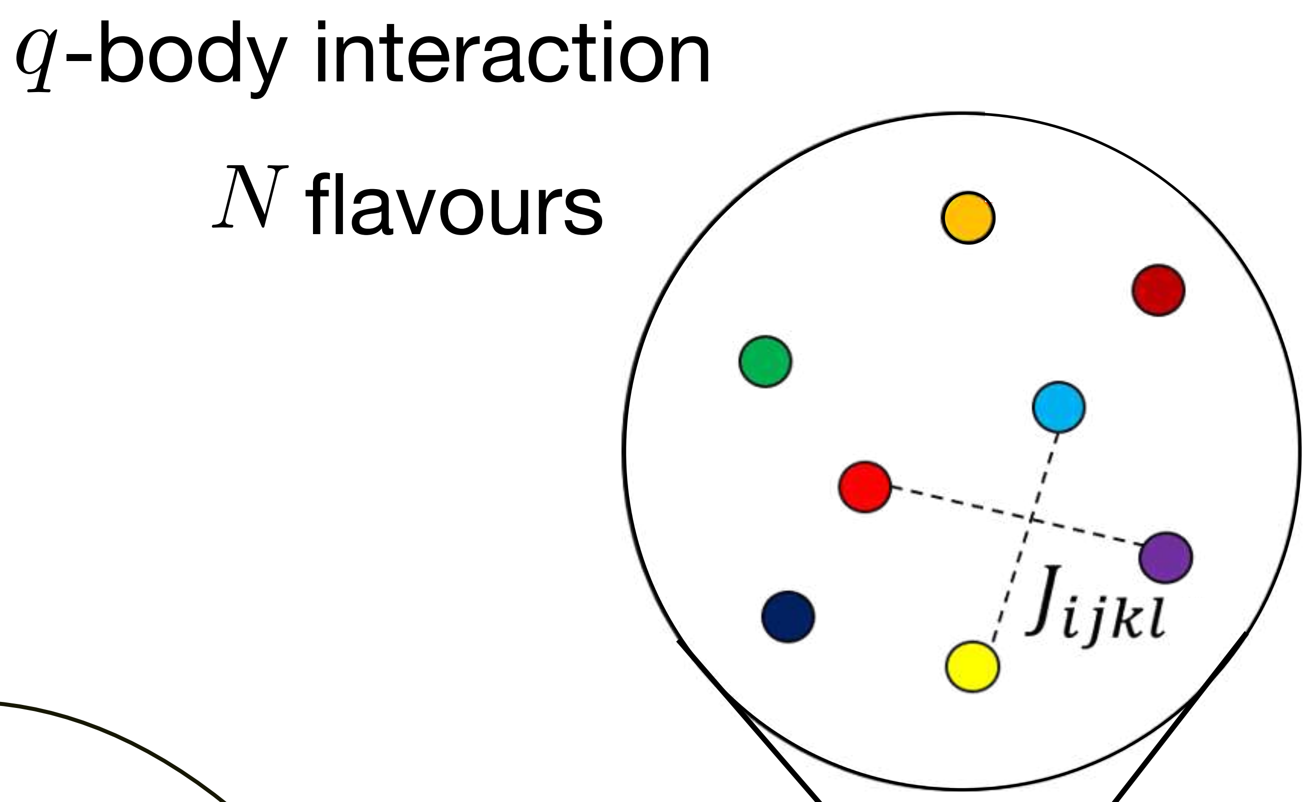
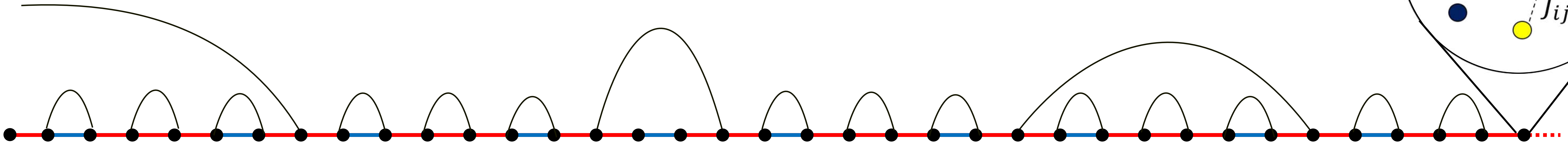
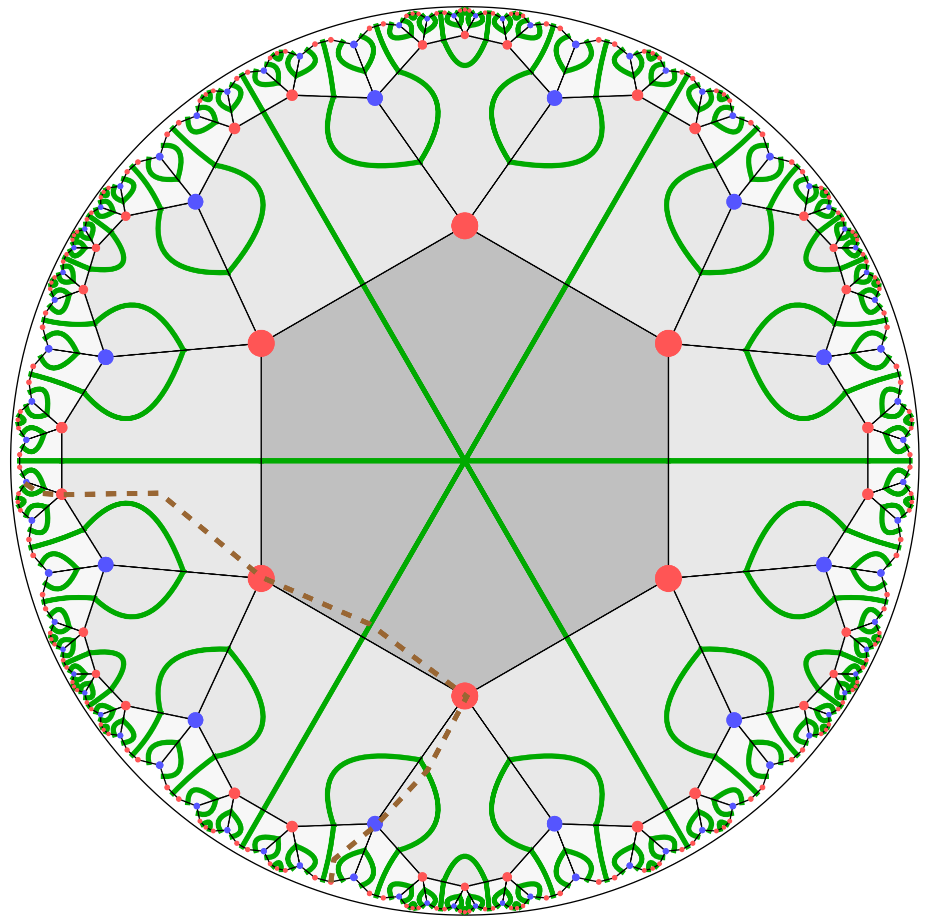
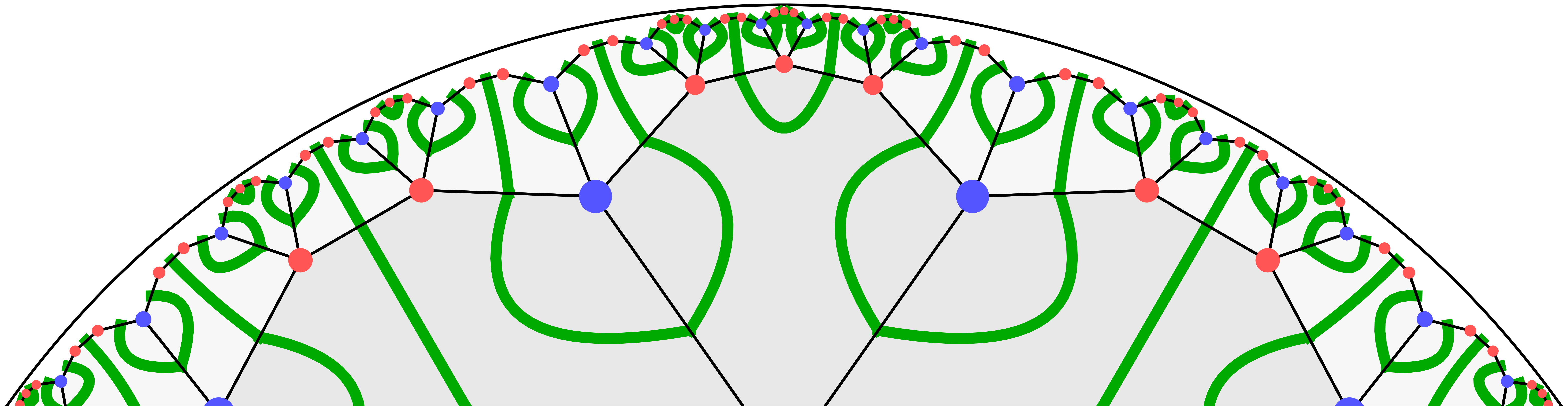


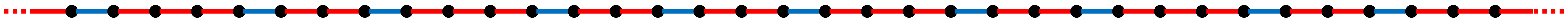




# SDRG on aperiodic SYK chain

[Basteiro, GDG, Erdmenger, Meyer, Xian, '25]

$N \to \infty \qquad q \to \infty \qquad \mu_a/\mu_b \ll 1$ SDRG



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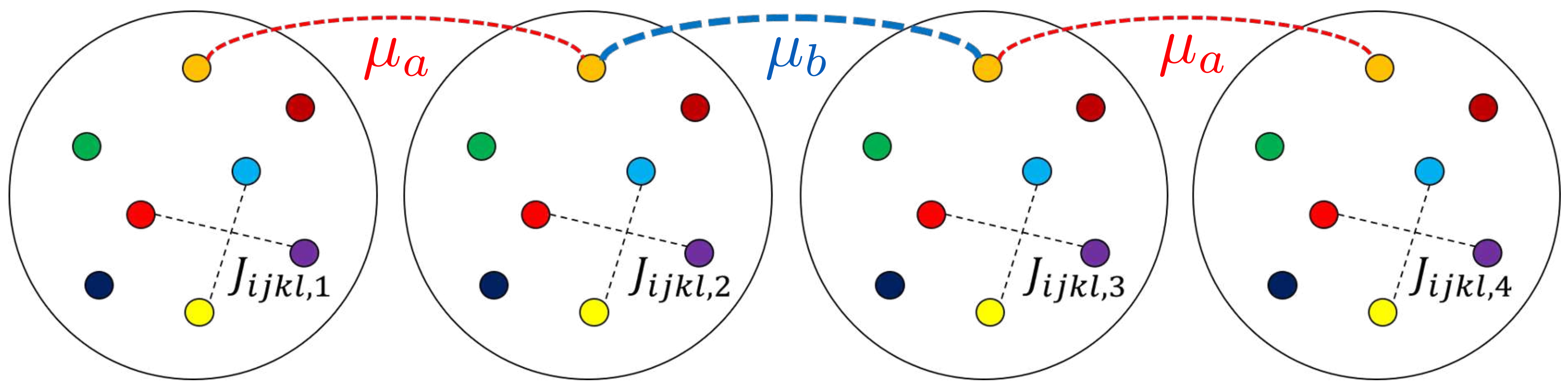



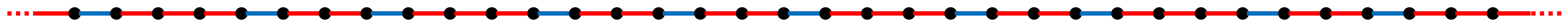

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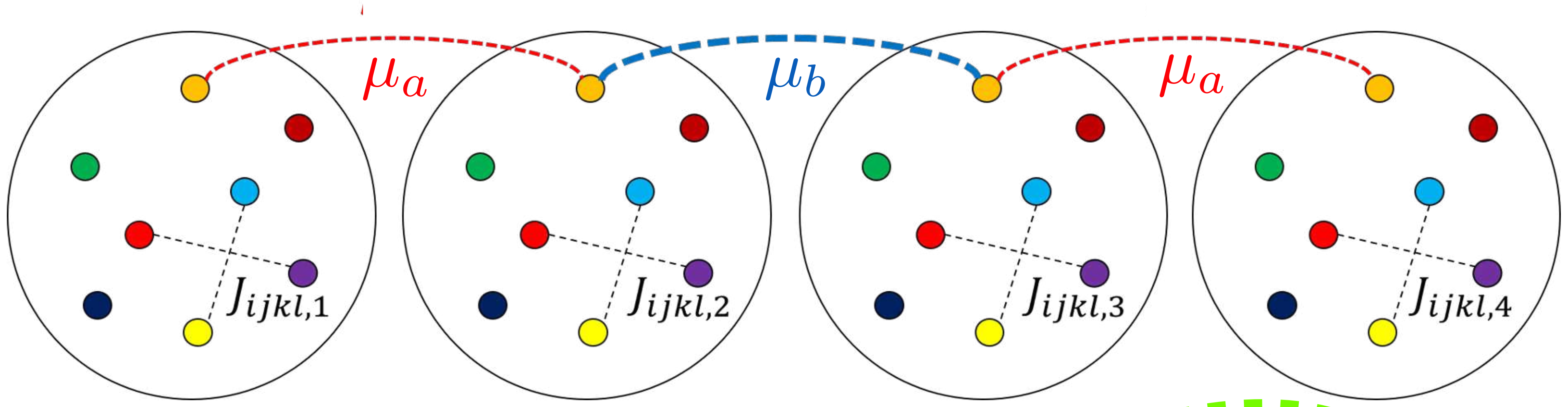
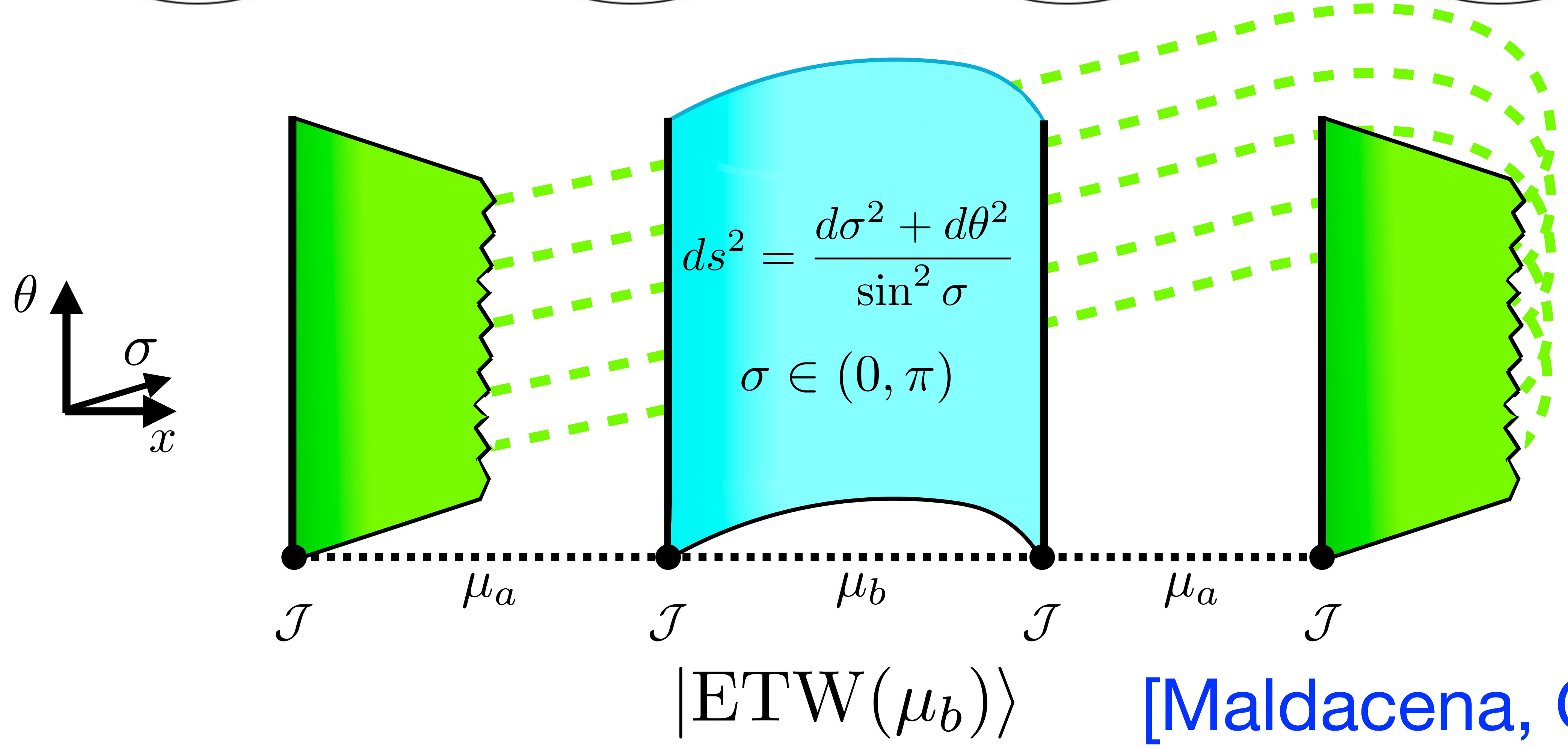
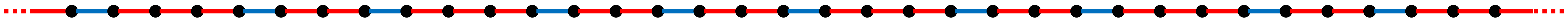



[Maldacena, Qi, '18]



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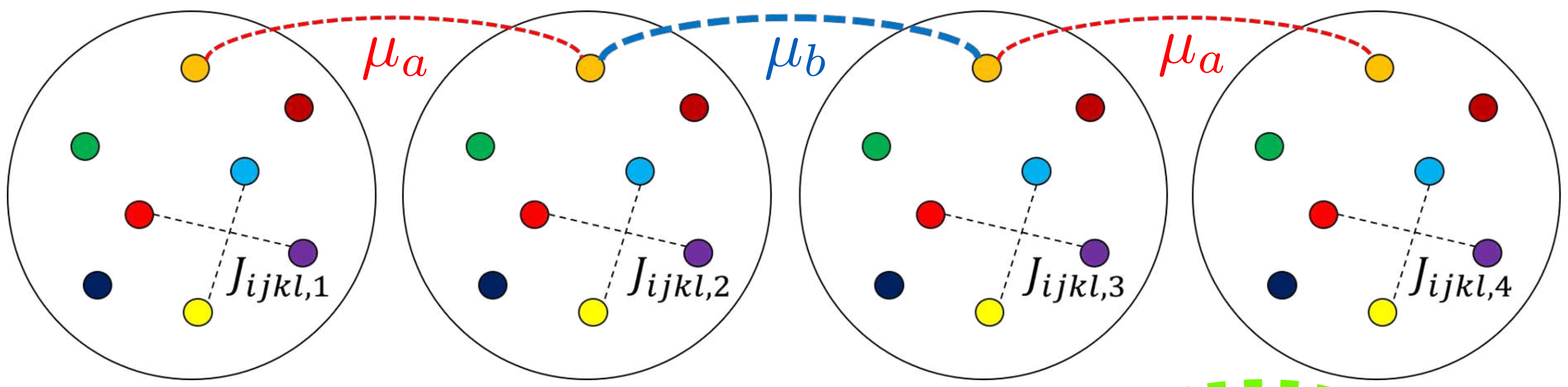


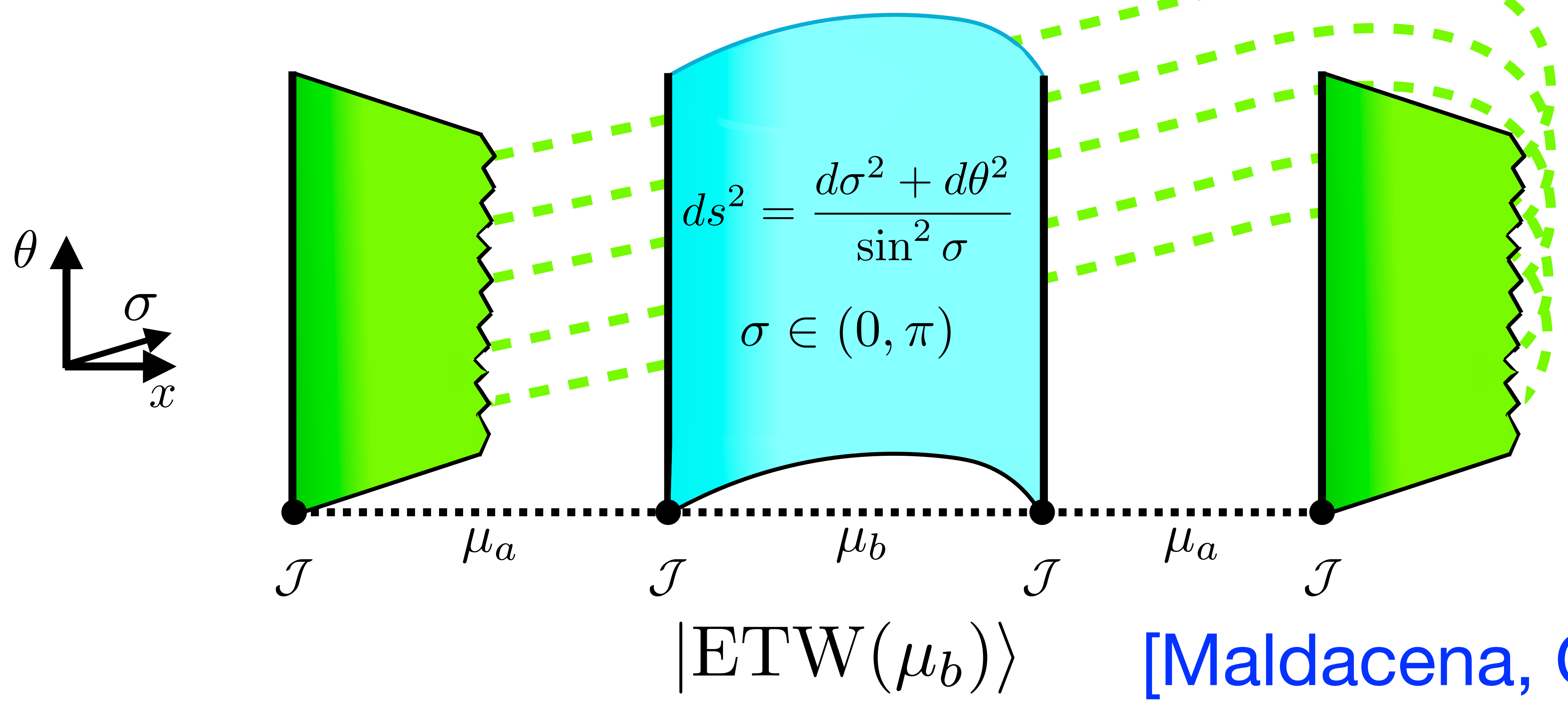


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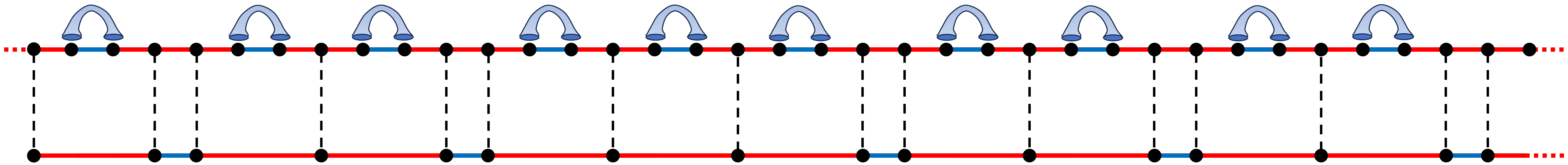

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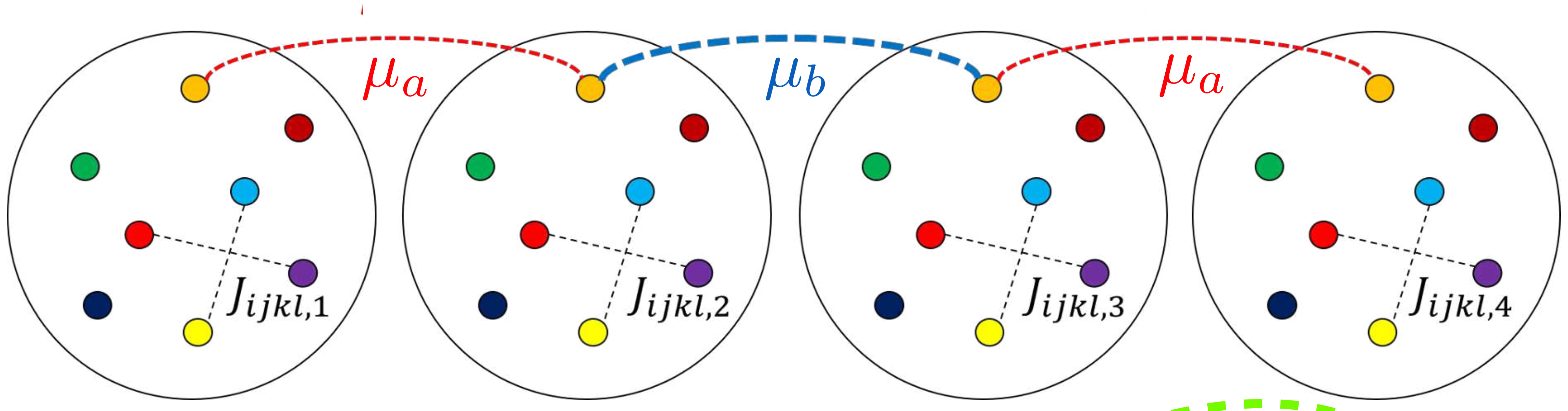


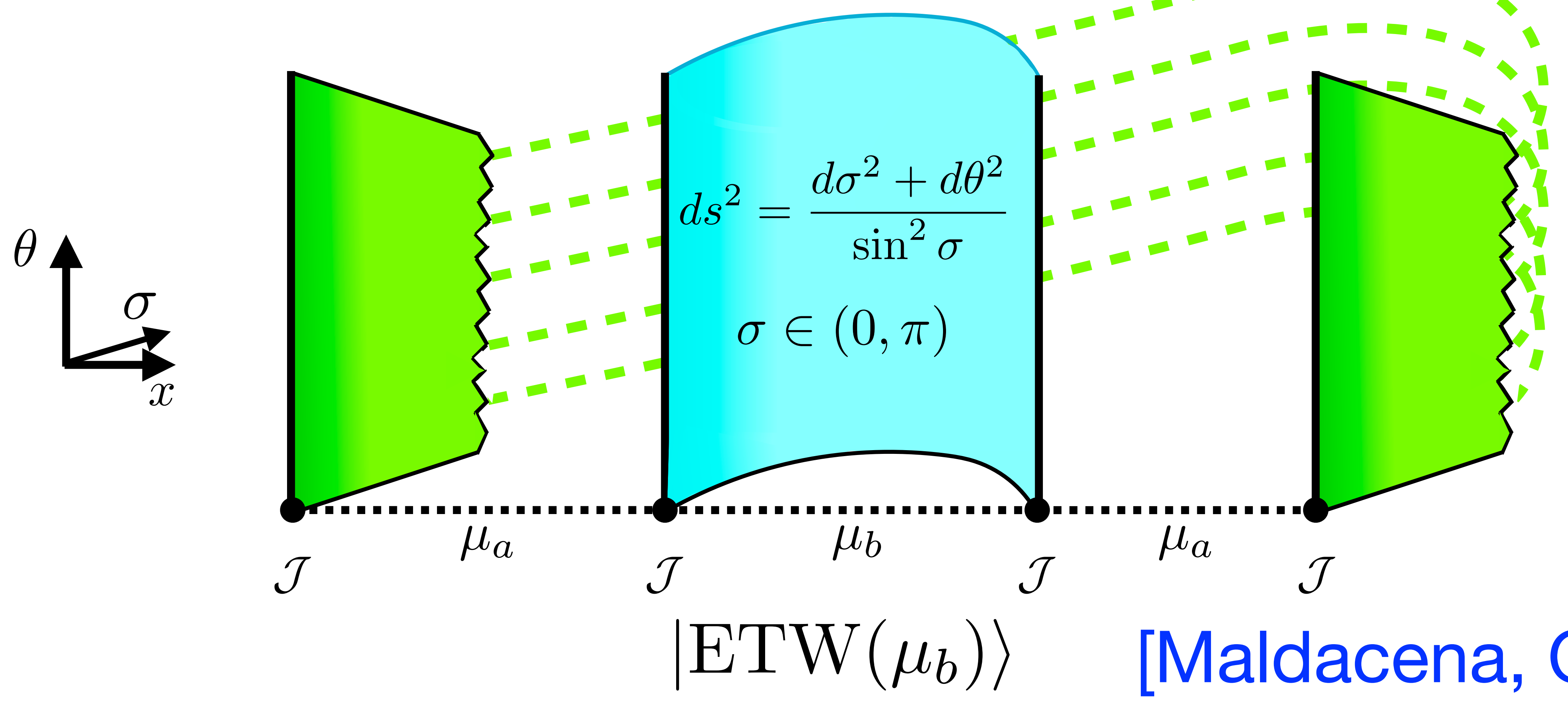


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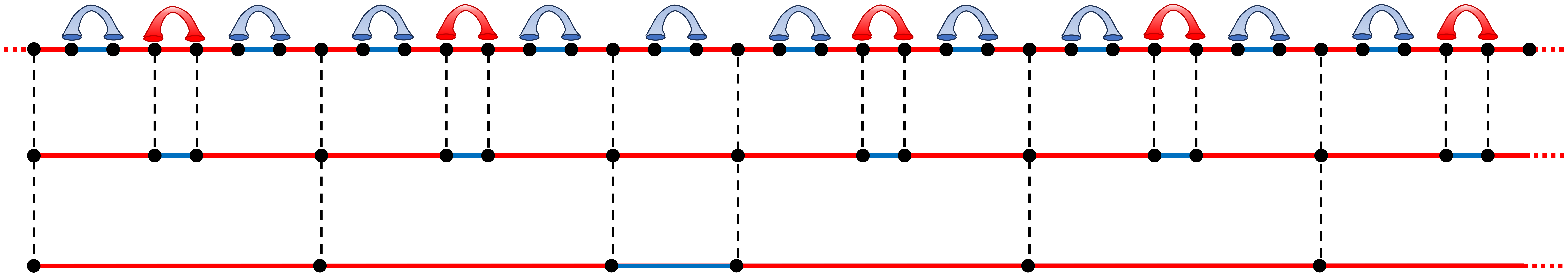

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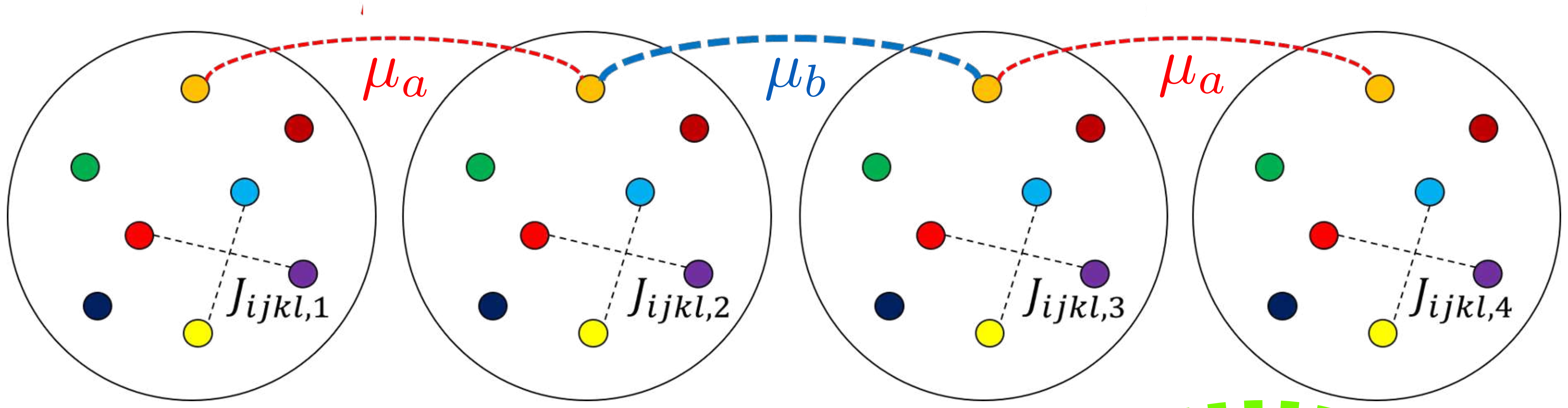
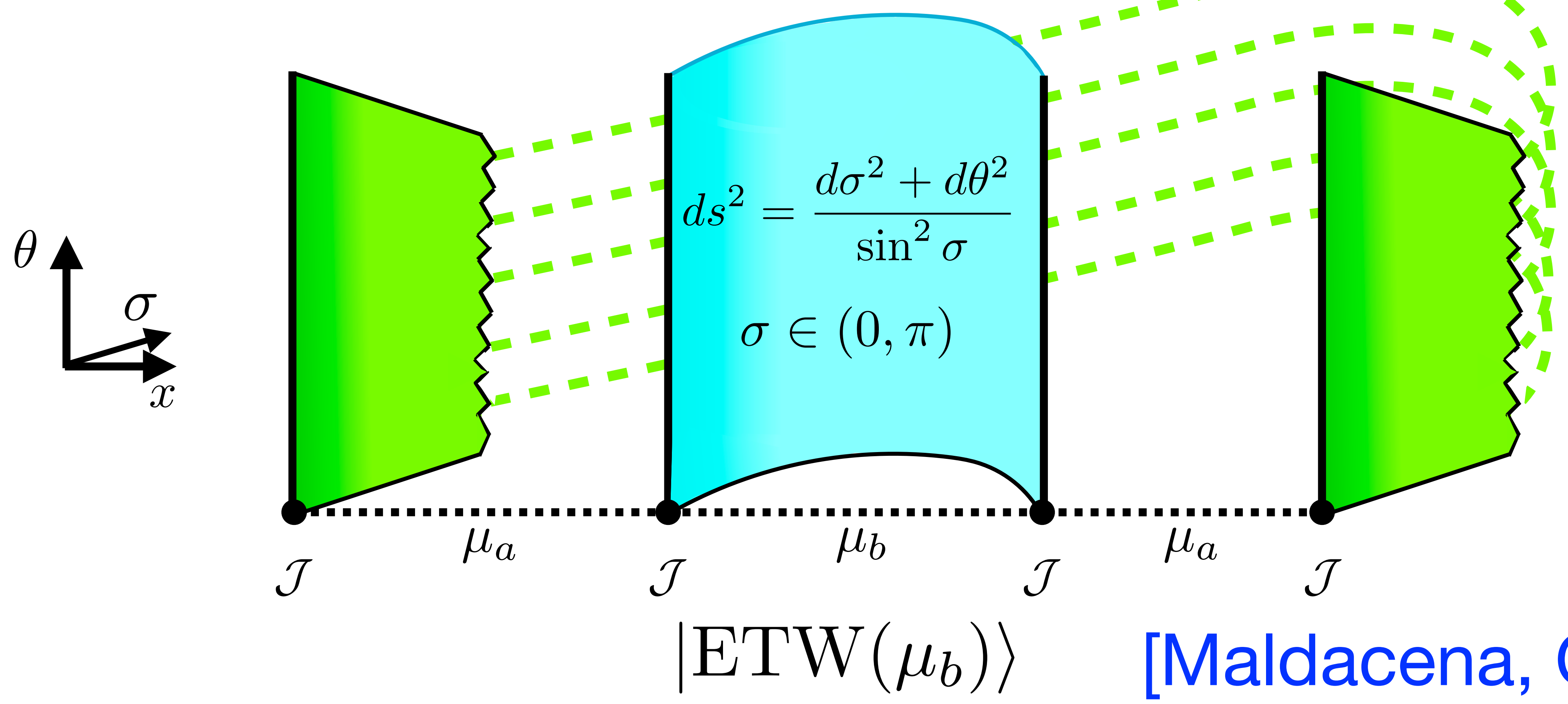
$N \to \infty \qquad q \to \infty \qquad \mu_a/\mu_b \ll 1 \xrightarrow{} \text{SDRG}$

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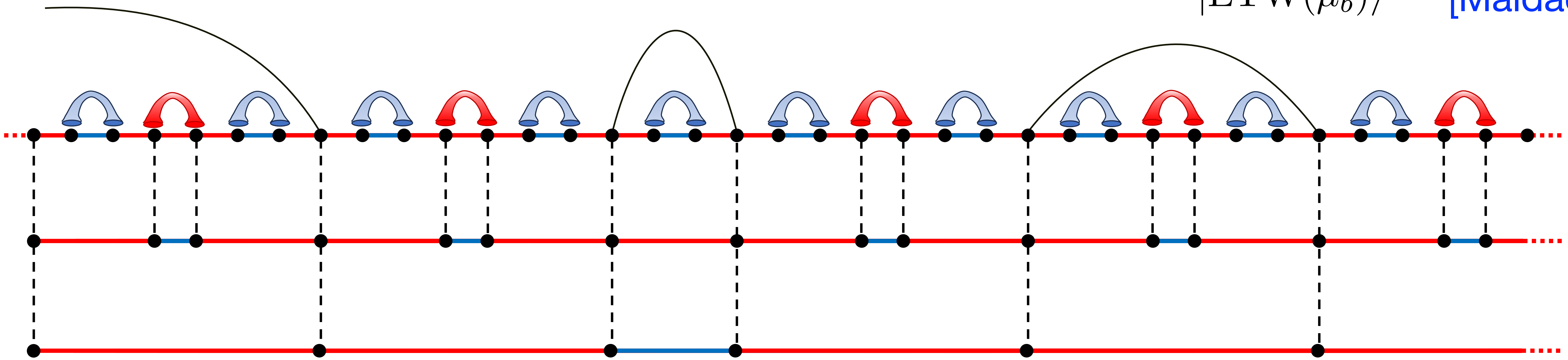

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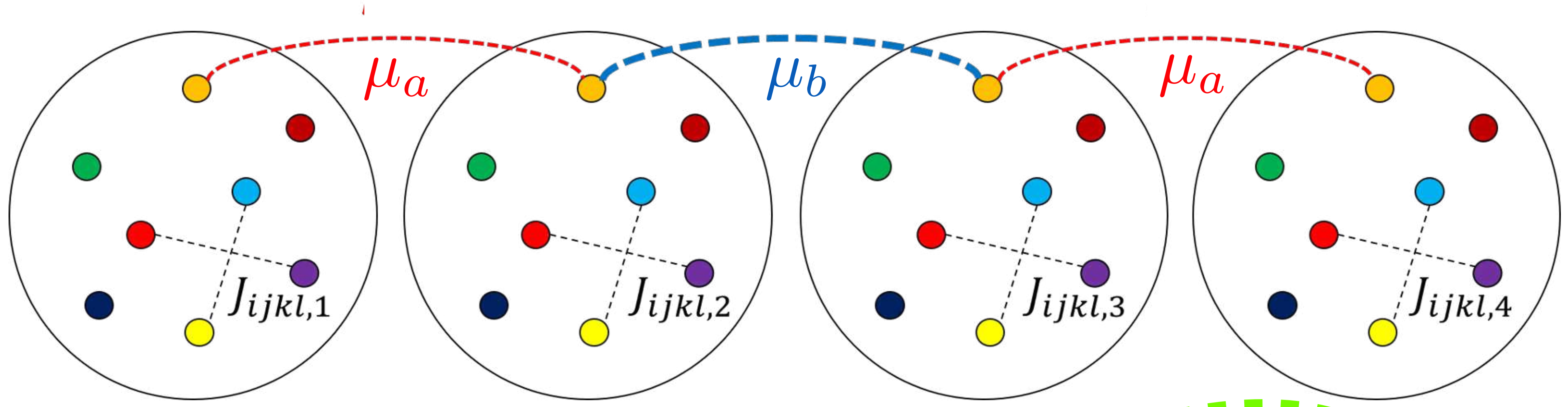
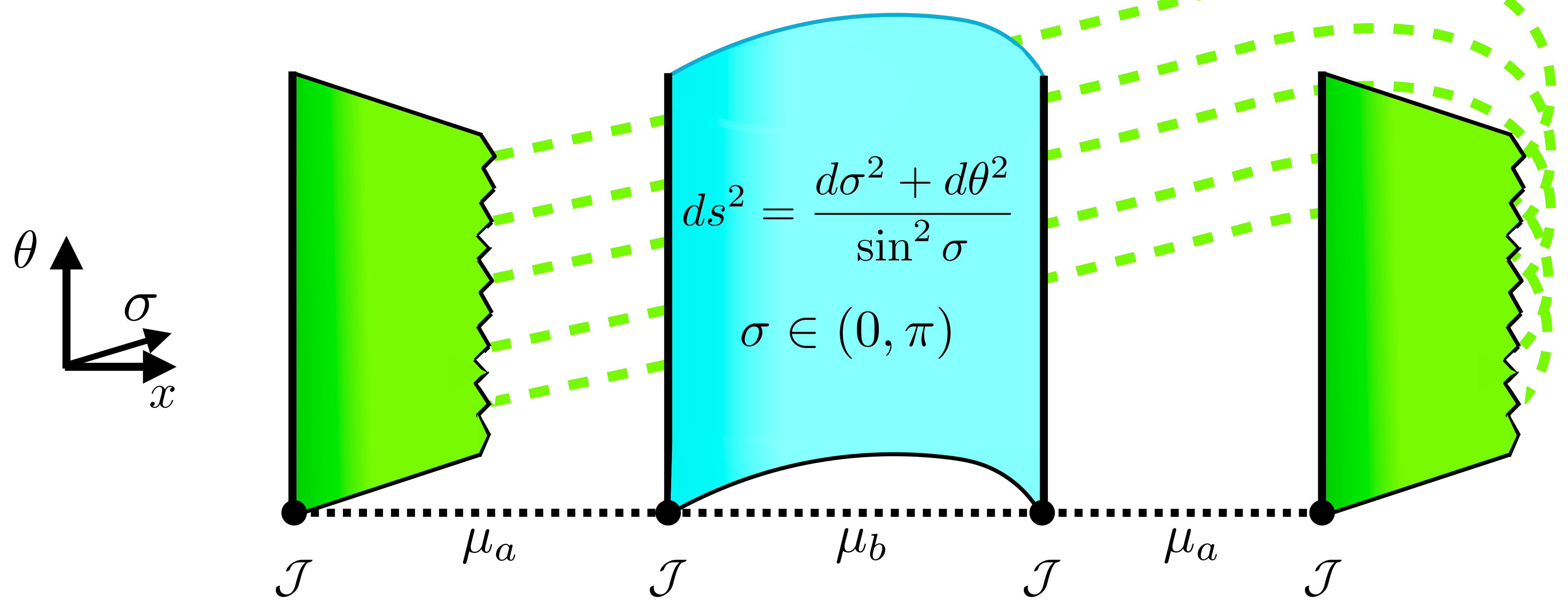
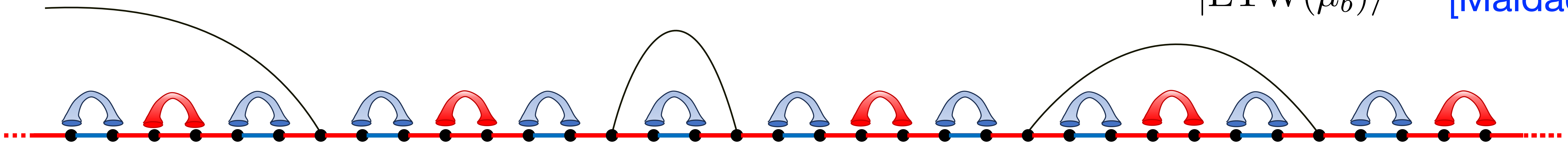



$|\mathrm{ETW}(\mu_b)\rangle$

[Maldacena, Qi, ’18]

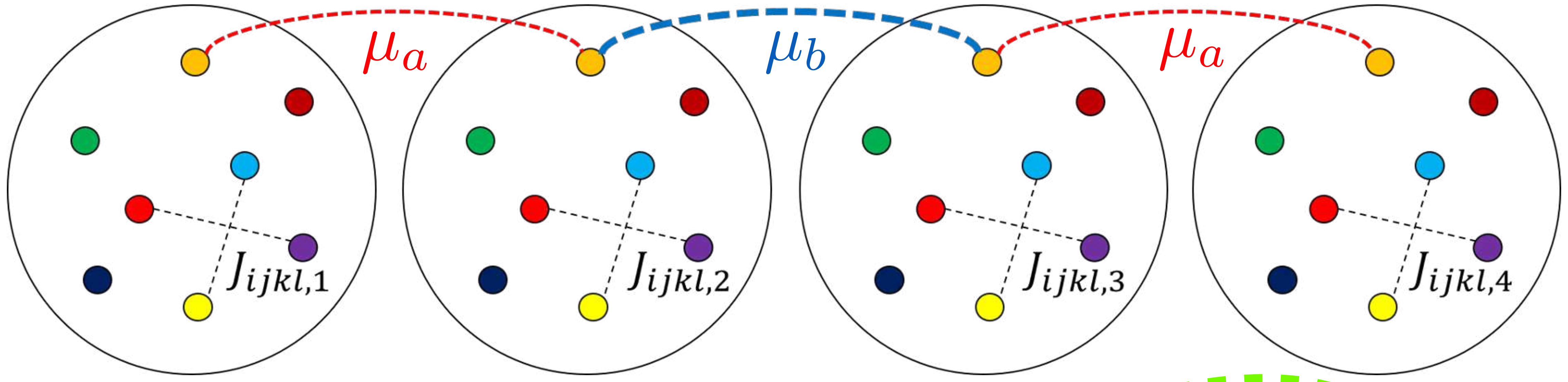


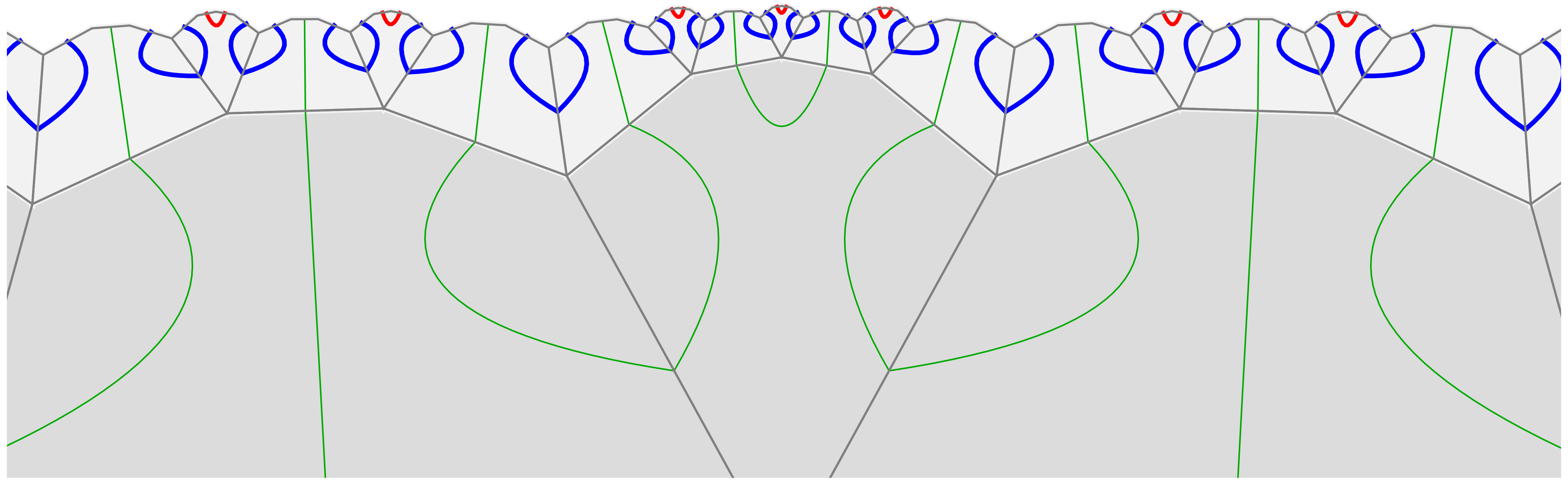
$$|\mathrm{GS}\rangle = \bigotimes_{s_1 \in G_1} |\mathrm{ETW}_{s_1}(\mu_b)\rangle \bigotimes_{s_2 \in G_2} |\mathrm{ETW}_{s_2}(\mu_a)\rangle \bigotimes_{s \in G_{i>2}} |\mathrm{EPR}_s\rangle$$

# SDRG on aperiodic SYK chain

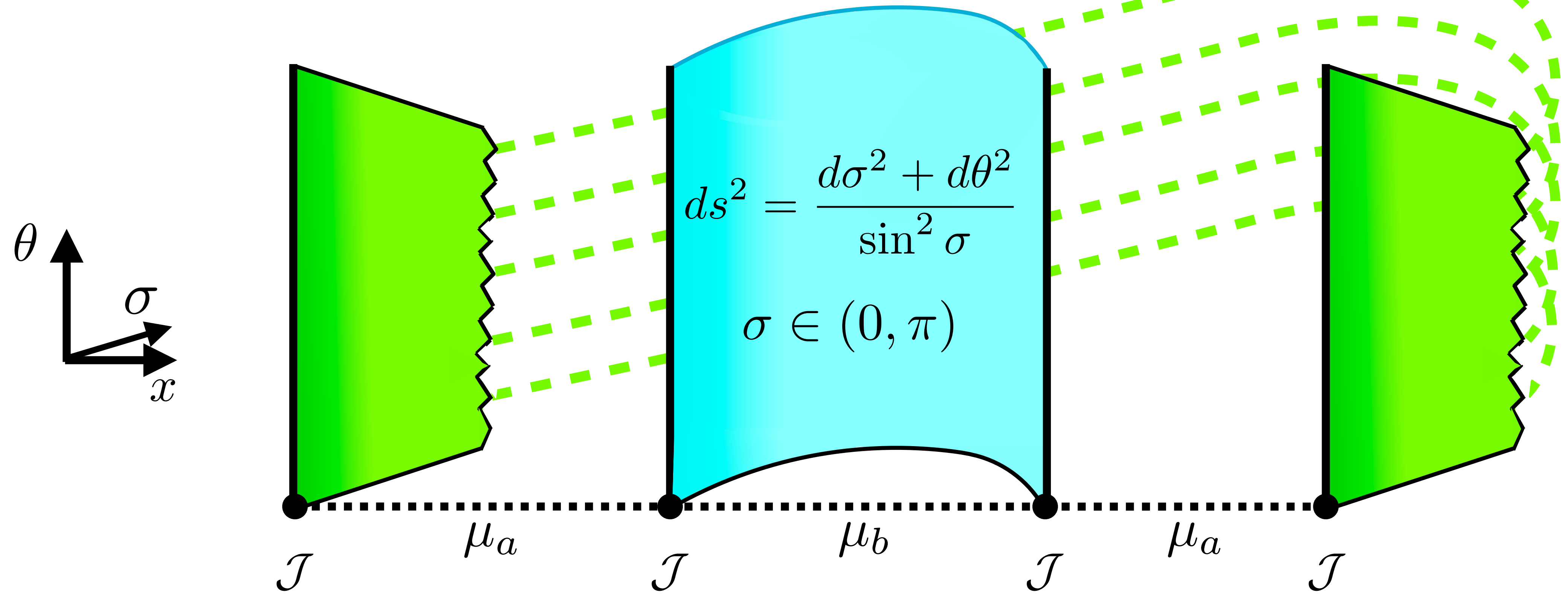
[Basteiro, GDG, Erdmenger, Meyer, Xian, '25]

$N \to \infty \quad q \to \infty \quad \mu_a/\mu_b \ll 1$ SDRG



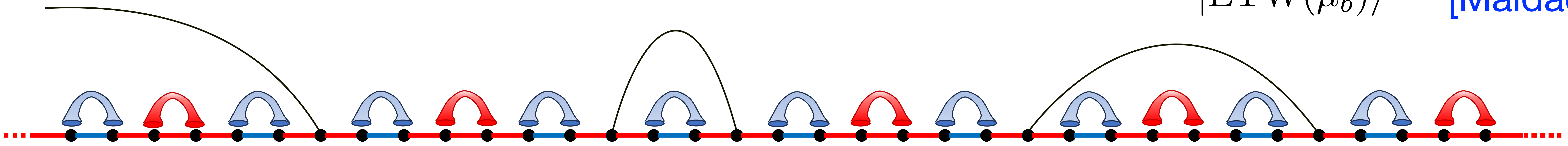



Metric description only close to the boundary




$|\mathrm{ETW}(\mu_b)\rangle$

[Maldacena, Qi, '18]



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# Outline

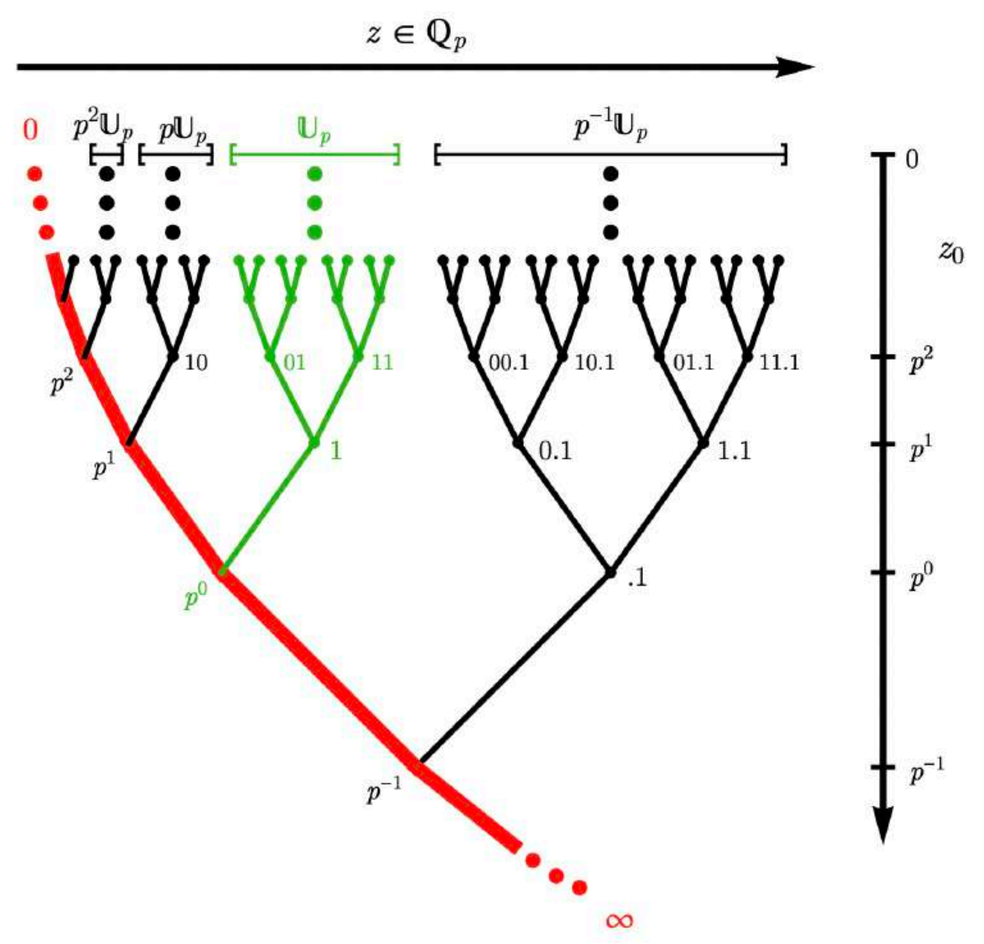
- Aperiodic spin chains at the boundary of hyperbolic tilings
- Ground state and entanglement properties
- Tensor networks as a bulk description from aperiodic chains
- Towards adding a metric in the bulk
- Conclusions and future perspectives

# Summary

- The inflation procedure identifies an aperiodic sequence at the boundary of $\{p, q\}$ tilings
- The ground state of a certain class of aperiodic chains is an aperiodic singlet phase
- Entanglement entropy of the chain through SDRG
- Exploiting SDRG we construct a tensor network in the bulk
- Adding SYK interactions, near-boundary regions with a sequence of factorized wormholes

# Future perspectives

Figure from [Gubser, Knaute, Parikh, Samberg, Witaszczyk ’16]




- Towards a bulk theory, edge-length-dynamics models in regular hyperbolic tilings [Gubser, Heydeman, Jepsen, Marcolli, Parikh, Saberi, Stoica, Trundy, ’17]

# Future perspectives

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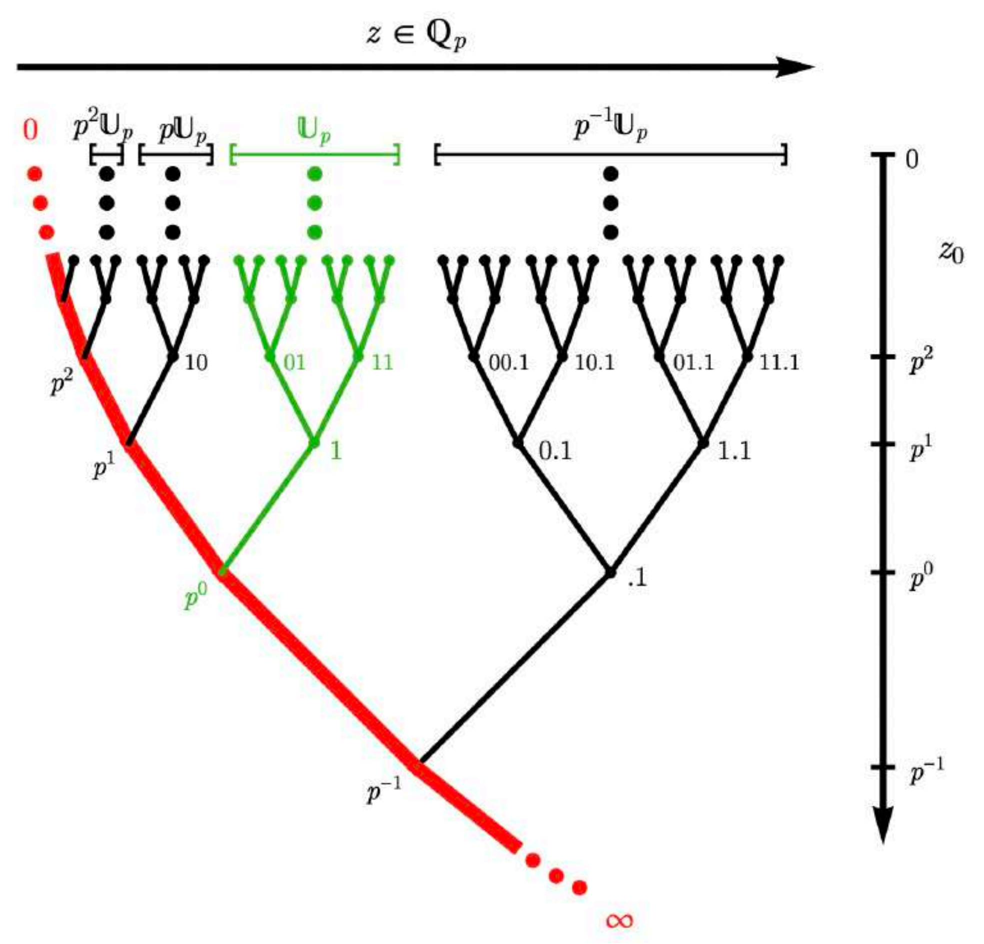


- Towards a bulk theory, edge-length-dynamics models in regular hyperbolic tilings [Gubser, Heydeman, Jepsen, Marcolli, Parikh, Saberi, Stoica, Trundy, ’17]
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# Future perspectives

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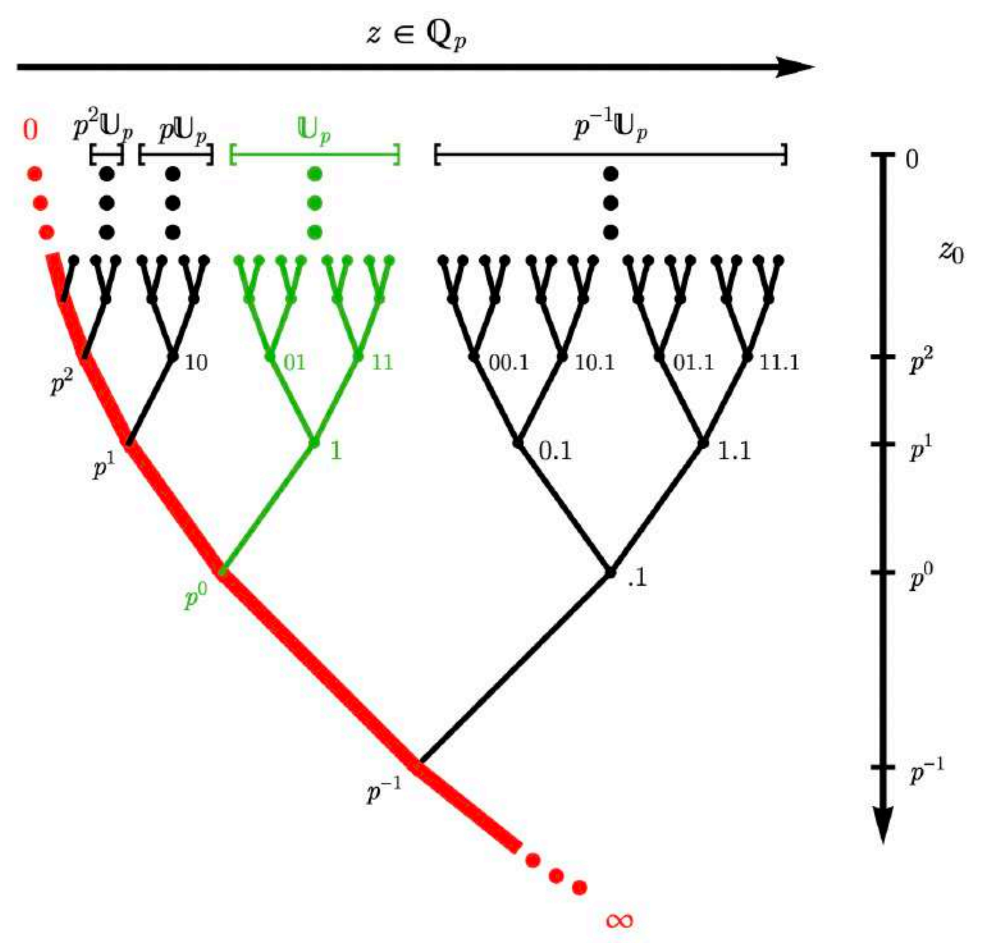

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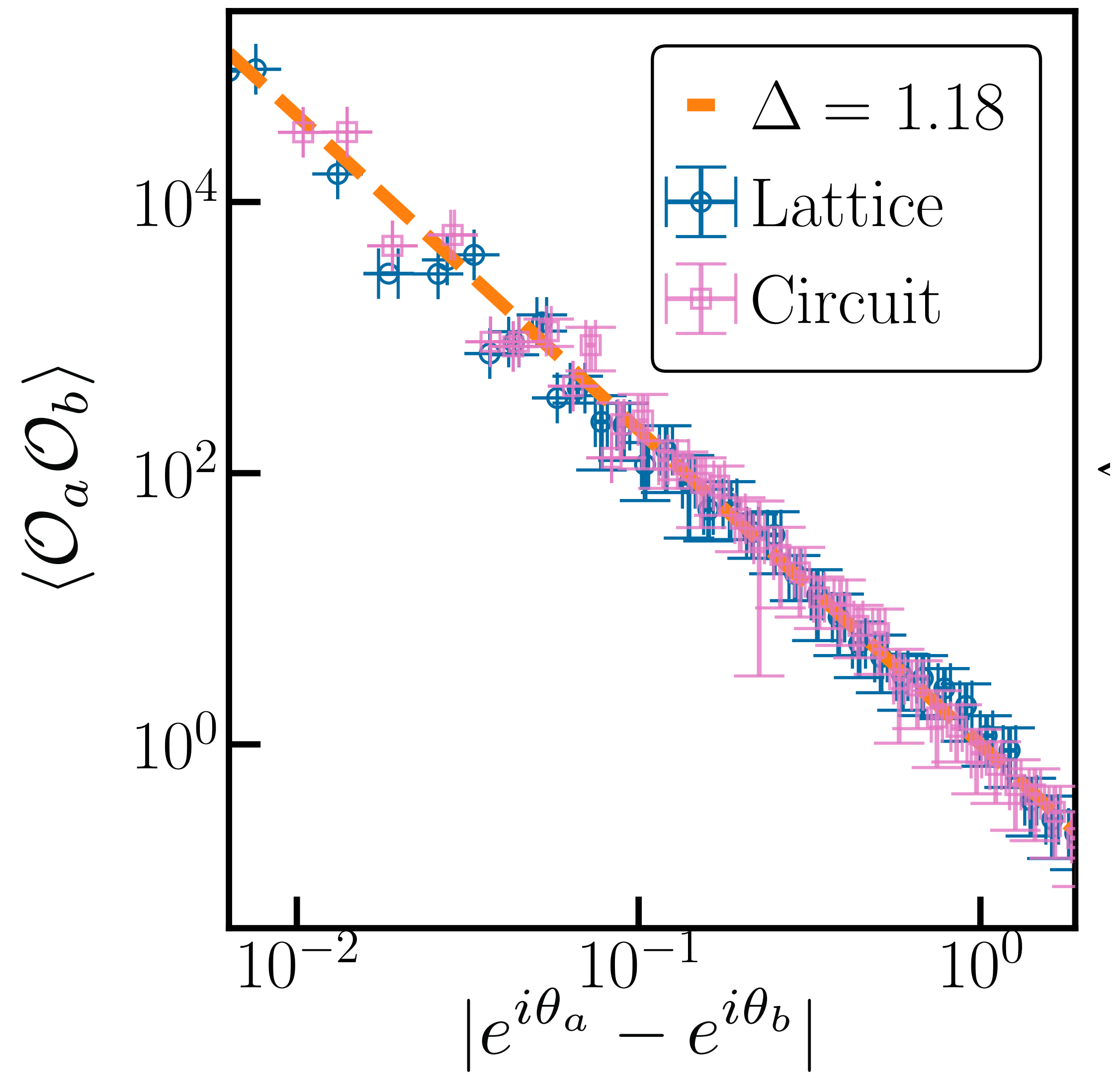
- Experimental realizations on circuit boards [Dey, Basteiro, *et al.* ’24]



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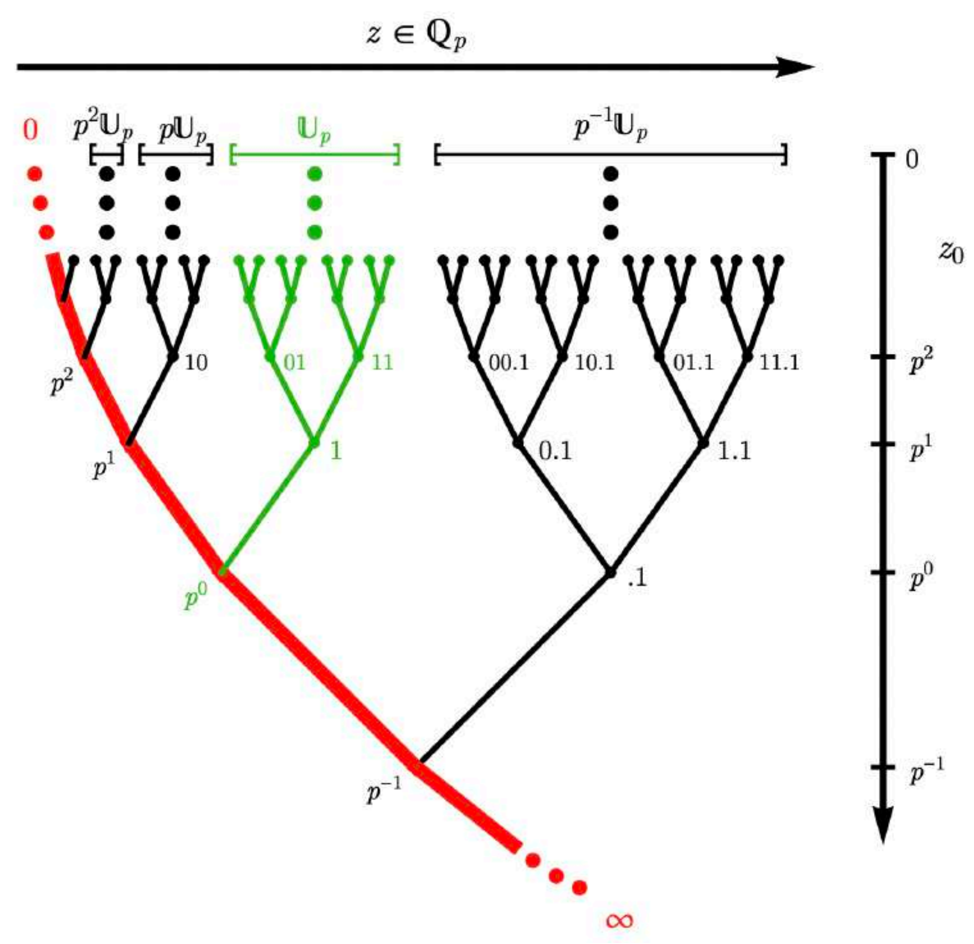

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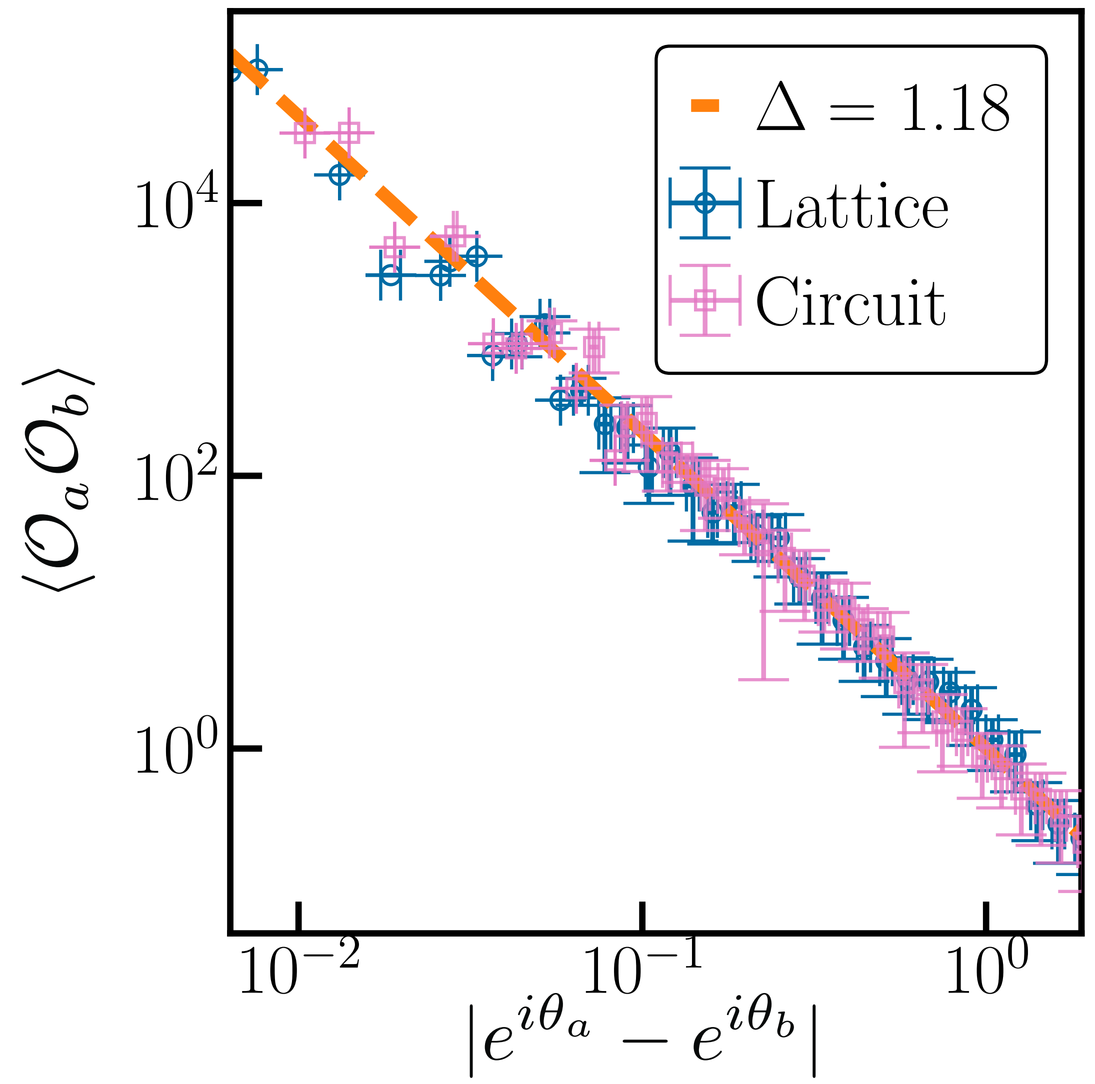


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Thank you for the attention!