Strongly correlated hyperbolic quantum matter

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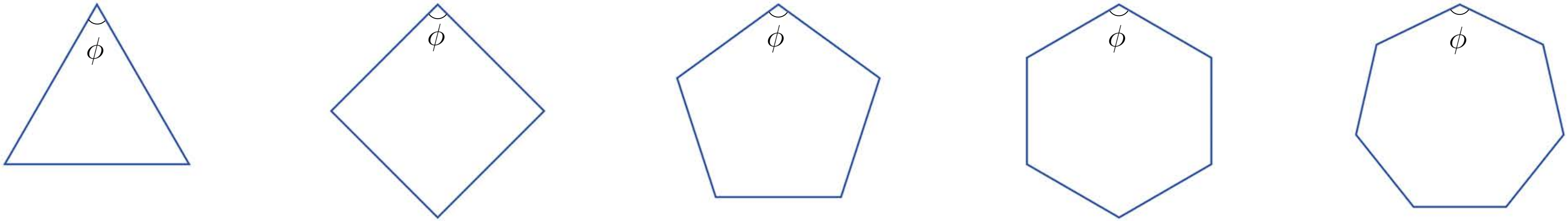
Quasicrystals in Fundamental Physics
International Centre for Mathematical Sciences (ICMS)
July 20, 2026

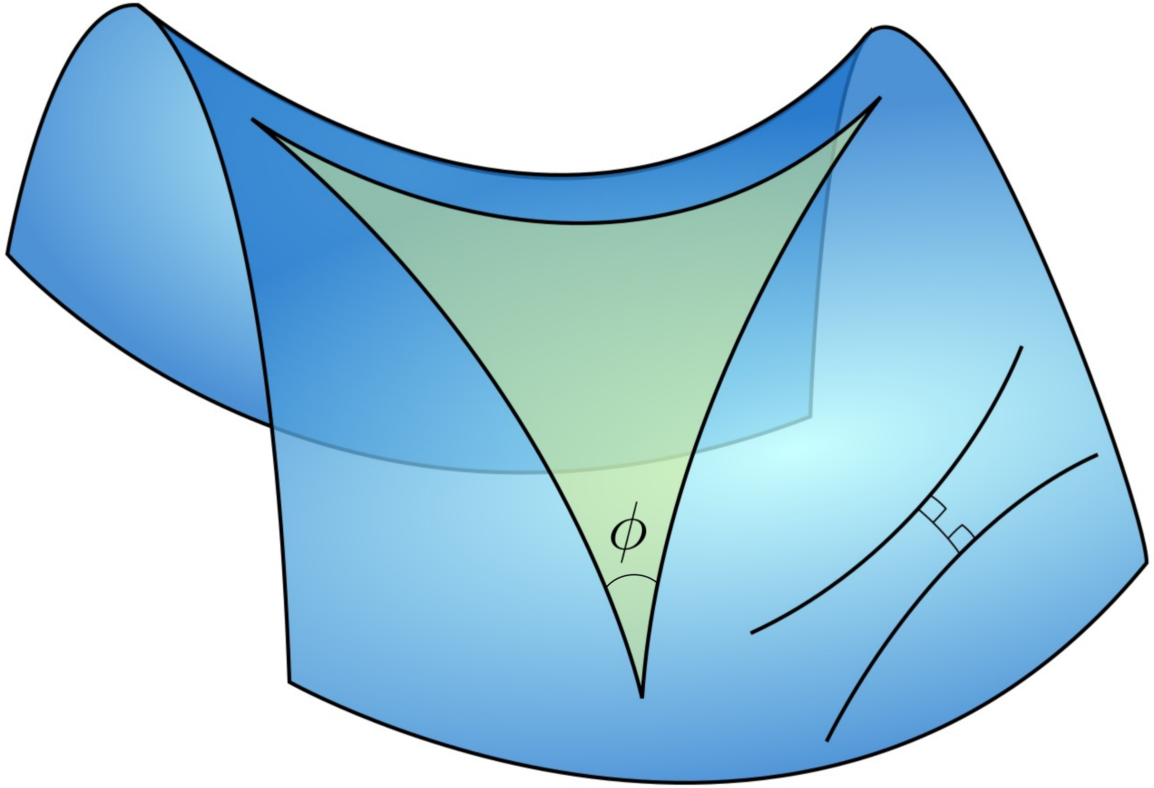
Regular polygons



$p = 3$ $\quad$ $p = 4$ $\quad$ $p = 5$ $\quad$ $p = 6$ $\quad$ $p = 7$

$$\sum_{i=1}^{p} \phi = p\phi = (p-2)\pi$$

Polygons in negative curvature



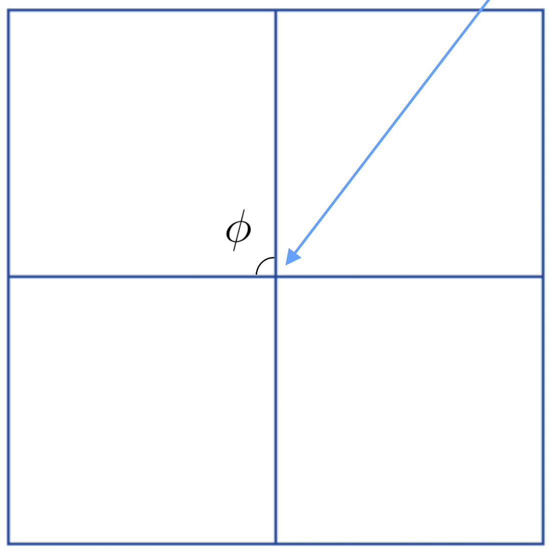
$$p\phi - (p-2)\pi = \kappa A$$

Gaussian curvature

$$p\phi < (p-2)\pi$$

Fitting polygons together

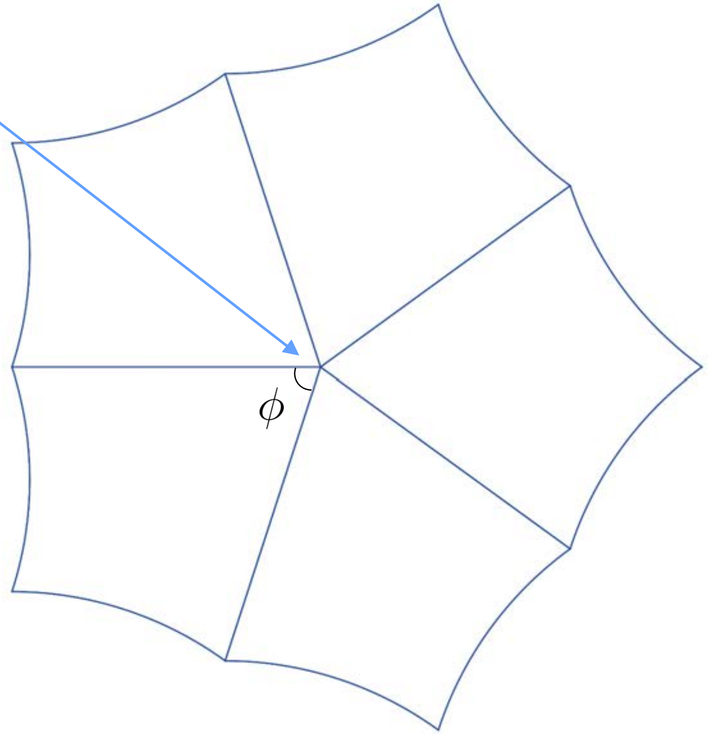
coordination number q



$\kappa = 0$

$$q\phi = 2\pi$$

$$p\phi - (p-2)\pi = \kappa A$$



$\kappa < 0$

Euclidean tessellations

$$\text{Schläfli symbol } \{p, q\}$$

Regular p-gons   Coordination q

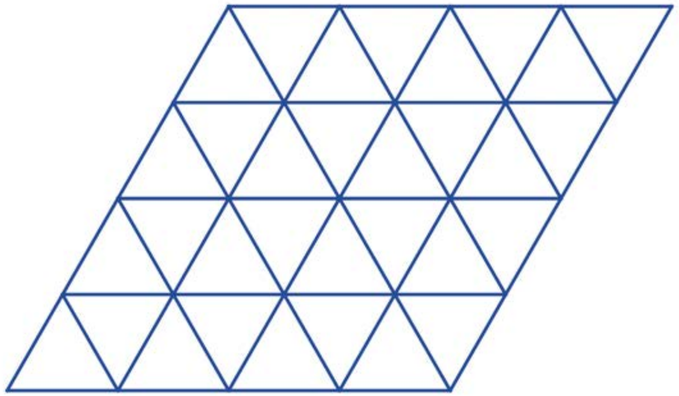
$$(p-2)(q-2) = 4$$

Euclidean tessellations

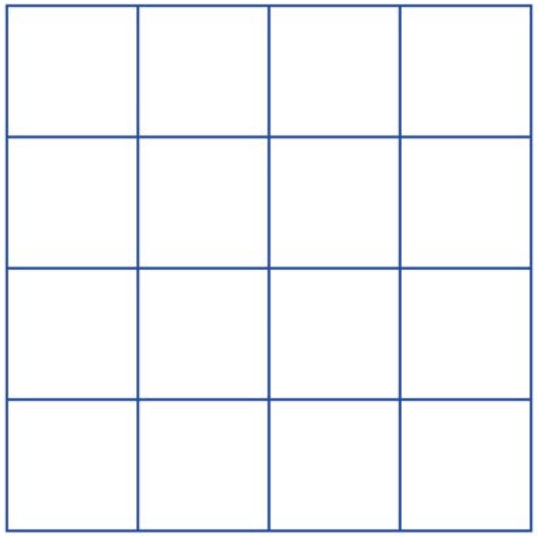
Schläfli symbol $\{p, q\}$

Regular p-gons   Coordination q

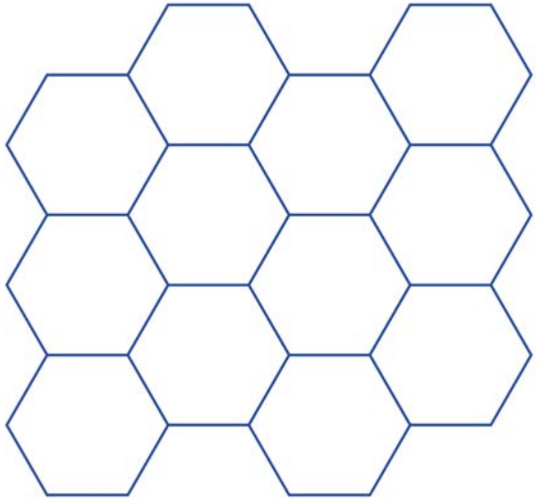
$$(p-2)(q-2) = 4$$



$\{3, 6\}$



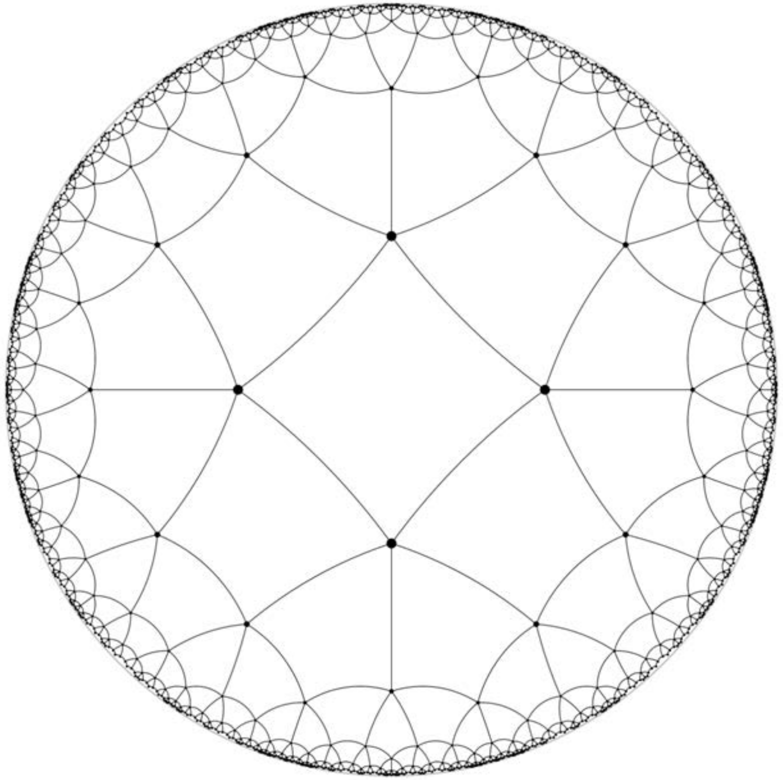
$\{4, 4\}$



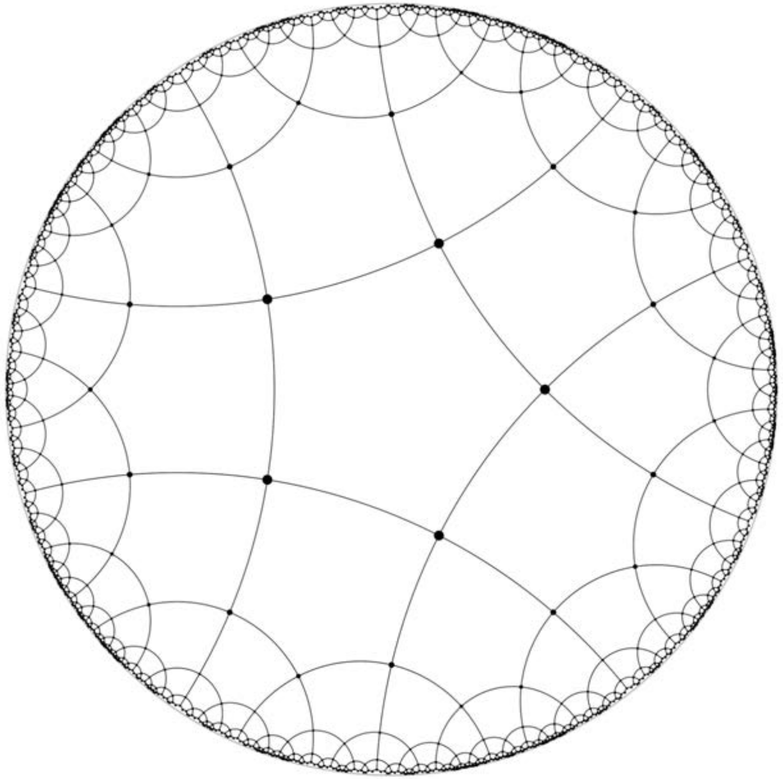
$\{6, 3\}$

Hyperbolic tessellations

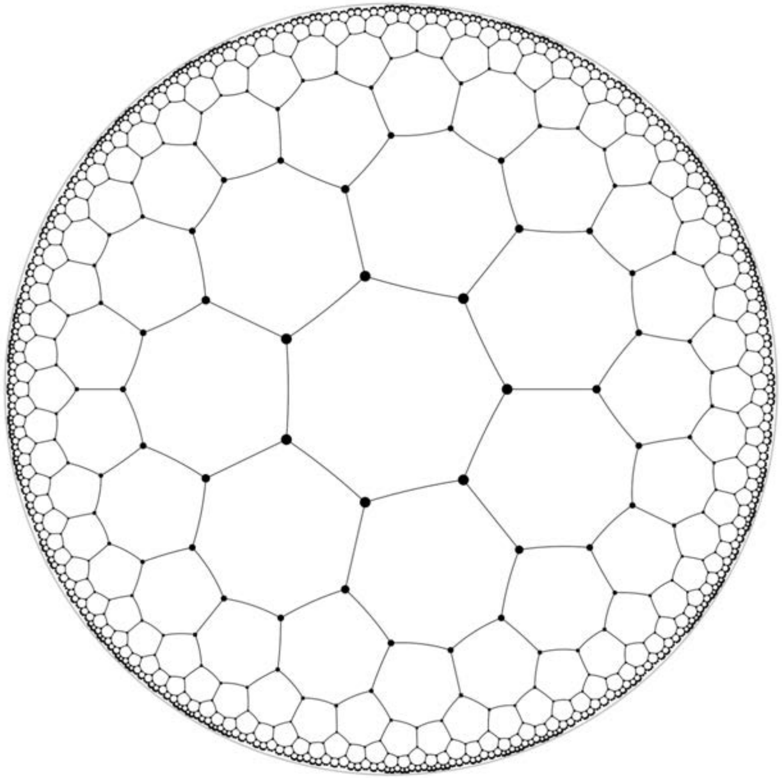
$\{p, q\}$ with $(p-2)(q-2) > 4$



$\{4, 5\}$



$\{5, 4\}$



$\{7, 3\}$

Poincaré disk representation

Plane of constant negative curvature:

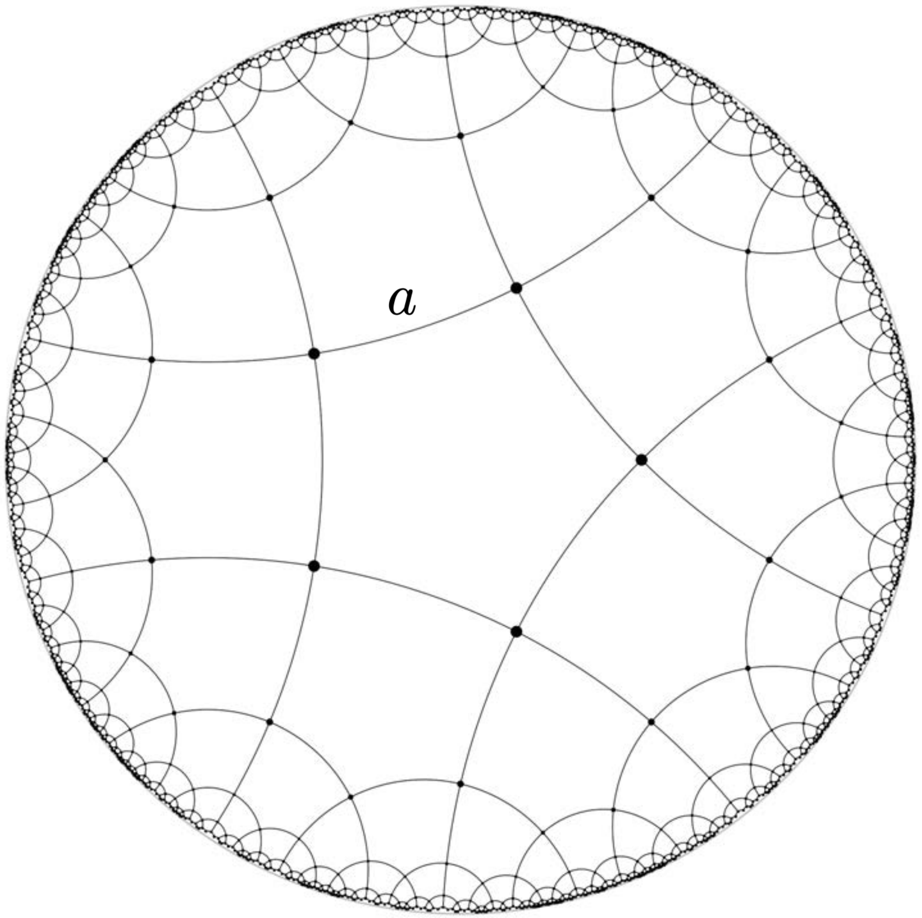
$$\mathbb{D} = \left\{x, y \in \mathbb{R} : x^2 + y^2 < 1\right\},$$

with metric

$$\mathrm{d}s^2 = \frac{4}{|\kappa|} \frac{\mathrm{d}x^2 + \mathrm{d}y^2}{\left(1 - x^2 - y^2\right)^2} \ .$$

Hyperbolic geometry implies:

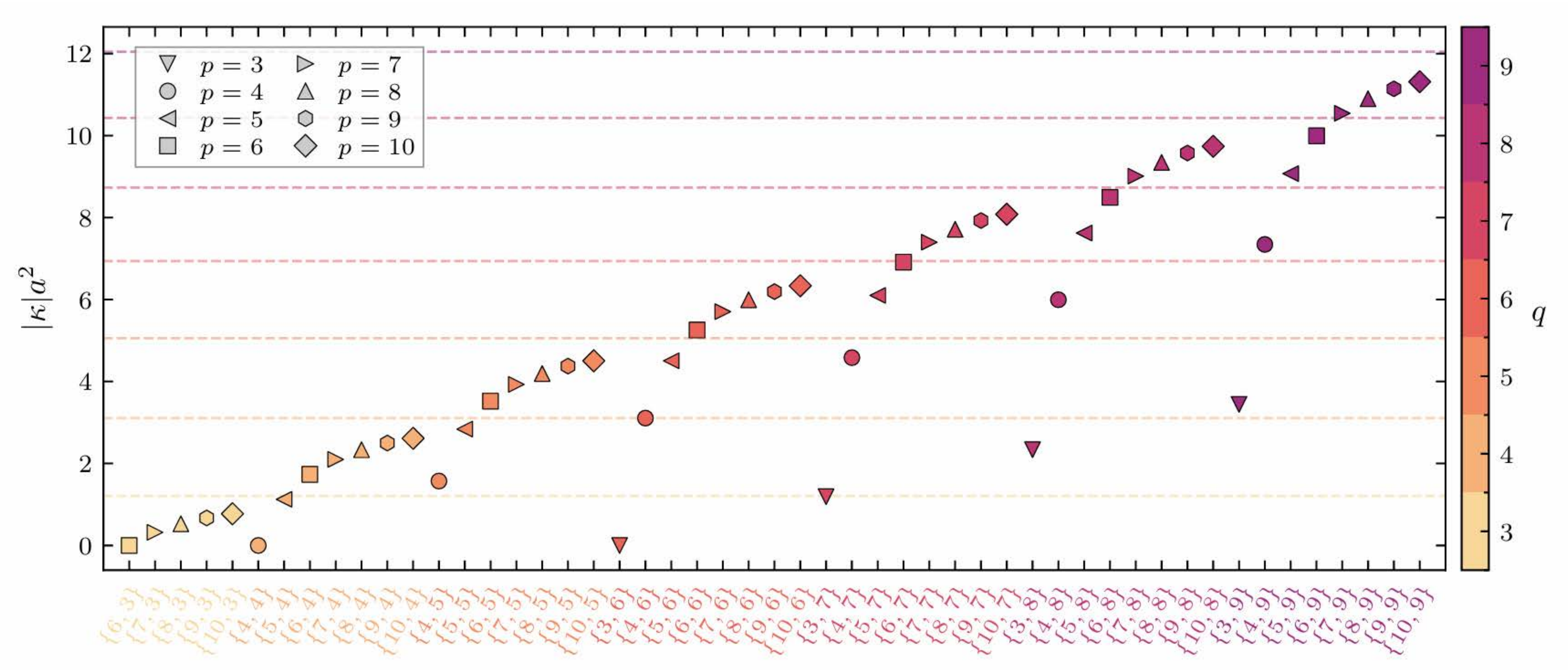
$$\cosh\left(\frac{1}{2}\sqrt{|\kappa| a^2}\right) = \frac{\cos\left(\pi/p\right)}{\sin\left(\pi/q\right)} \ .$$



$$\{p, q\} = \{5, 4\}$$

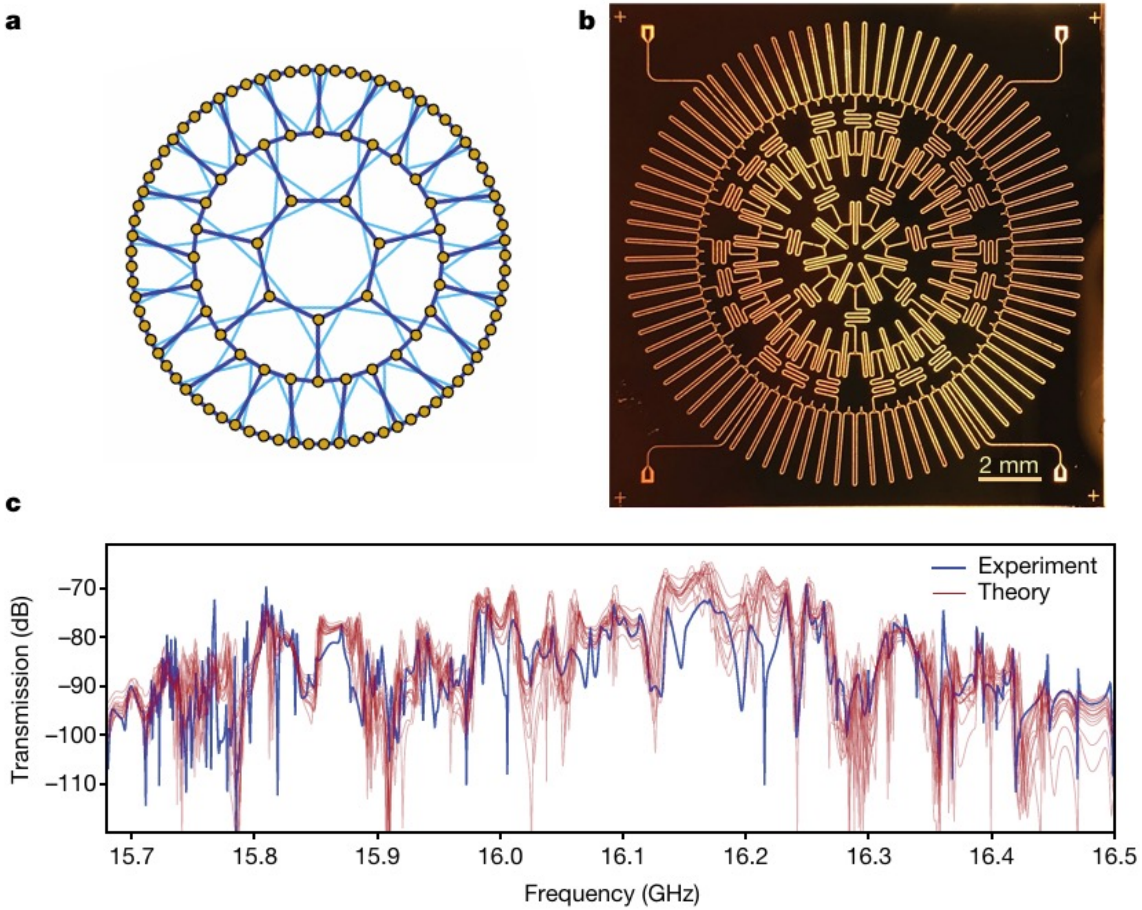
"Intrinsic" curvature scale

$$\cosh\left(\frac{1}{2}\sqrt{|\kappa|a^2}\right) = \frac{\cos(\pi/p)}{\sin(\pi/q)}$$



Hyperbolic matter

- Geometry beyond Euclidean crystals: curvature becomes an additional degree of freedom.
- Lattice ≠ atomic crystal: circuit QED, topolectric, photonic, and programmable architectures can realize graph connectivity directly.
- Hyperbolic lattices appear naturally in AdS/CFT tensor-network and quantum-error-correction constructions.
- Literature on single-particle systems is relatively mature, strongly correlated phenomena such as magnetism, superconductivity, Hubbard physics, and spin liquids are still developing.



A. J. Kollár *et al.*, Nature **571**, 45-50 (2019)

Outline

- Long-range magnetic order at finite temperature in 2D hyperbolic lattices

  [A. Hickey & JM, arXiv:2606.27422]

- Hyperbolic spin liquids

  [P. M. Lenggenhager, S. Dey, T. Bzdušek, & JM, Phys. Rev. Lett. 135, 076604 (2025)]

- Lieb-Schultz-Mattis constraints for hyperbolic lattices

  [G. Shankar & JM, arXiv:2605.15974]

Long-range magnetic order at finite temperature in 2D hyperbolic lattices

arXiv:2606.27422

Alexander Hickey
(U. Alberta)

Hyperbolic Heisenberg model

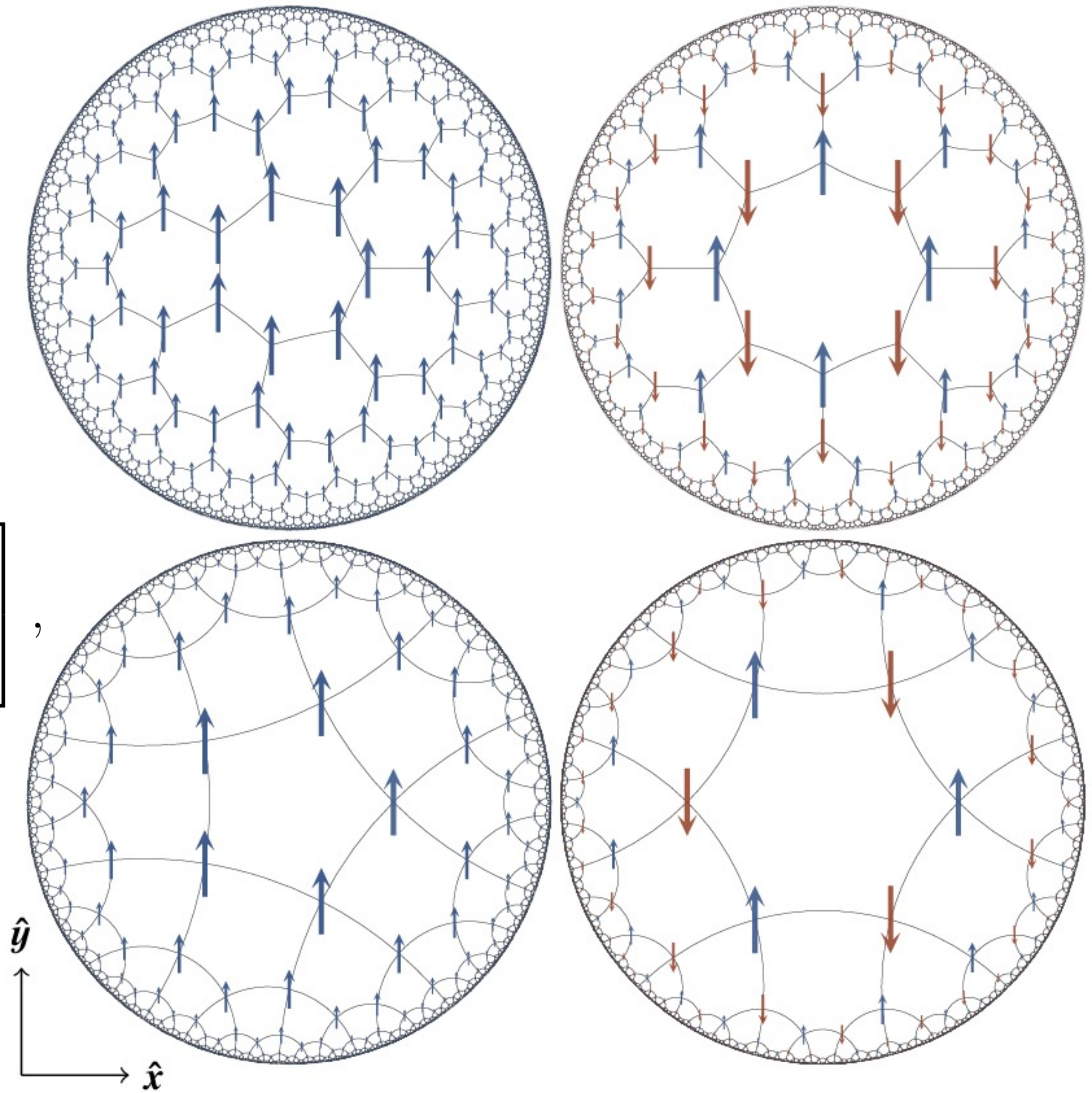
$$H = J \sum_{\langle i,j \rangle} \boldsymbol{S}_i \cdot \boldsymbol{S}_j$$

Holstein-Primakoff transformation

$$\boldsymbol{S}_i = \left(S - a_i^\dagger a_i\right) \hat{\boldsymbol{e}}_{i,0} + \sqrt{S} \left[\left(1 - \frac{a_i^\dagger a_i}{2S}\right)^{1/2} a_i \hat{\boldsymbol{e}}_{i,-} + \text{H.c.} \right],$$

$$H = \sum_{n=0}^{\infty} S^{2-n/2} H_n$$

We study the low energy, bulk properties of the hyperbolic Heisenberg antiferromagnet.



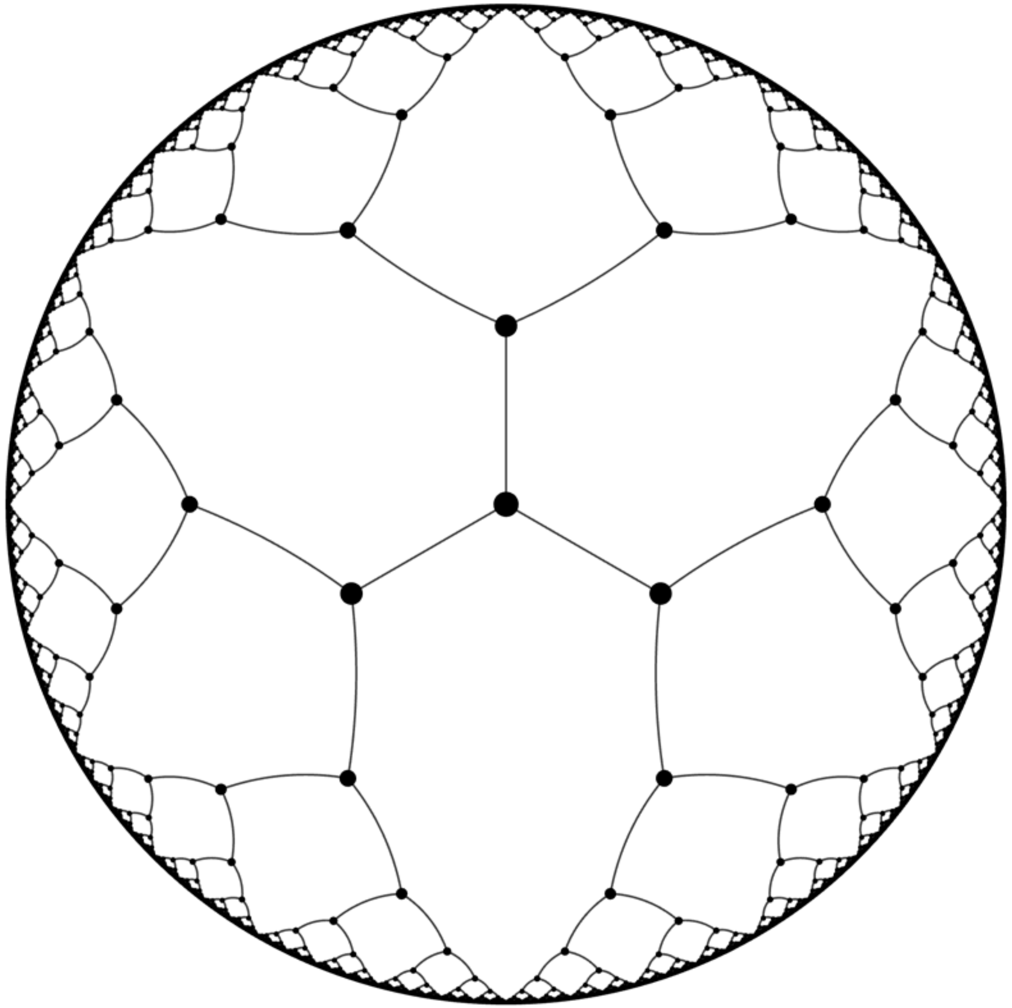
$J < 0$ $\qquad$ $J > 0$

Bethe lattice limit

The limit $\{p \to \infty, q\}$ is the infinite regular tree graph – the Bethe lattice.

Resolvent of the adjacency matrix:

$$\left[(z - \boldsymbol{A})^{-1}\right]_{ij} = \frac{\alpha(z)^{d(i,j)}}{z - q\alpha(z)}$$



$q = 3$

Continuum limit

O(3) Nonlinear sigma model:

$$S_{\mathrm{AFM}}[\boldsymbol{n}] = \frac{\rho_s}{2} \int \mathrm{d}\tau \mathrm{d}^2 x \sqrt{|g|} \left[\frac{1}{c^2} \left(\partial_\tau \boldsymbol{n} \right)^2 + g^{ab} \partial_a \boldsymbol{n} \cdot \partial_b \boldsymbol{n} \right],$$

$$\Delta_{\mathrm{AFM}} = \frac{JSq}{2\sqrt{2}} \sqrt{|\kappa| a^2}.$$

$$\langle S_r^+ S_0^- + S_r^- S_0^+ \rangle \sim \frac{e^{-\sqrt{|\kappa|}\, r}}{\sqrt{r}}.$$

Euclidean overview

- Ground state (T=0) is Néel ordered, low energy subspace is Goldstone mode $\omega_{\boldsymbol{k}} \sim c|\boldsymbol{k}|$.
- Correlations (T=0) are long-range (massless):

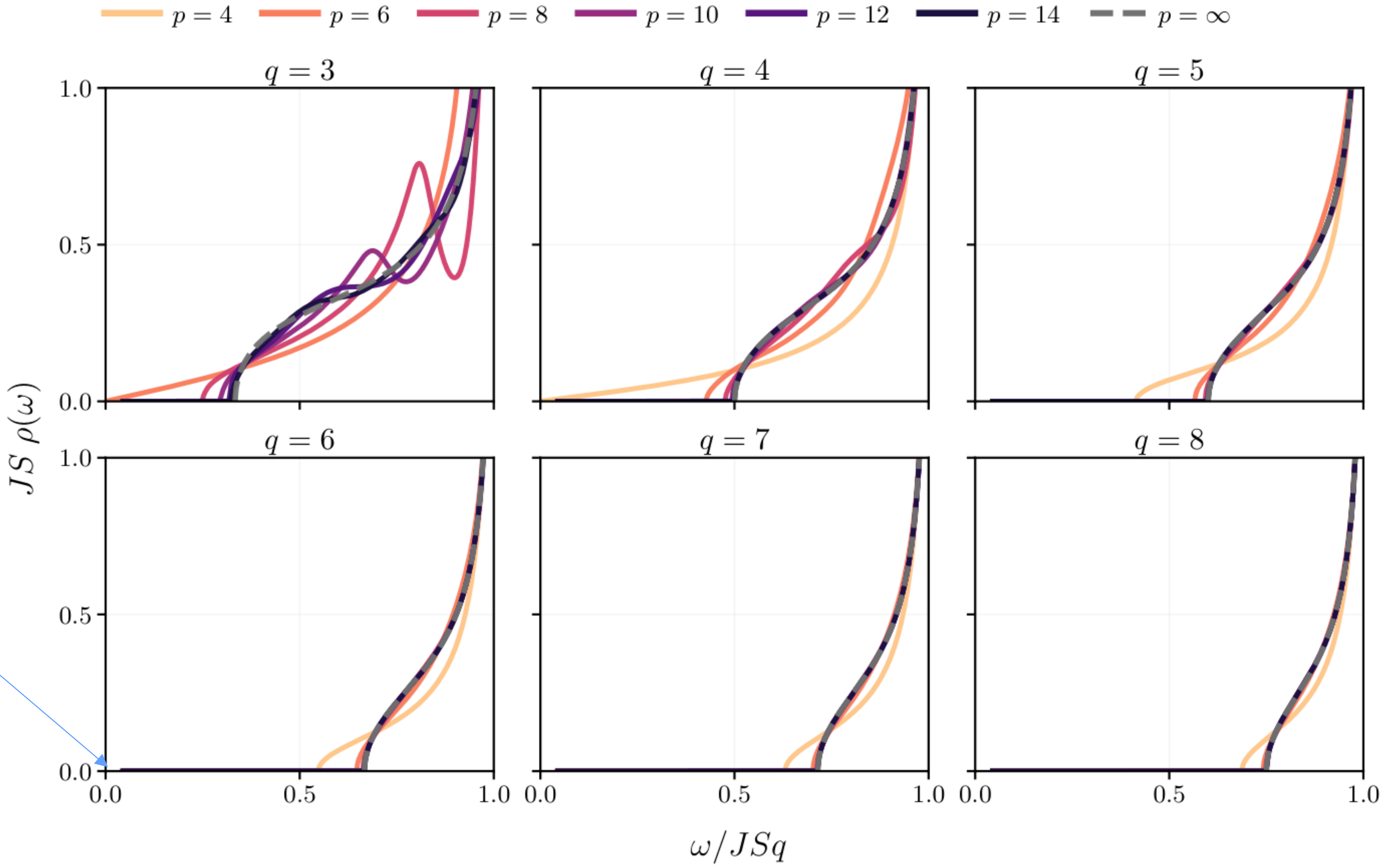
$$C_{\perp}(r_{ij}) = \frac{1}{2S}\langle S_i^+ S_j^- + S_i^- S_j^+ \rangle \sim \frac{1}{r_{ij}} \quad .$$

- Finite temperature fluctuations destroy magnetic order (Mermin-Wagner theorem):

$$\langle \boldsymbol{S}_i \rangle \sim S - \delta m(0) - \frac{\mathcal{C}_0 T}{S} \int_0^{\omega_*} \frac{\mathrm{d}\omega}{\omega}.$$

Bulk density of states

Global symmetry (Goldstone) mode contributes O(1/N) at zero energy.



Spectral gap

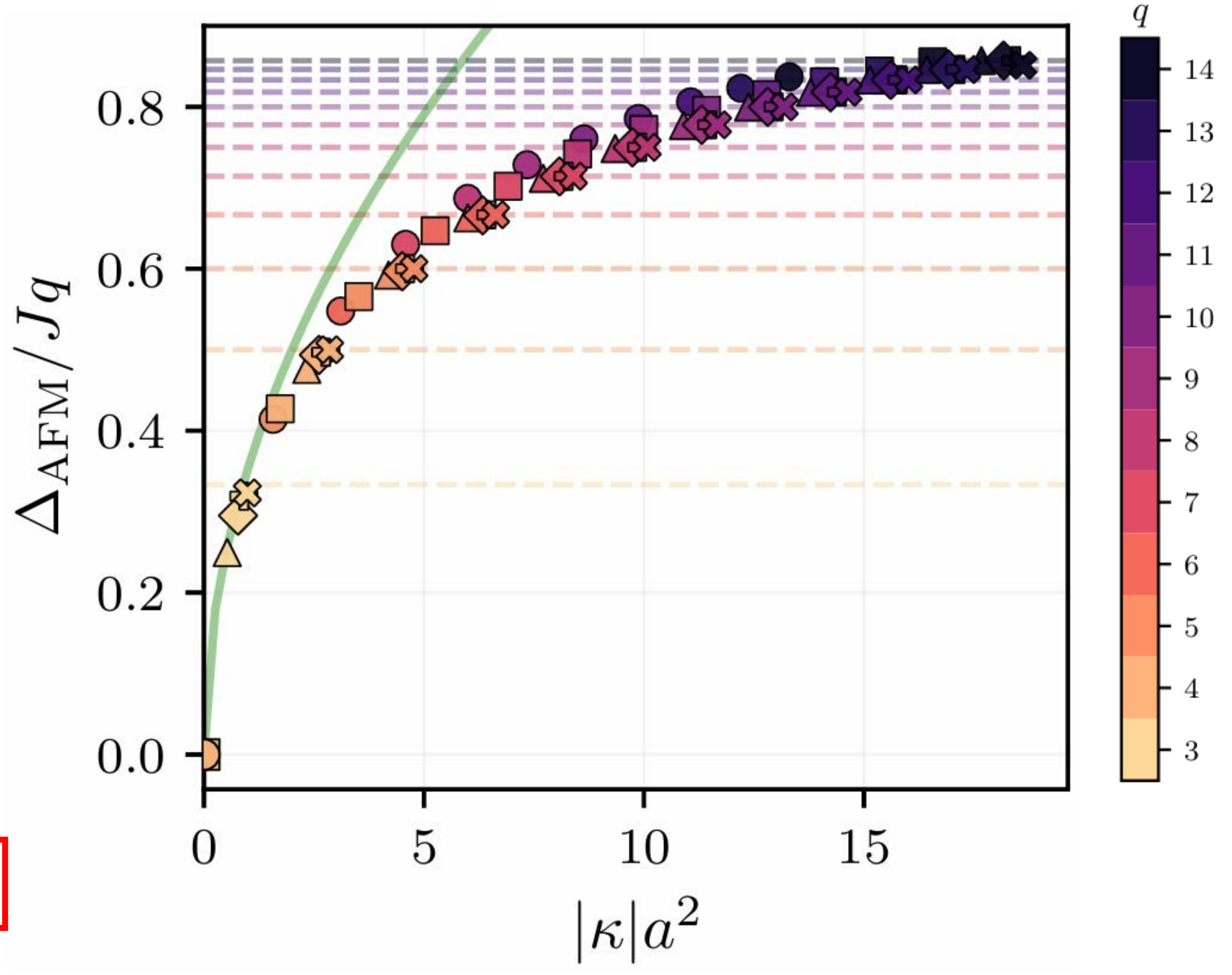
q-regular Bethe lattice:

$$\Delta_{\mathrm{AFM}} = J(q-2).$$

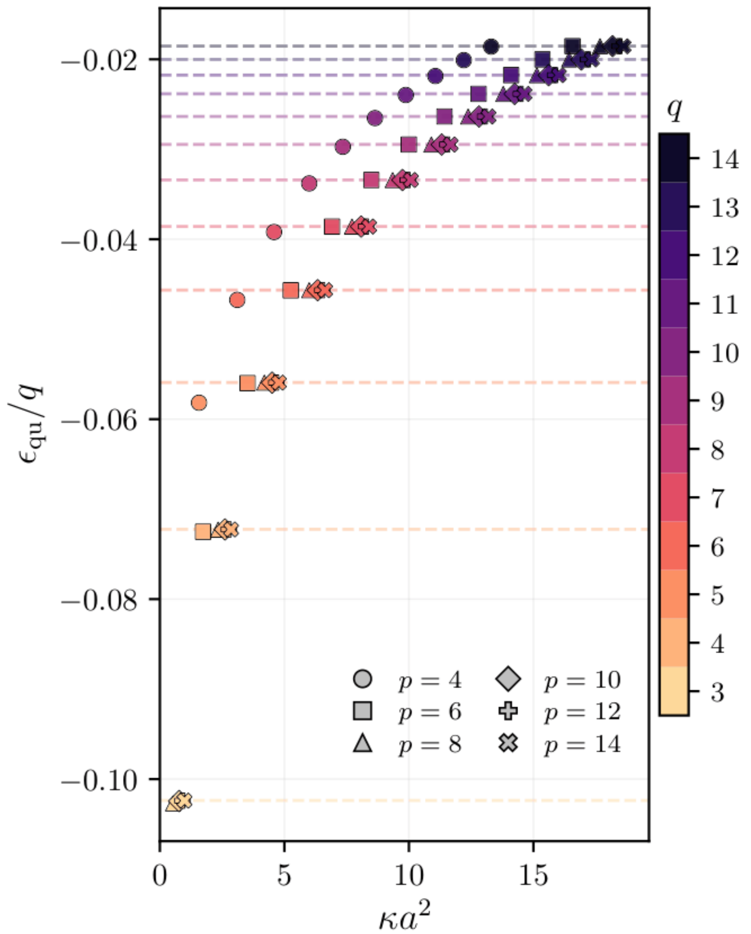
Continuum:

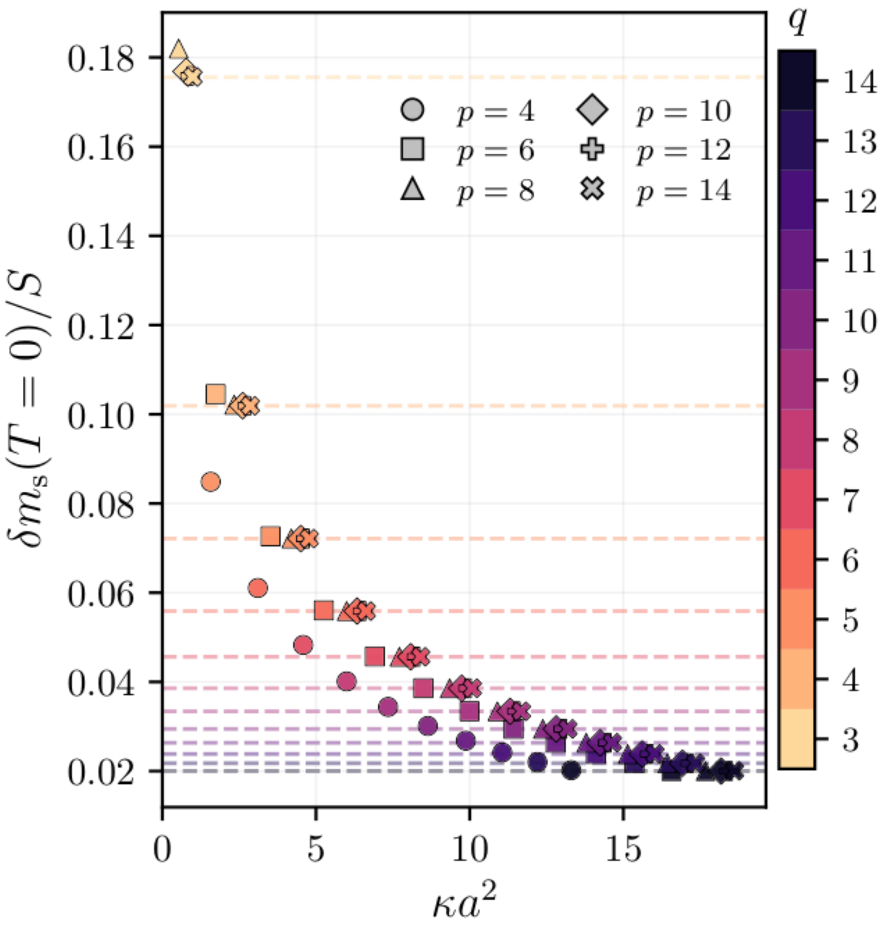
$$\Delta_{\mathrm{AFM}} = \frac{Jq}{2\sqrt{2}}\sqrt{|\kappa|a^2}.$$

Curvature induces magnon gap



Zero-point fluctuations





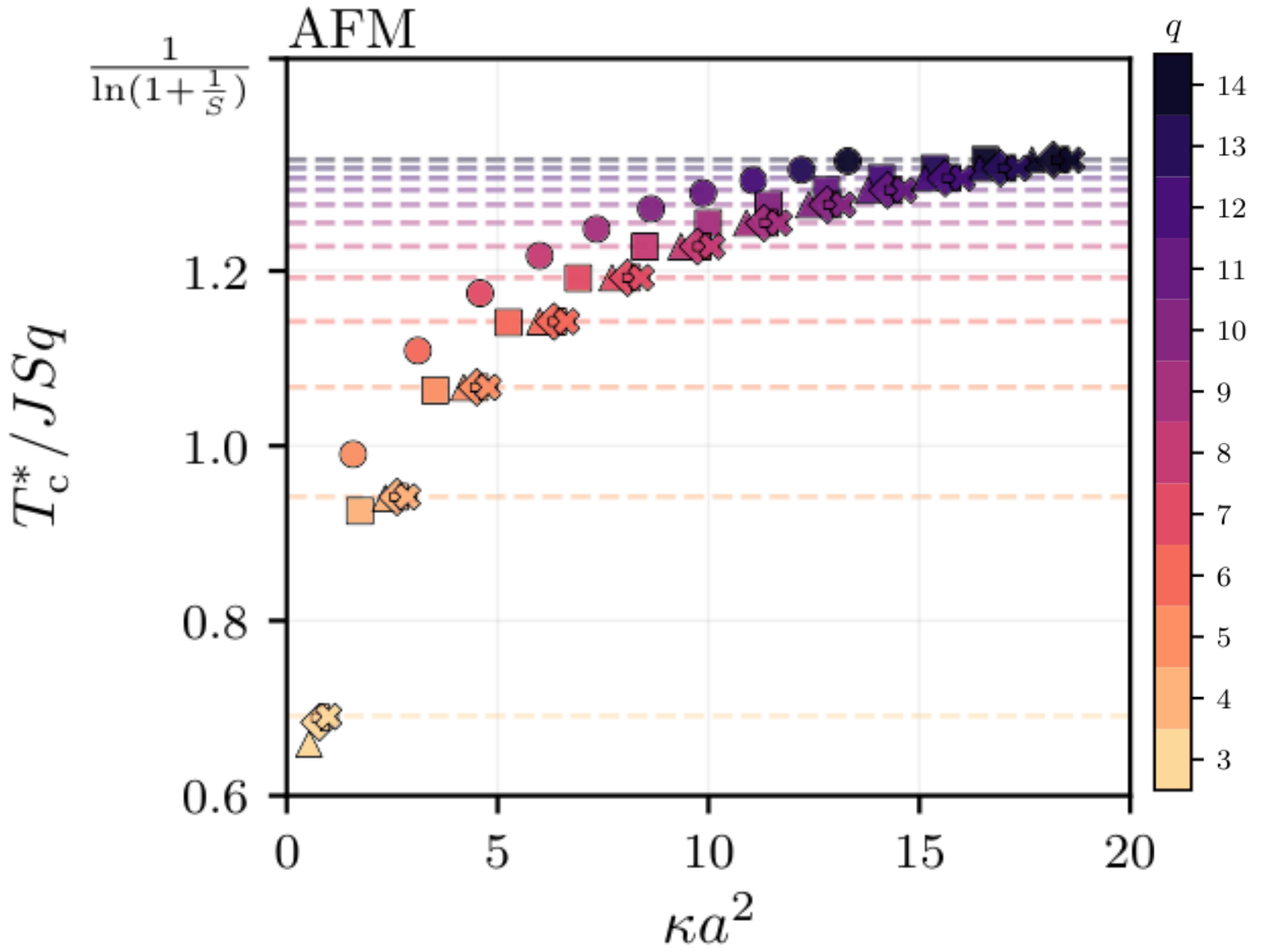
Curvature reduces quantum fluctuations

Critical temperature

Solve for $T = T_c^*$, such that $\langle \boldsymbol{S}_i \rangle = 0$.

Infrared fluctuations are no longer divergent:

$$\langle \boldsymbol{S}_i \rangle \sim S - \delta m(0) - \mathcal{C}\,\frac{\sqrt{\pi}}{2}\left(\frac{T}{S}\right)^{3/2} e^{-S\Delta/T}.$$



Transverse correlations

$$\langle S_r^+ S_0^- + S_r^- S_0^+ \rangle \sim \frac{e^{-r/\xi_\perp}}{\sqrt{r}}.$$

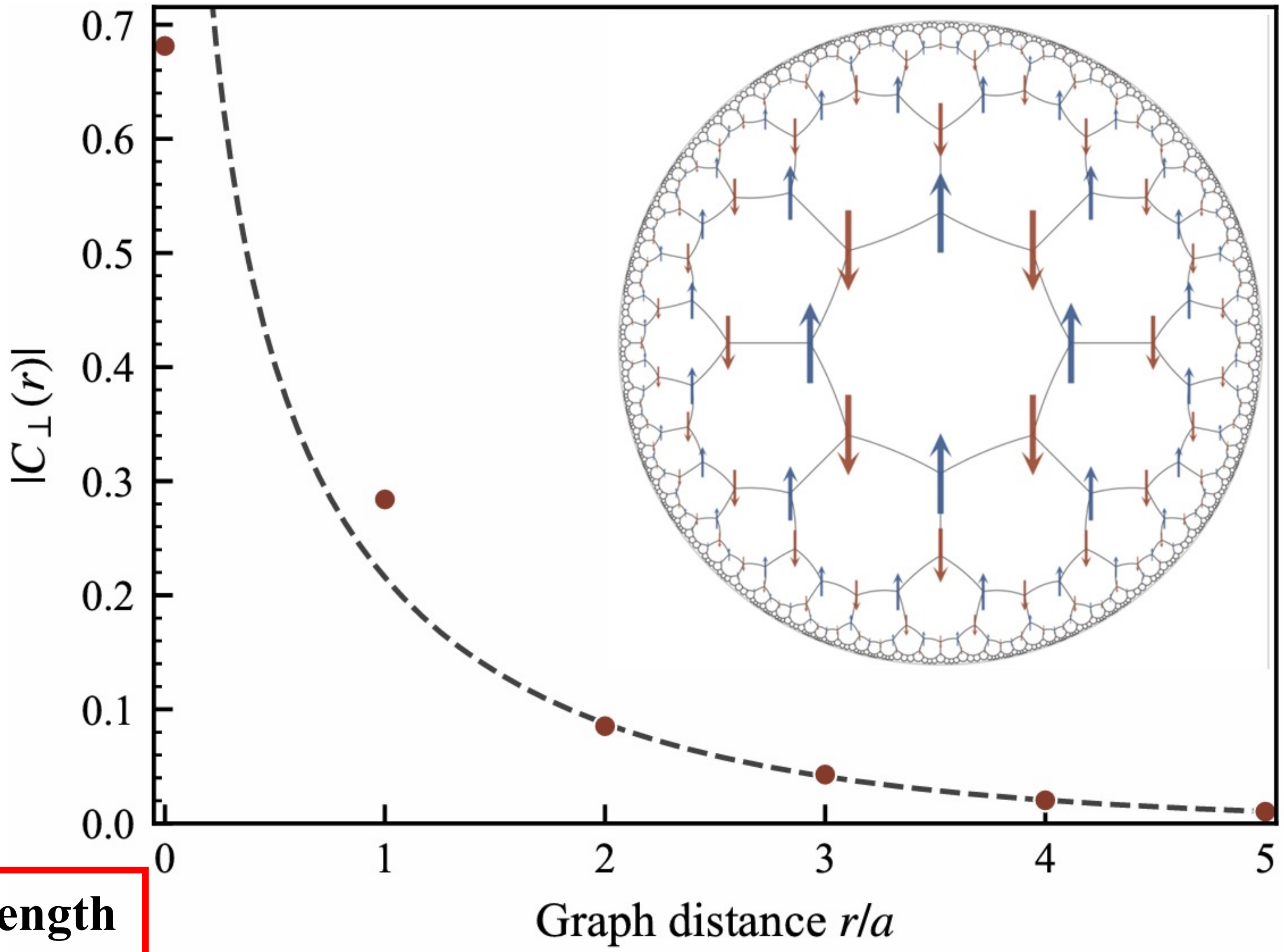
$\{8,3\}$: $\xi_\perp / a \approx 1.8$

Bethe lattice: $\xi_\perp / a = \dfrac{1}{\ln(q-1)}$

Continuum: $\xi_\perp / a = \dfrac{1}{\sqrt{\kappa a^2}}$

Transverse correlations have finite correlation length

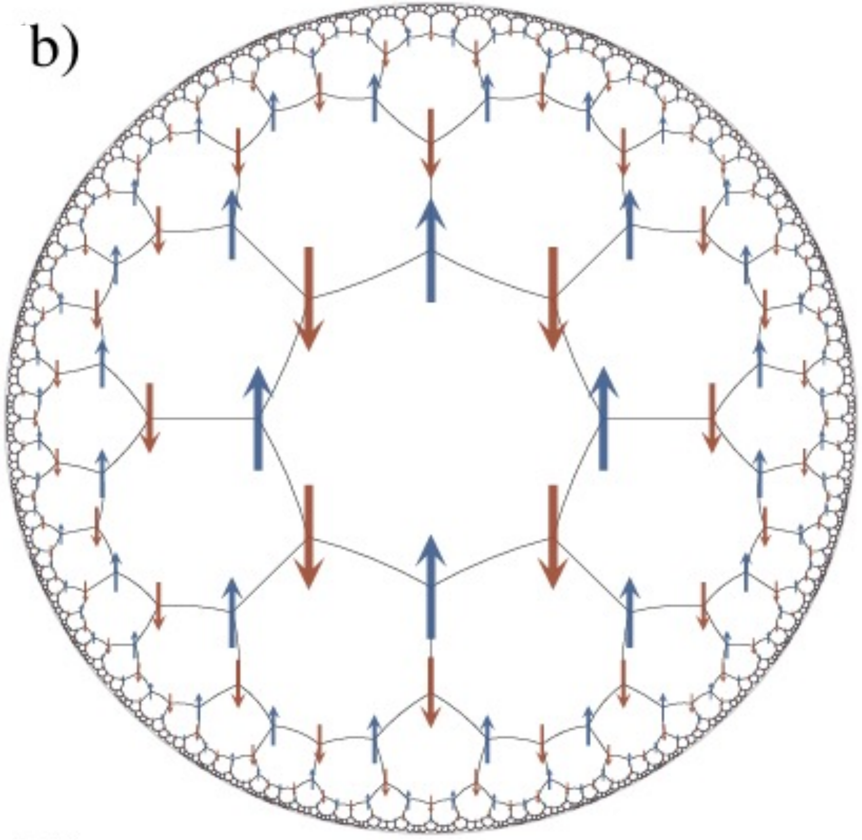
$\{8,3\}$



Transverse correlation length

$$\mathcal{N}(r) \sim \lambda^r = e^{r \ln \lambda}$$

$$C_\perp(r) \sim \frac{1}{\sqrt{r}} e^{-r/\xi_\perp}$$



$$\mathcal{S}^\perp_{\text{bulk}} = \sum_r \mathcal{N}(r)\, C_\perp(r) \sim \sum_r \frac{1}{\sqrt{r}} \exp\left[r \left(\ln\lambda - \frac{1}{\xi_\perp} \right) \right]$$

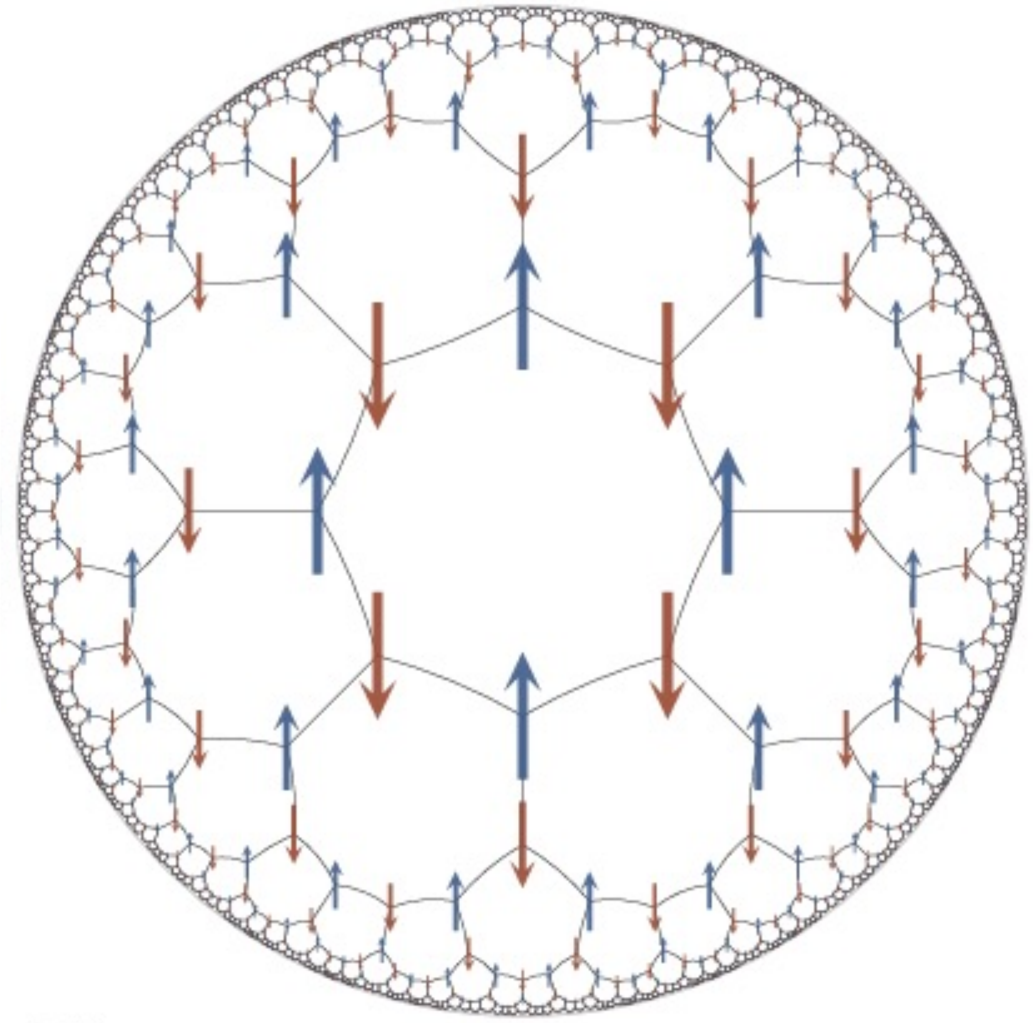
Diverges when:

$$\xi_\perp \geq \frac{1}{\ln \lambda}$$

Summary [Part I]

- Hyperbolic geometry changes the infrared phase space of spin waves, allowing finite-temperature magnetic order with spontaneously broken SO(3) symmetry.
- Negative curvature induces a bulk spectral gap, short-ranged transverse correlations, and reduced quantum fluctuations.

arXiv:2606.27422



Hyperbolic spin liquids

Phys. Rev. Lett. 135, 076604 (2025)

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(ÉNS)

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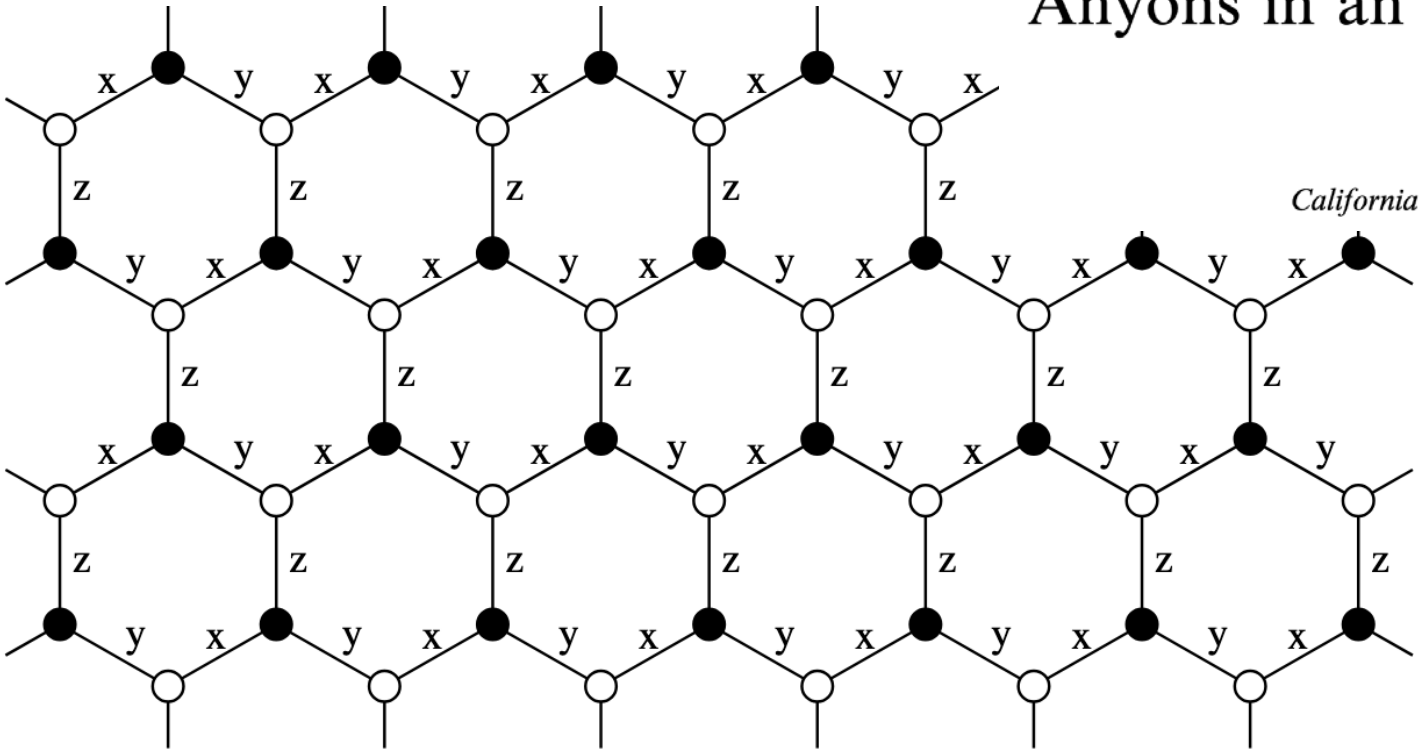
Kitaev spin liquid

Ann. Phys. 321, 2 (2006)

Anyons in an exactly solved model and beyond

Alexei Kitaev *

California Institute of Technology, Pasadena, CA 91125, USA



$$H = -J_x \sum_{x\text{-links}} \sigma_j^x \sigma_k^x - J_y \sum_{y\text{-links}} \sigma_j^y \sigma_k^y - J_z \sum_{z\text{-links}} \sigma_j^z \sigma_k^z$$

Kitaev spin liquid

Ann. Phys. 321, 2 (2006)

Anyons in an exactly solved model and beyond

Alexei Kitaev *

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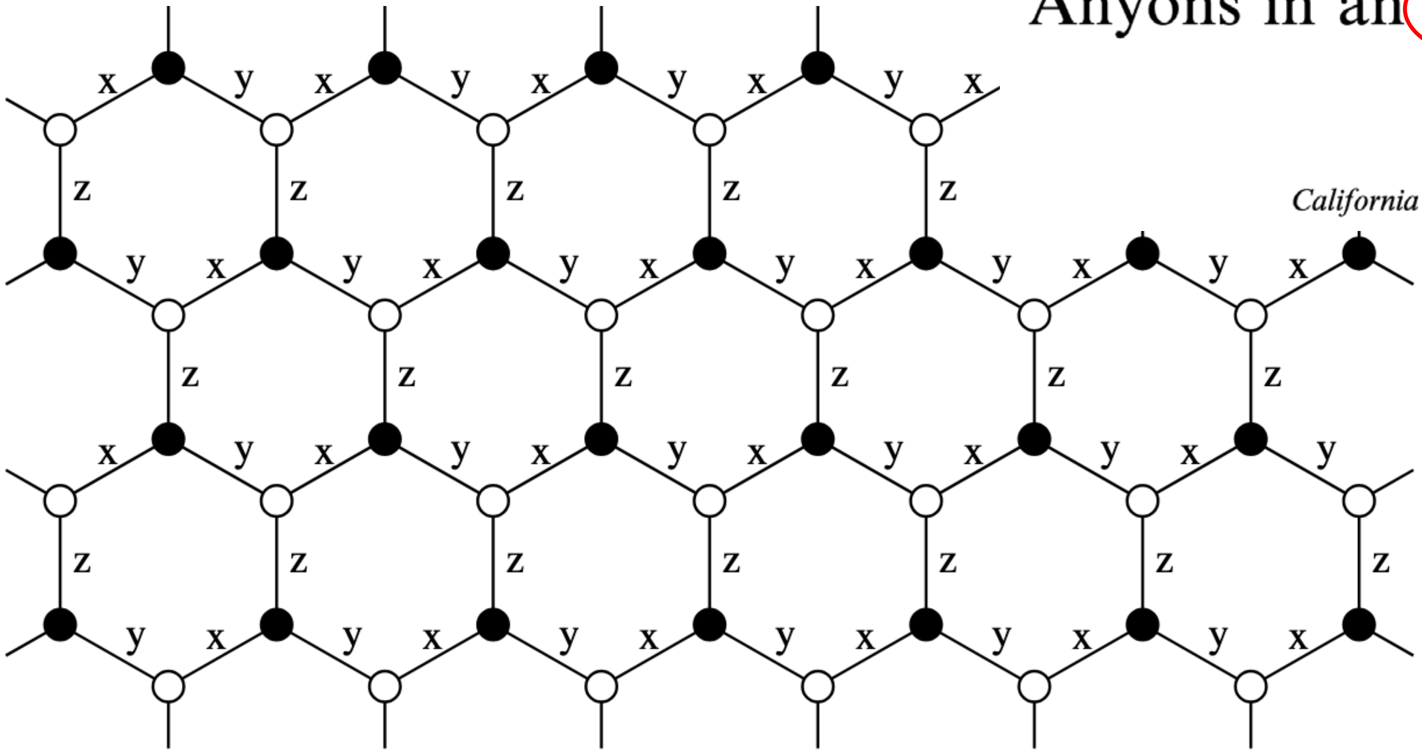
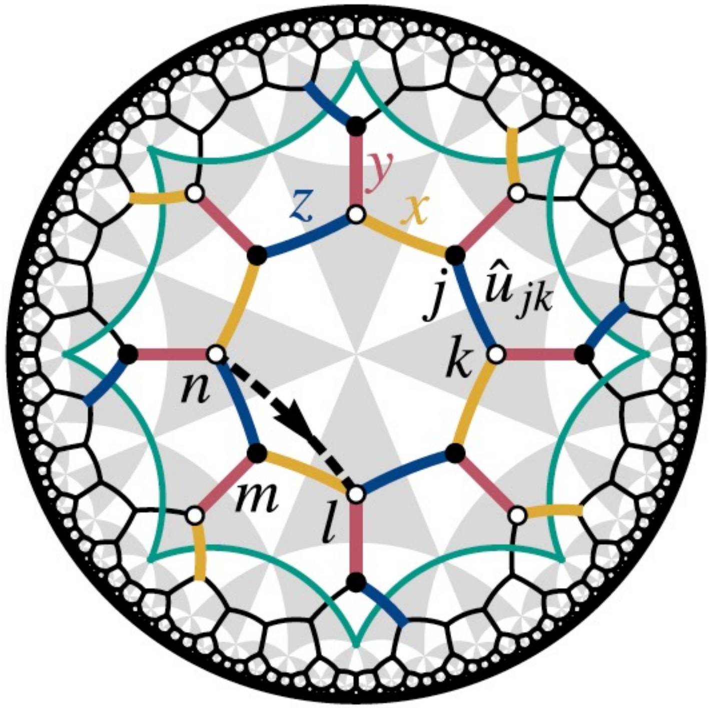


- Mapping to free fermions: 3-edge coloring
- Solution of the flux optimization problem: Lieb's lemma

$$H = -J_x \sum_{x\text{-links}} \sigma_j^x \sigma_k^x - J_y \sum_{y\text{-links}} \sigma_j^y \sigma_k^y - J_z \sum_{z\text{-links}} \sigma_j^z \sigma_k^z$$

Hyperbolic Kitaev model

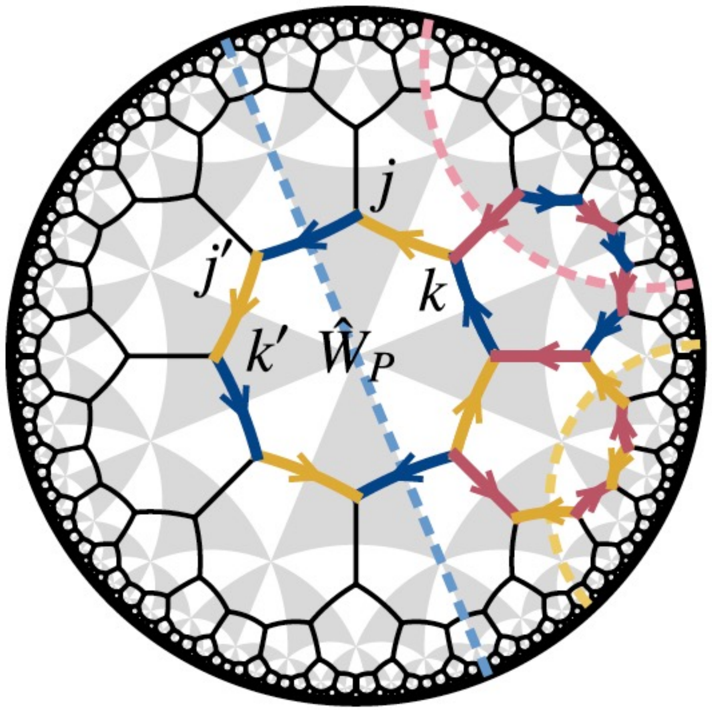
$$\hat{\mathcal{H}} = -\sum_{\langle j,k\rangle_\alpha} J_\alpha \hat{\sigma}_j^\alpha \hat{\sigma}_k^\alpha$$



- Generalize Kitaev model to bipartite $\{p,3\}$ lattices (p even): 3-edge colorable (Kőnig's theorem)

Hyperbolic Kitaev model

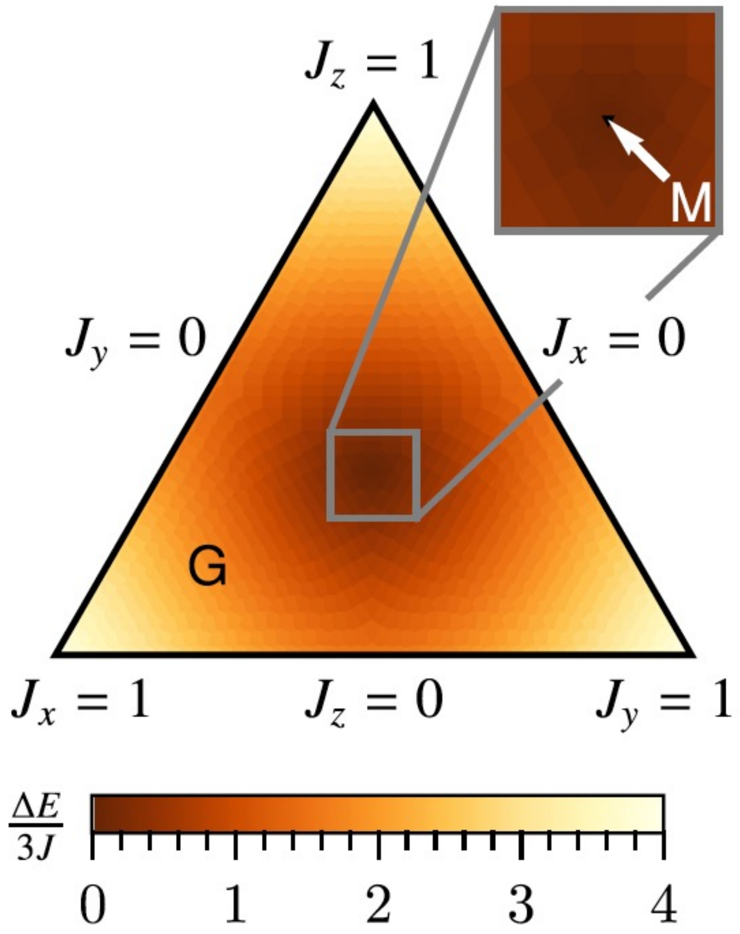
$$\hat{\mathcal{H}} = \sum_{\langle j,k\rangle_\alpha} J_\alpha \hat{u}_{jk} \mathrm{i} \hat{c}_j \hat{c}_k$$



- Generalize Kitaev model to bipartite $\{p,3\}$ lattices (p even): 3-edge colorable (Kőnig's theorem)
- "Symmetric" (Kekulé-like) 3-edge coloring admits **extensive number** of (hyperbolic) reflection symmetries
- Allows for **exact** solution of flux optimization problem via Lieb's lemma:

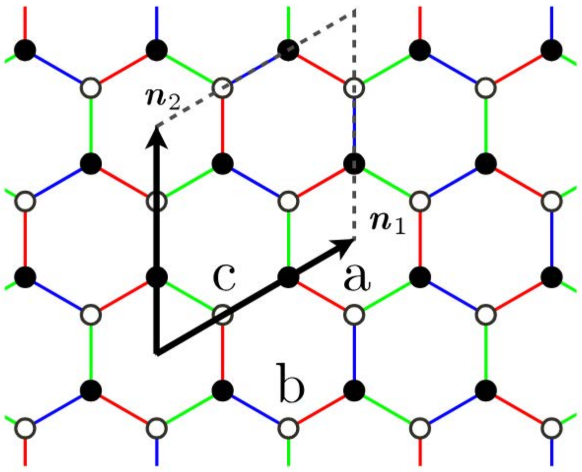
$$W_P = -(-1)^{p/2}$$

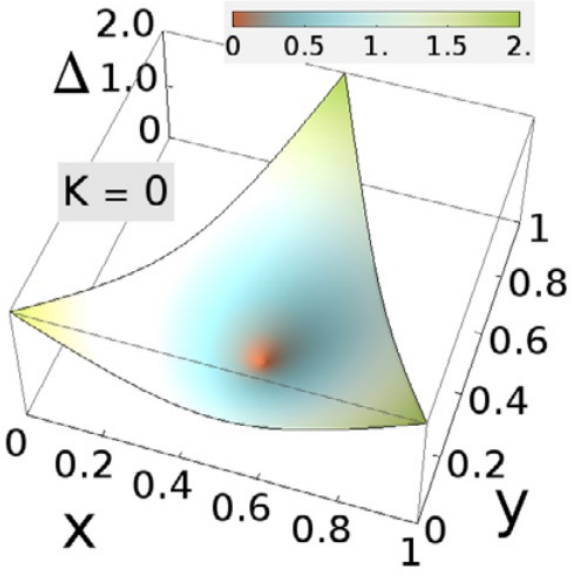
Phase diagram



fermion gap

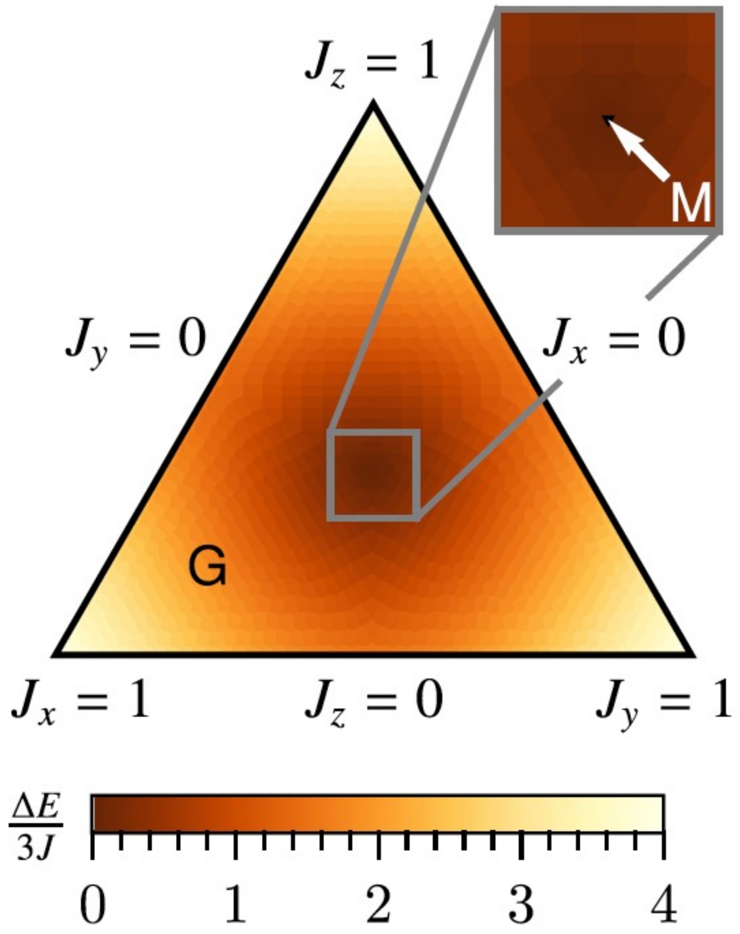
- Similar to "type-III" (Kekulé) 3-edge coloring on honeycomb lattice



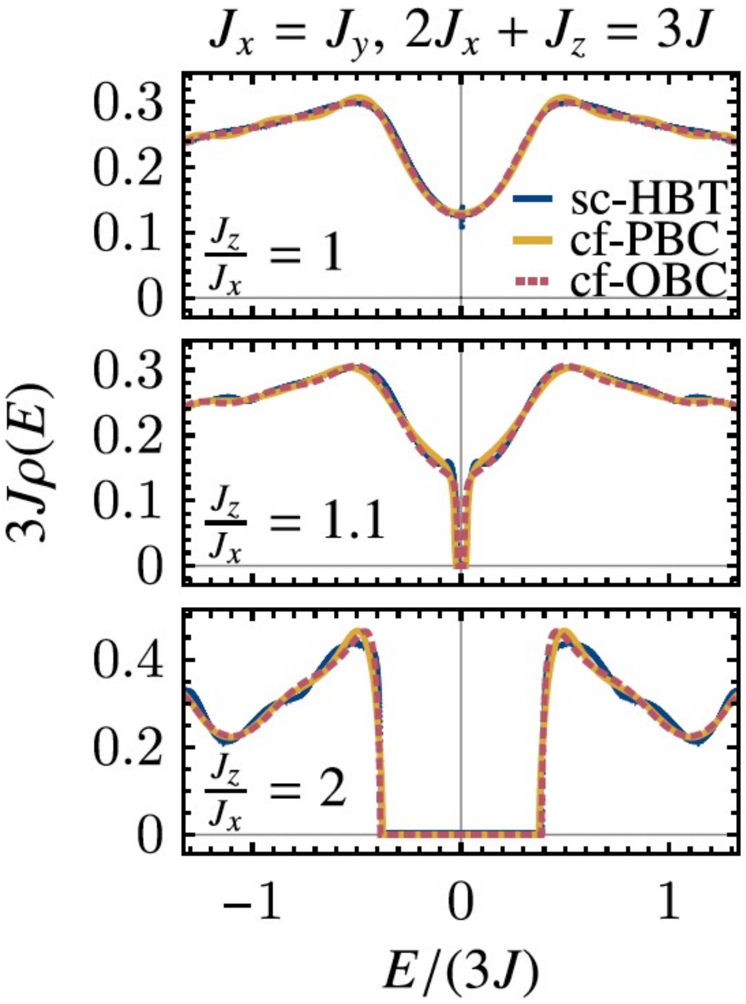


Kamfor, Dusuel, Vidal, Schmidt, J. Stat. Mech. P08010 (2010)

Phase diagram

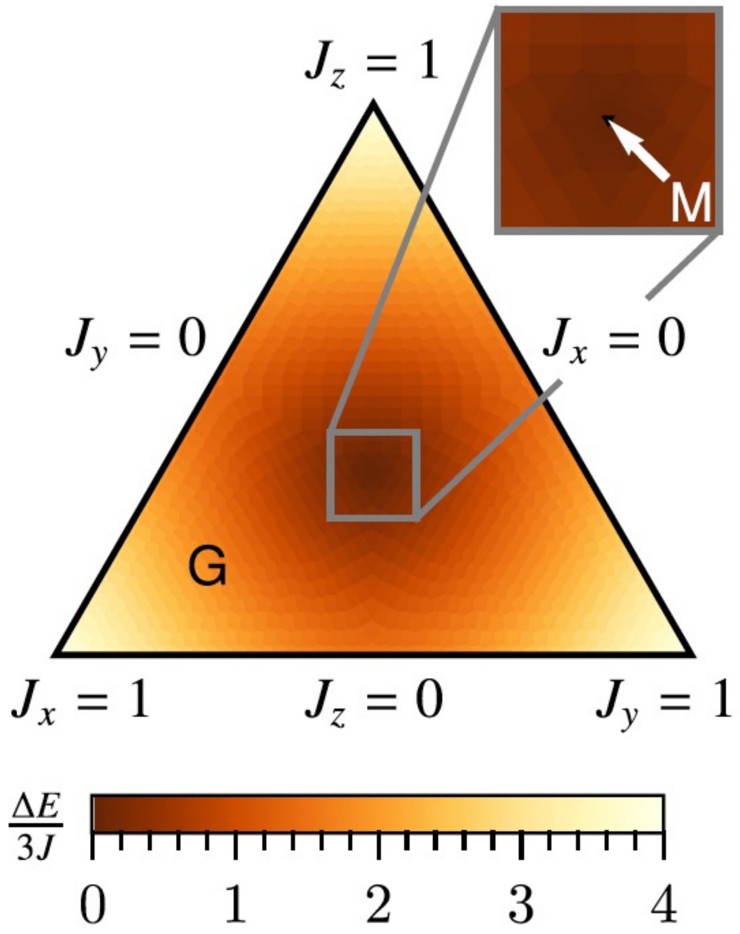


fermion gap

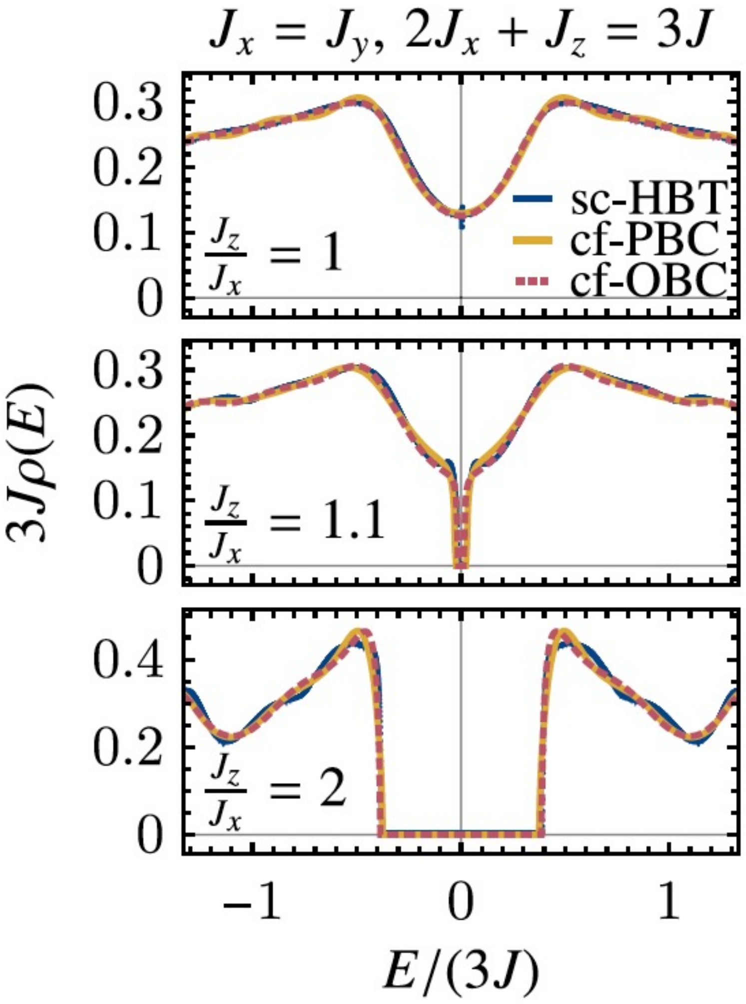


- Unlike honeycomb lattice, gapless isotropic point has **finite** density of states ("Majorana metal")

Phase diagram

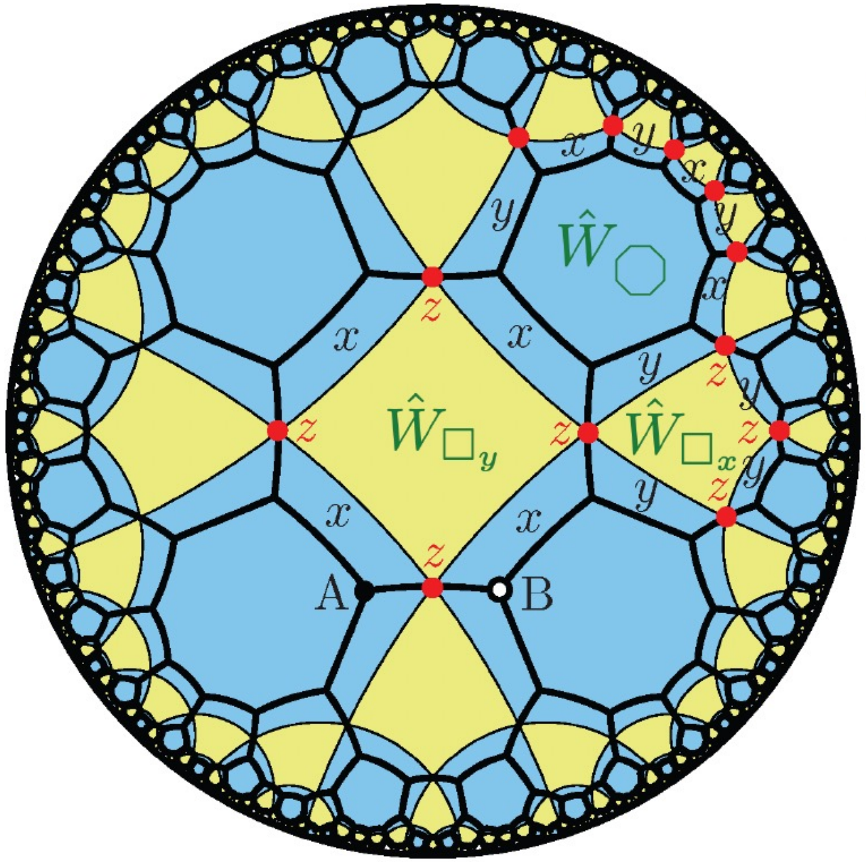


fermion gap



- Unlike honeycomb lattice, gapless isotropic point has **finite** density of states ("Majorana metal")
- Metallic behavior due to "non-Abelian Bloch states" *(Rayan & JM, PNAS '22)*

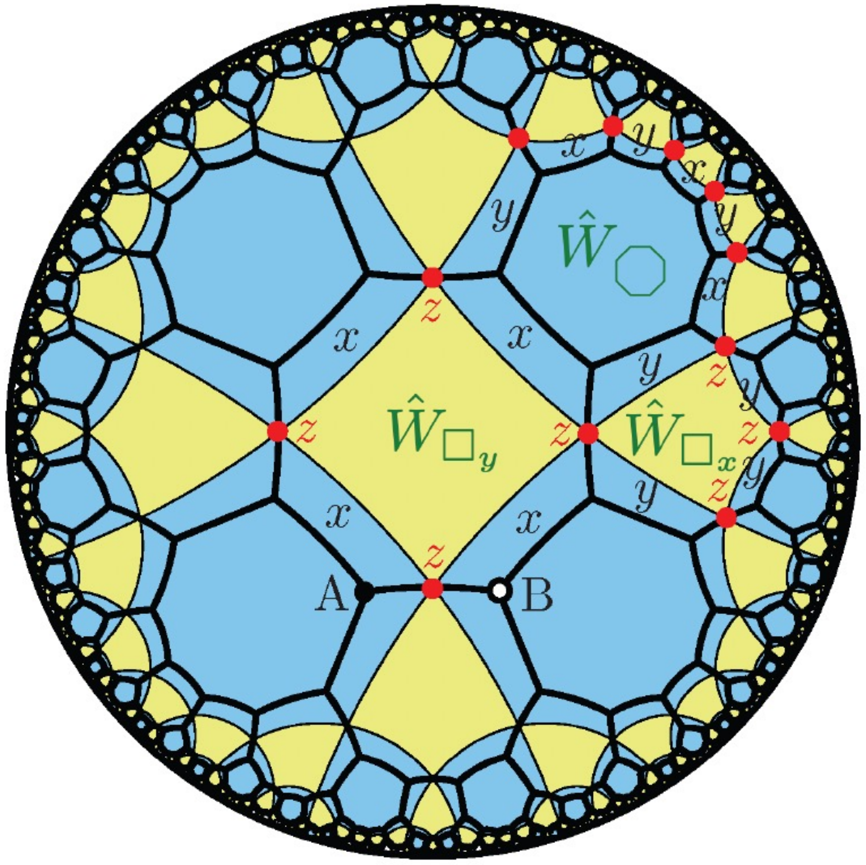
Z_2 spin liquid

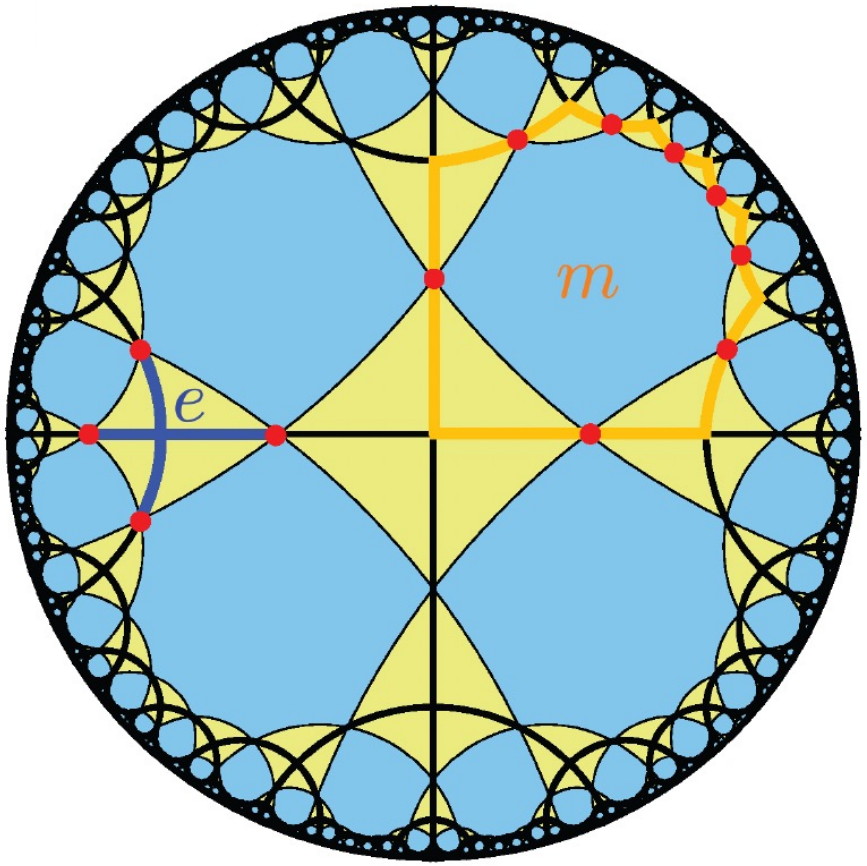


$$\hat{\mathcal{H}}_{\text{eff}} = \frac{5}{16}\frac{J_y^4}{J_z^3}\sum_{\square_x}\hat{W}_{\square_x} + \frac{5}{16}\frac{J_x^4}{J_z^3}\sum_{\square_y}\hat{W}_{\square_y} + \frac{5}{2048}\frac{J_x^4 J_y^4}{J_z^7}\sum_{\bigcirc}\hat{W}_{\bigcirc}$$

- $J_z \gg J_x, J_y$ limit maps to "Wen plaquette model" on (8,4,8,4) Archimedean lattice
- Vortex gap $\Delta/J_z \sim J_x^4 J_y^4/J_z^8$, compared to $\Delta/J_z \sim J_x^2 J_y^2/J_z^4$ on honeycomb lattice

Z_2 spin liquid

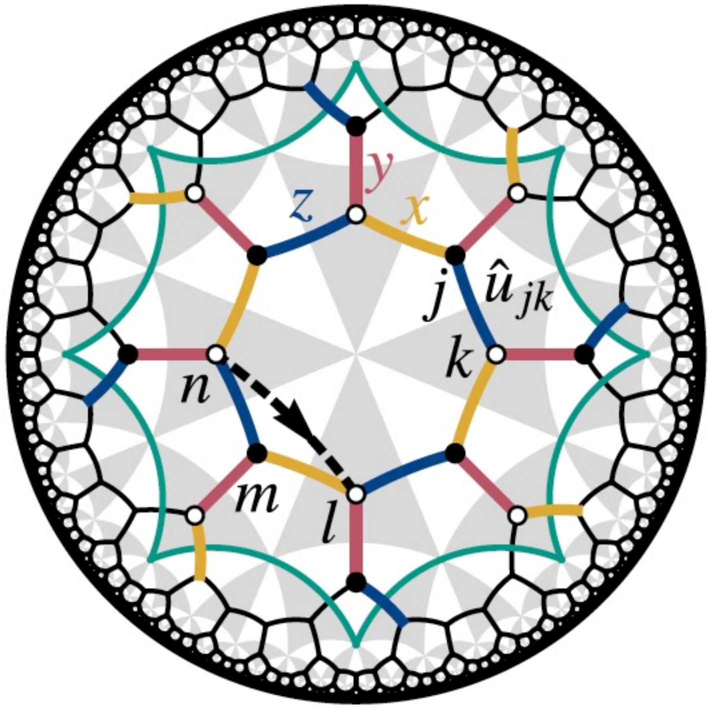


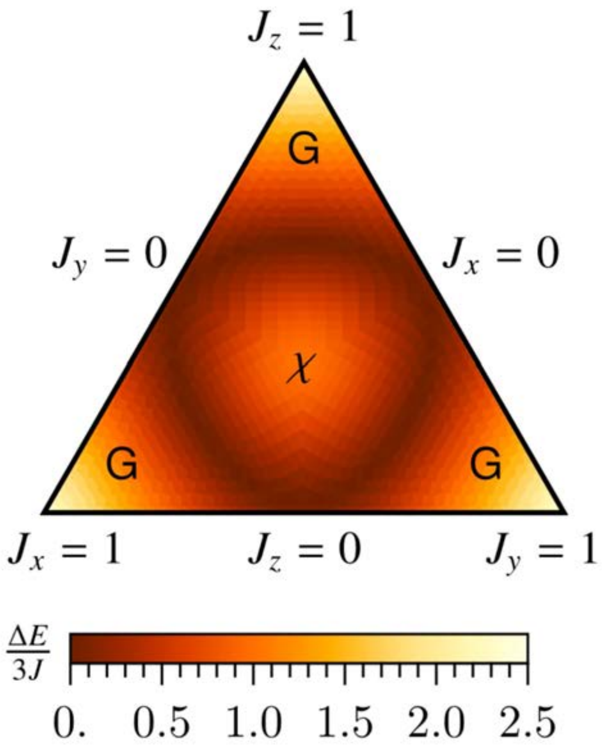


maps to Z_2 toric code on {8,4} lattice

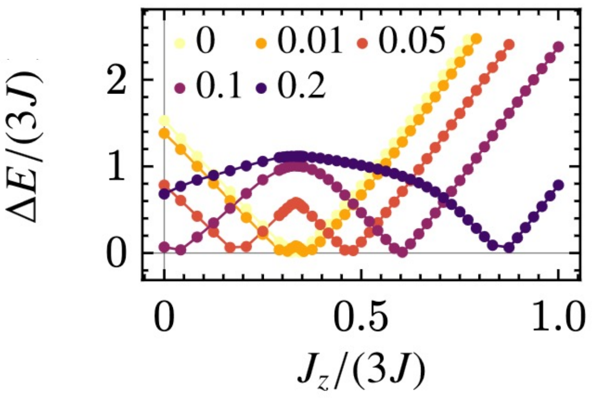
Breaking time-reversal symmetry

$$\hat{\mathcal{H}} = -\sum_{\langle j,k\rangle_\alpha} J_\alpha \hat{\sigma}_j^\alpha \hat{\sigma}_k^\alpha - K \sum_{[lmn]^+_{\alpha\beta\gamma}} \varepsilon_{\alpha\beta\gamma} \hat{\sigma}_l^\alpha \hat{\sigma}_m^\beta \hat{\sigma}_n^\gamma \quad \Rightarrow \quad \hat{\mathcal{H}} = \sum_{\langle j,k\rangle_\alpha} J_\alpha \hat{u}_{jk} \mathrm{i}\hat{c}_j \hat{c}_k + K \sum_{[lmn]^+_{\alpha\beta\gamma}} \hat{u}_{lm}\hat{u}_{mn} \mathrm{i}\hat{c}_l \hat{c}_n$$



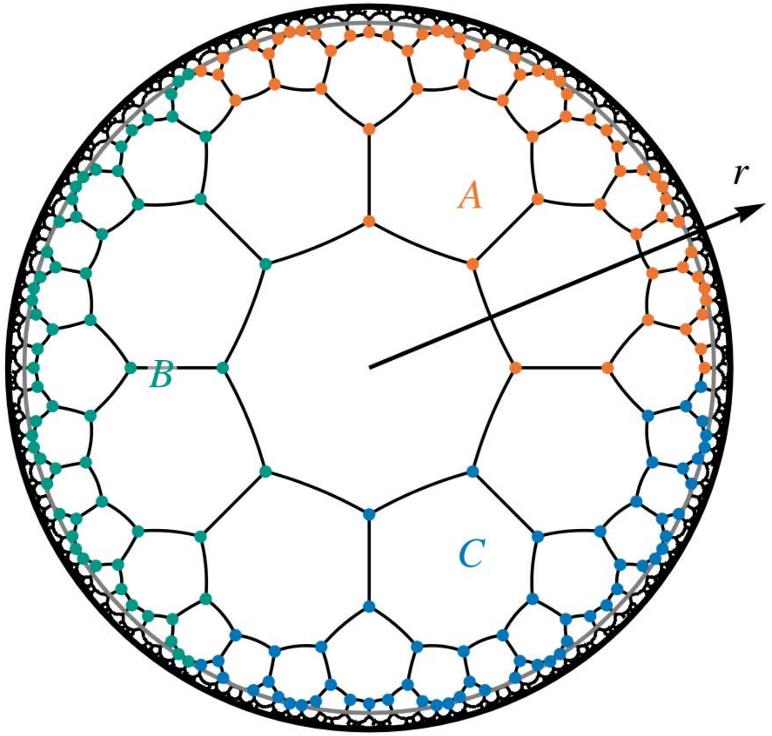


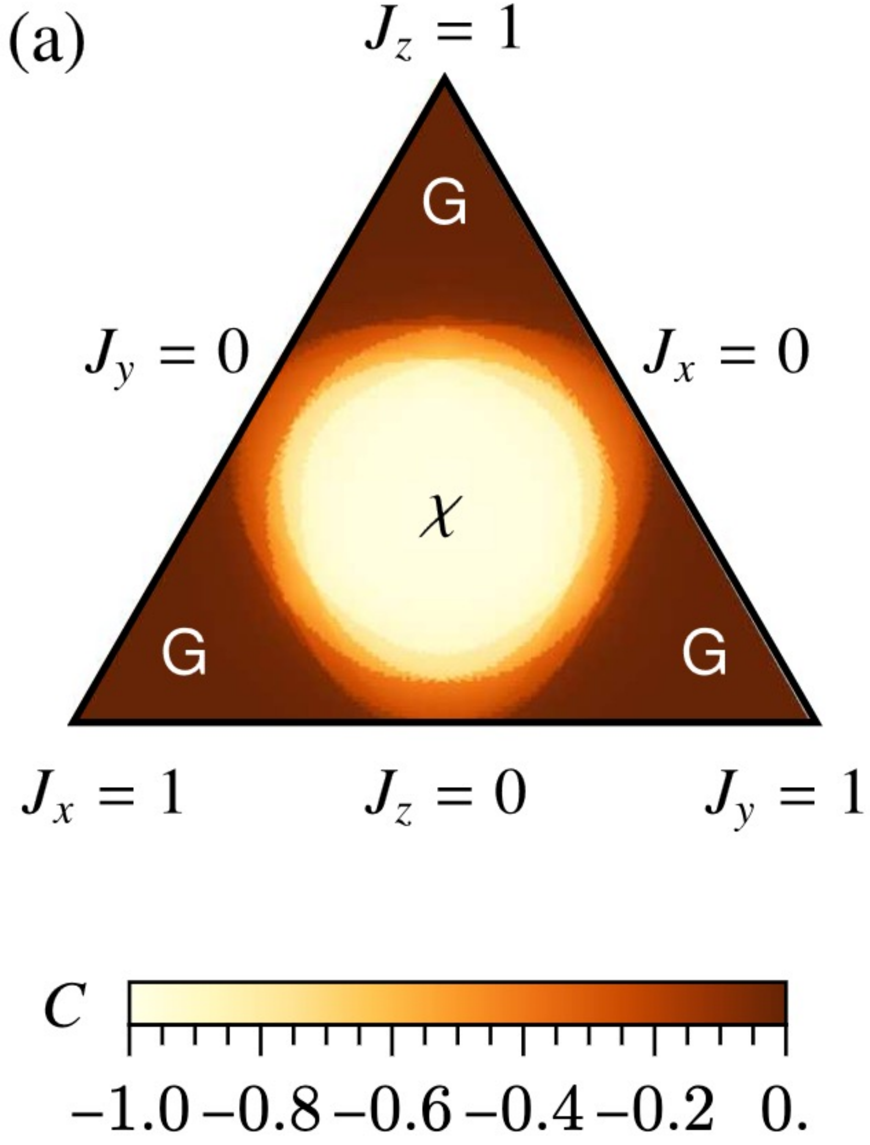
- 3-spin interaction breaks time-reversal symmetry and gaps out Majorana metal



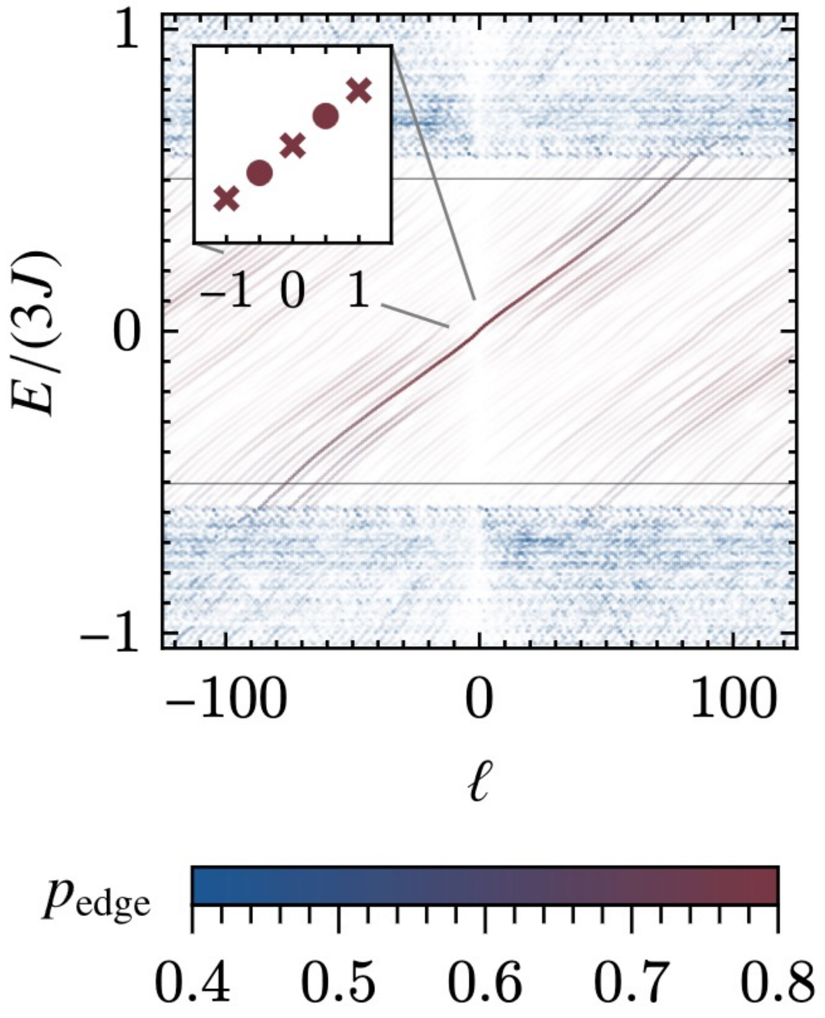
Real-space Chern number

$$C = 12\pi \mathrm{i} \sum_{j\in A} \sum_{k\in B} \sum_{l\in C} \left(P_{jk}P_{kl}P_{lj} - P_{jl}P_{lk}P_{kj}\right)$$



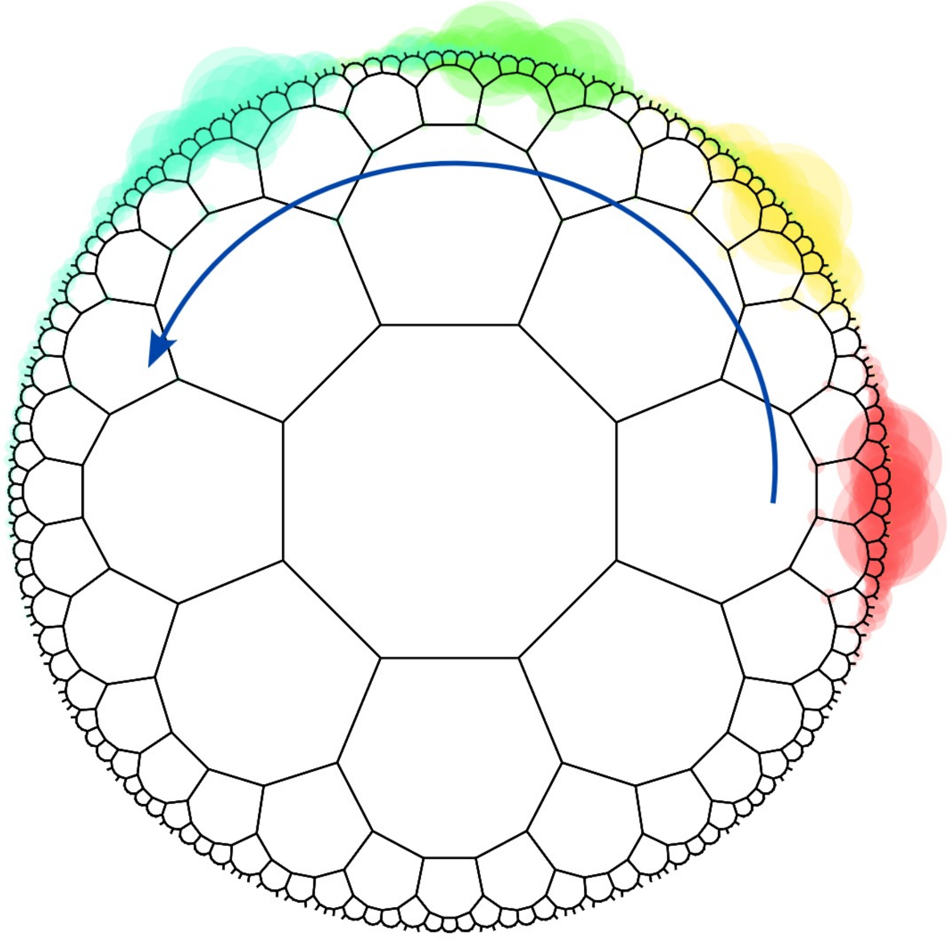


Chiral Majorana edge states



- In-gap boundary modes with chiral angular momentum dispersion $E \propto \ell$ with half-integer quantization $\ell \in \mathbb{Z} + \frac{1}{2}$
- Inserting vortex at center of the disk shifts angular momentum by 1/2

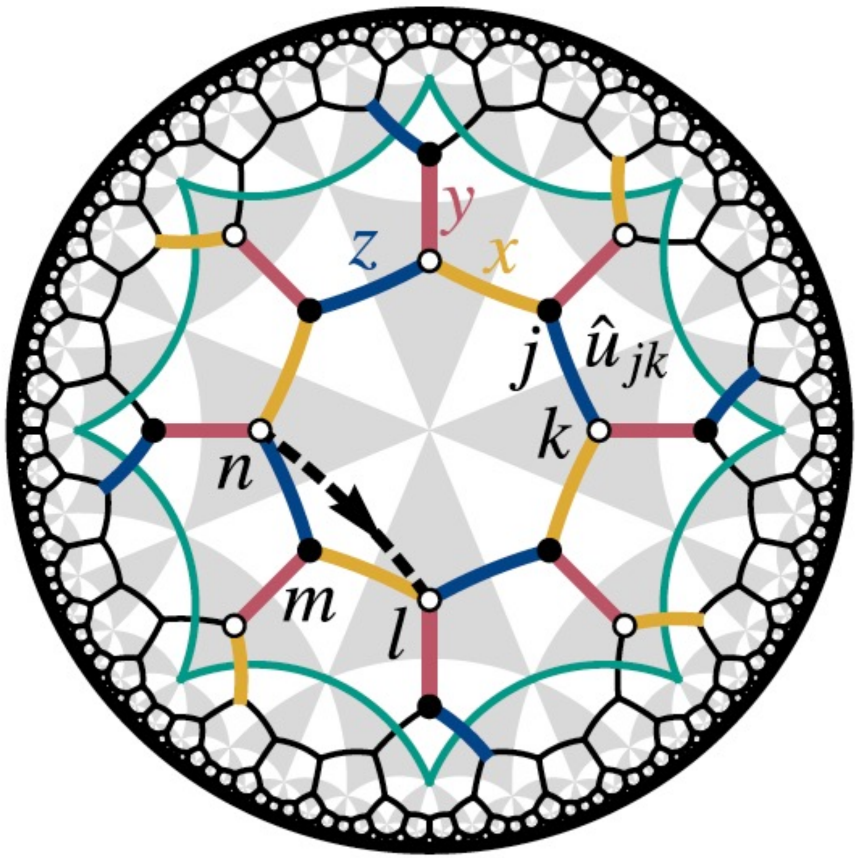
Chiral Majorana edge states



- In-gap boundary modes with chiral angular momentum dispersion $E \propto \ell$ with half-integer quantization $\ell \in \mathbb{Z} + \frac{1}{2}$
- Inserting vortex at center of the disk shifts angular momentum by 1/2
- Wave packet propagates with definite chirality

Summary [Part II]

- Kitaev model can be generalized to $\{p,3\}$ hyperbolic lattices
- p even (bipartite):
  - symmetric 3-edge coloring exists, ground-state flux known through Lieb's theorem
  - Gapped/gapless Z_2 spin liquids
- p odd (non-bipartite):
  - 3-edge coloring exists in principle (Tait's theorem)
  - Odd cycles give chiral spin liquid with *spontaneous* T-breaking *(Dusel et al., PRL 2025; Vidal & Mosseri, PRB 2025)*



Phys. Rev. Lett. 135, 076604 (2025)

Lieb-Schultz-Mattis constraints for hyperbolic lattices

arXiv:2605.15974

G. Shankar
(Yale U.)

Lieb-Schultz-Mattis theorem

- General constraint on ground states of quantum many-body systems with **global U(1) symmetry (charge / S_z conservation)** + **lattice translation symmetry**
- "*A trivial (unique, symmetric, gapped) ground state is forbidden at fractional filling / spin per unit cell*"
- Proved in $d = 1$ by Lieb, Schultz, & Mattis (1961)
- Extended to $d > 1$ by Oshikawa (2000) and Hastings (2004)

Lieb-Schultz-Mattis theorem

- *“A trivial (unique, symmetric, gapped) ground state is forbidden at fractional filling / spin per unit cell”*
- What is the ground-state physics at fractional filling / spin per unit cell?

gapless

e.g. metal, spin-1/2 AF Heisenberg chain

gapped, degenerate

broken translation symmetry

e.g. CDW insulator, spin-1/2 Majumdar-Ghosh chain (dimerized)

Lieb-Schultz-Mattis theorem

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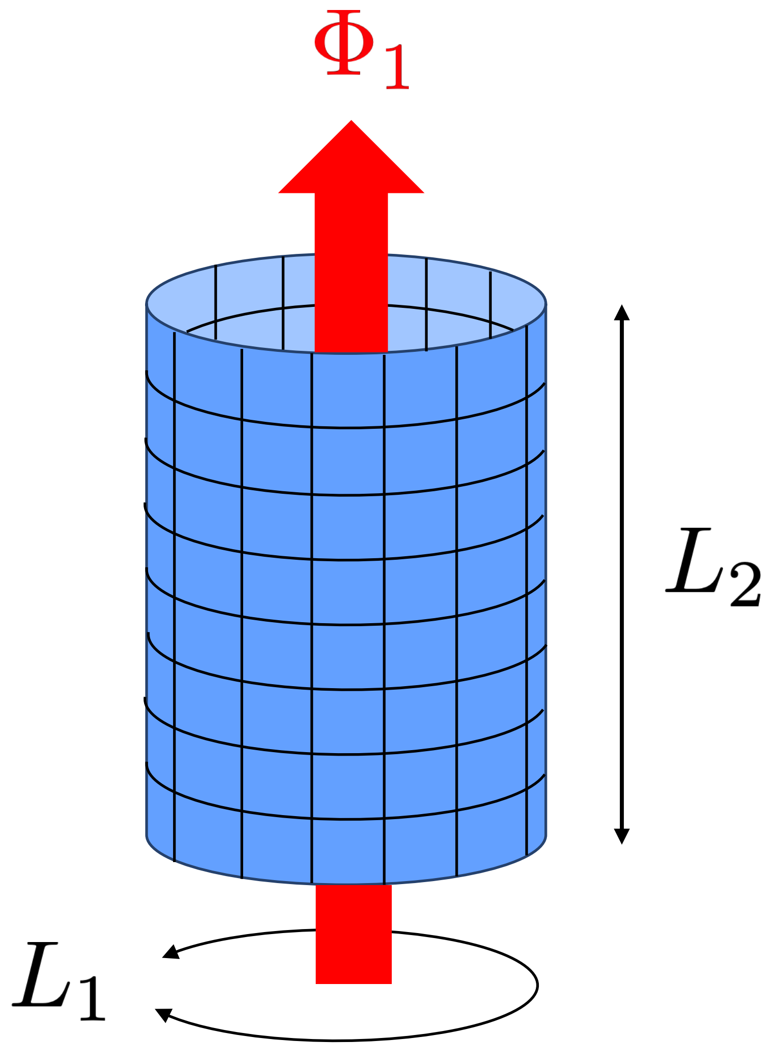
topological order

e.g. FQHE, Z_2 spin liquid

Oshikawa's topological argument

$$H = -t \sum_{r} \sum_{j=1,2} e^{-iA_j} b_r^\dagger b_{r+e_j} + \text{h.c.} + H_{\text{int}}[n]$$

- Hardcore bosons / spinless fermions at filling ν



Oshikawa, PRL 84, 1535 (2000)

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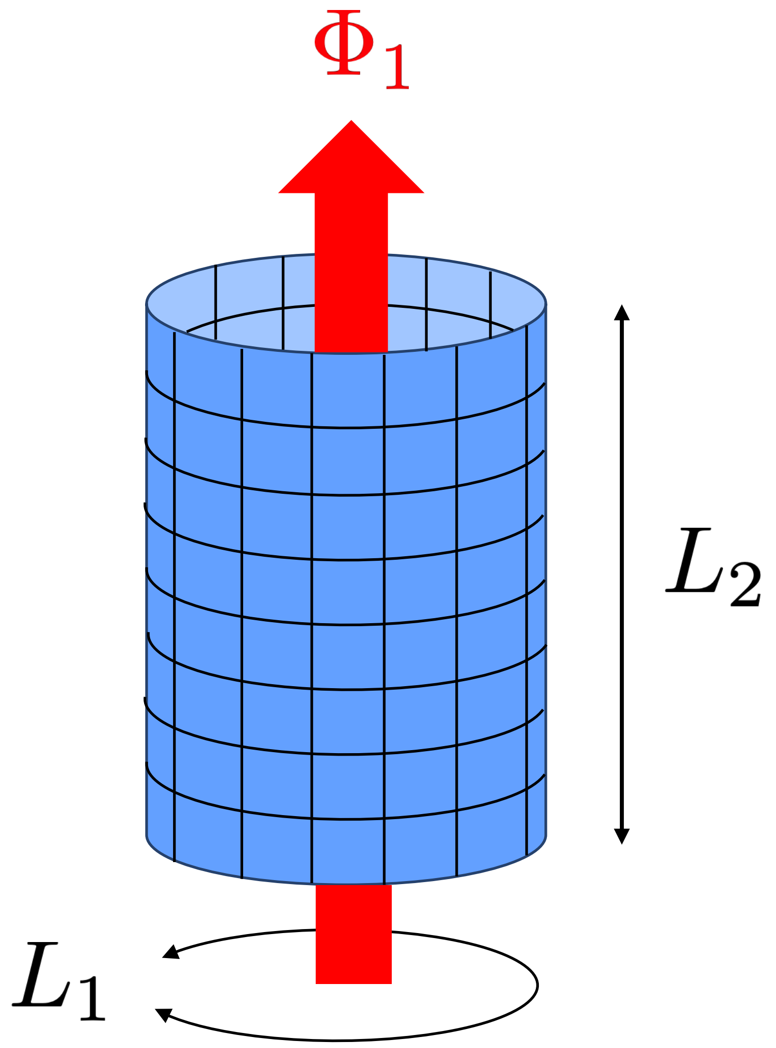

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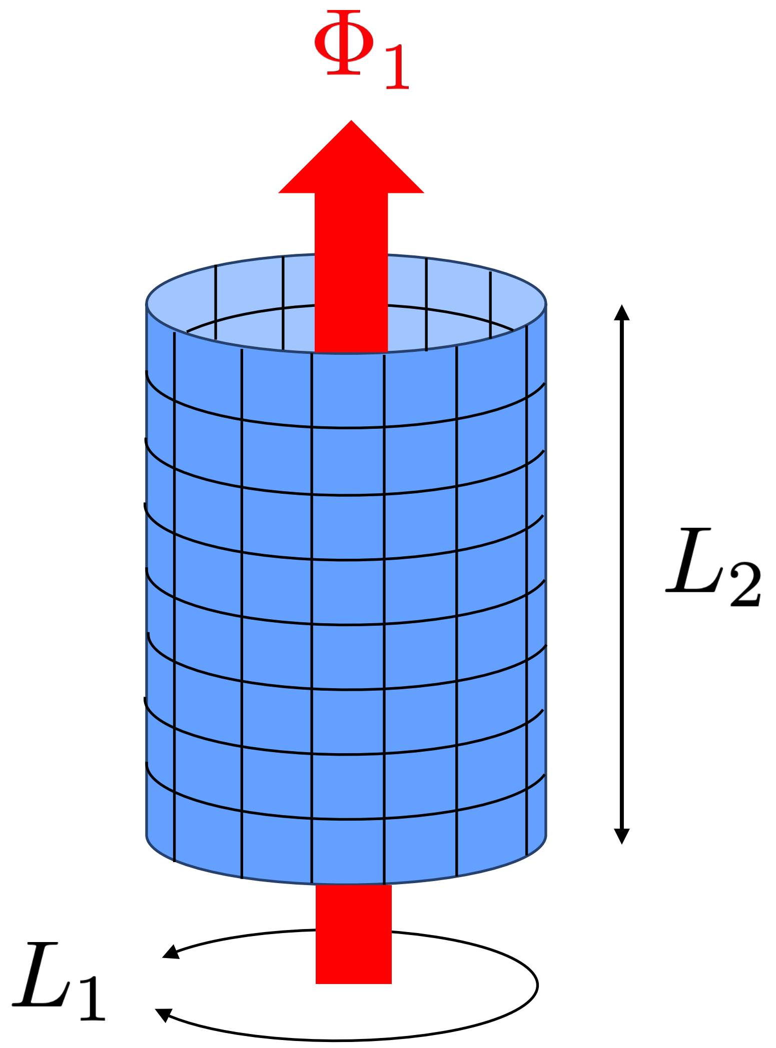

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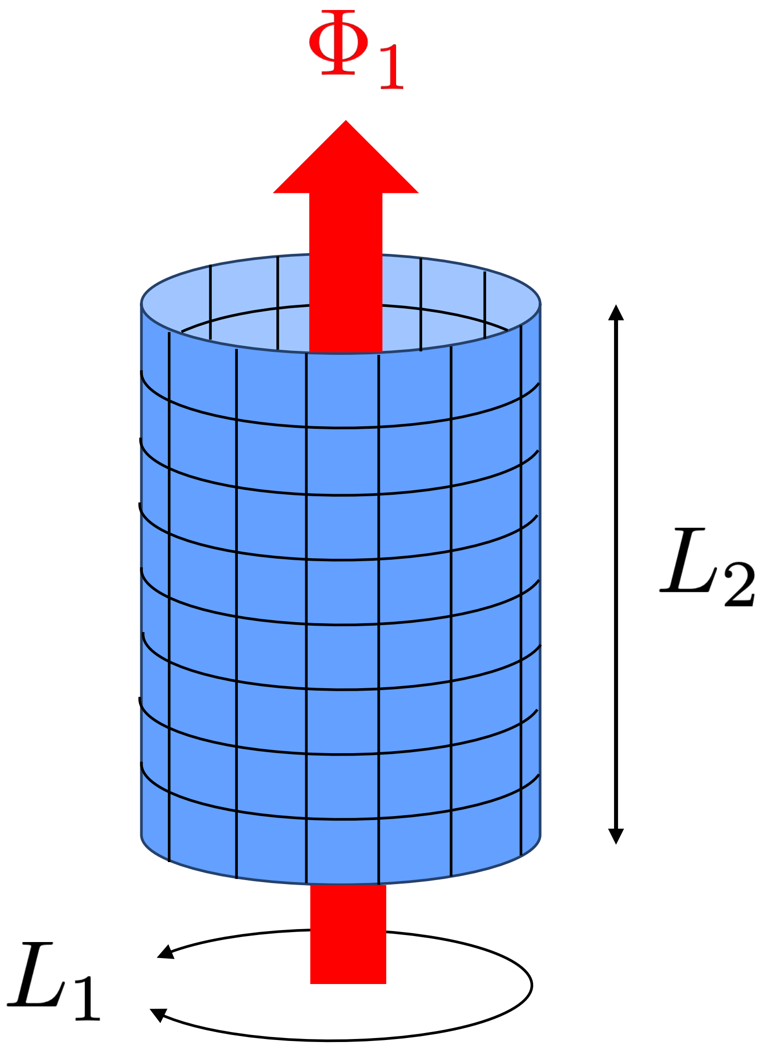

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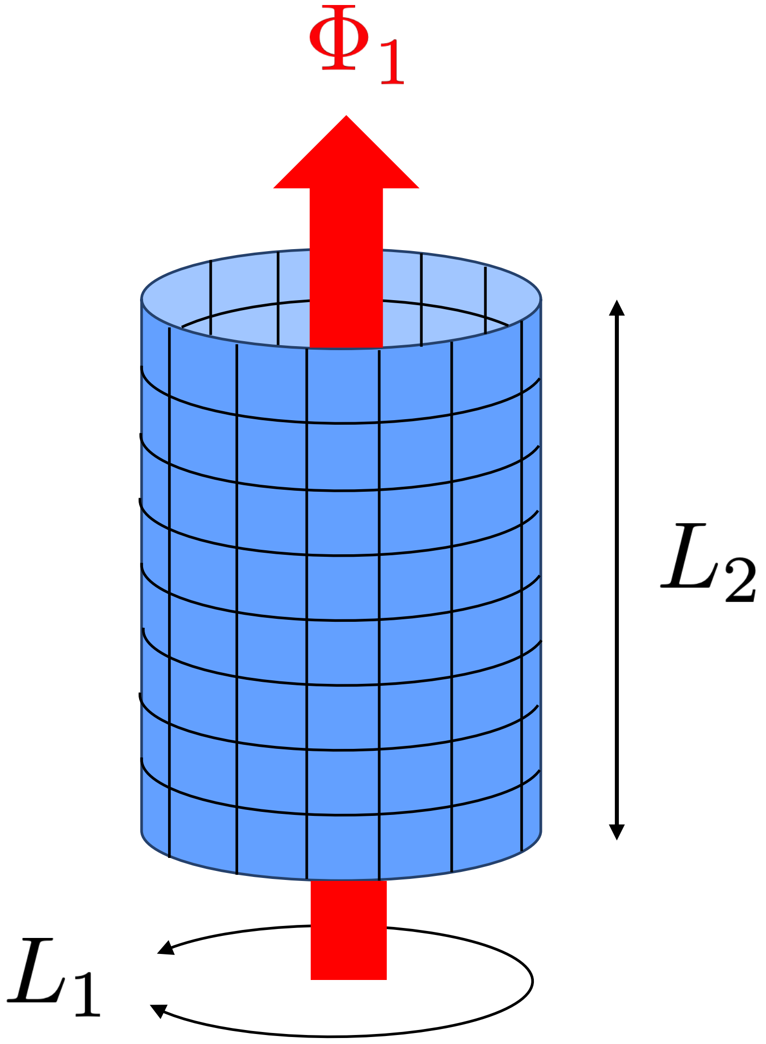
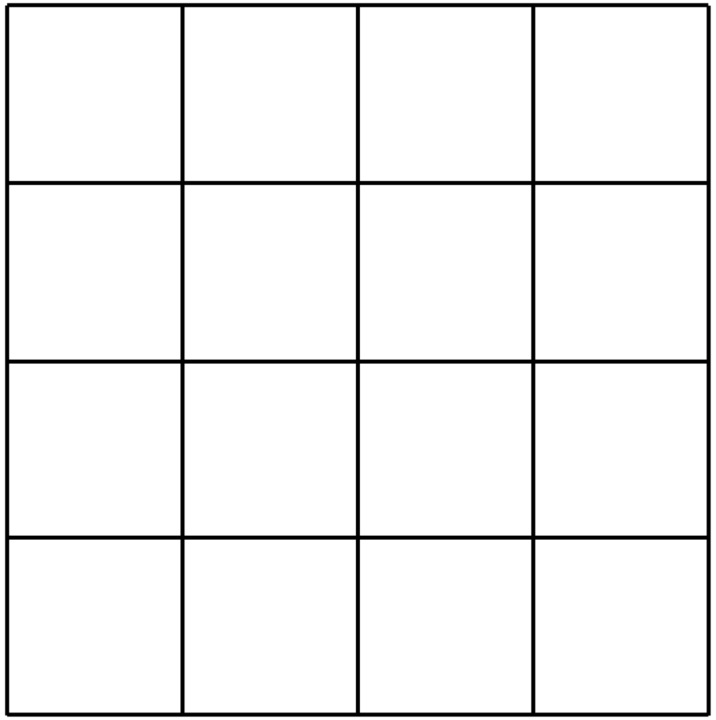


Oshikawa, PRL 84, 1535 (2000)

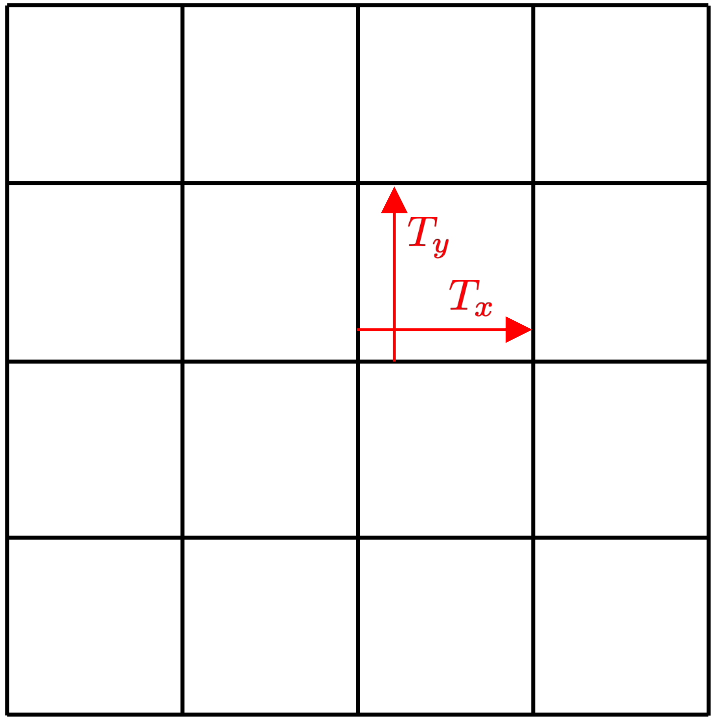
Euclidean crystallography



$$\mathcal{G} = p4m$$

- Space group G: full symmetry group (infinite), discrete subgroup of Euclidean group $E(2)$

Euclidean crystallography

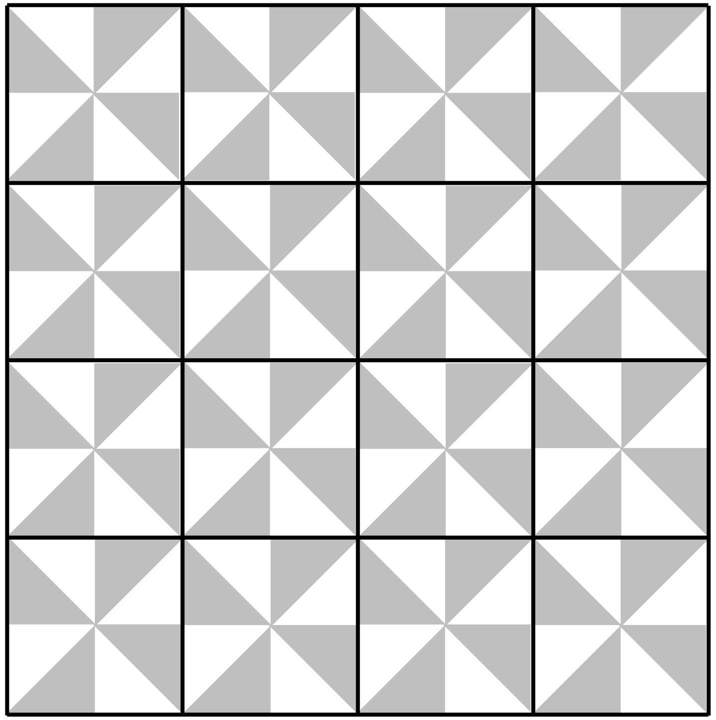


$$\mathcal{G} = p4m$$

$$\mathcal{T} = \langle T_x, T_y : T_x T_y T_x^{-1} T_y^{-1} \rangle \cong \mathbb{Z}^2$$

- Space group G: full symmetry group (infinite), discrete subgroup of Euclidean group $E(2)$
- **Translation group** $\boldsymbol{T}$: torsion-free normal subgroup of G (infinite)

Euclidean crystallography



$$\mathcal{G} = p4m = \Delta(2,4,4)$$

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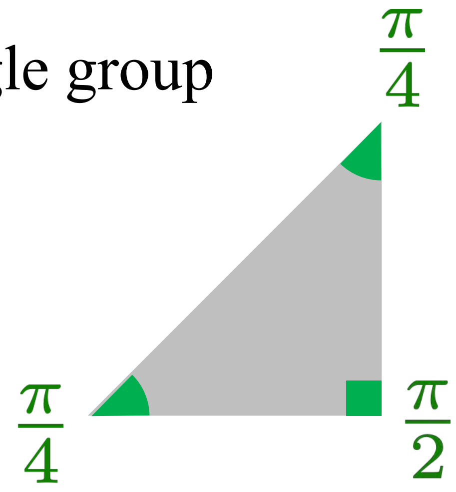
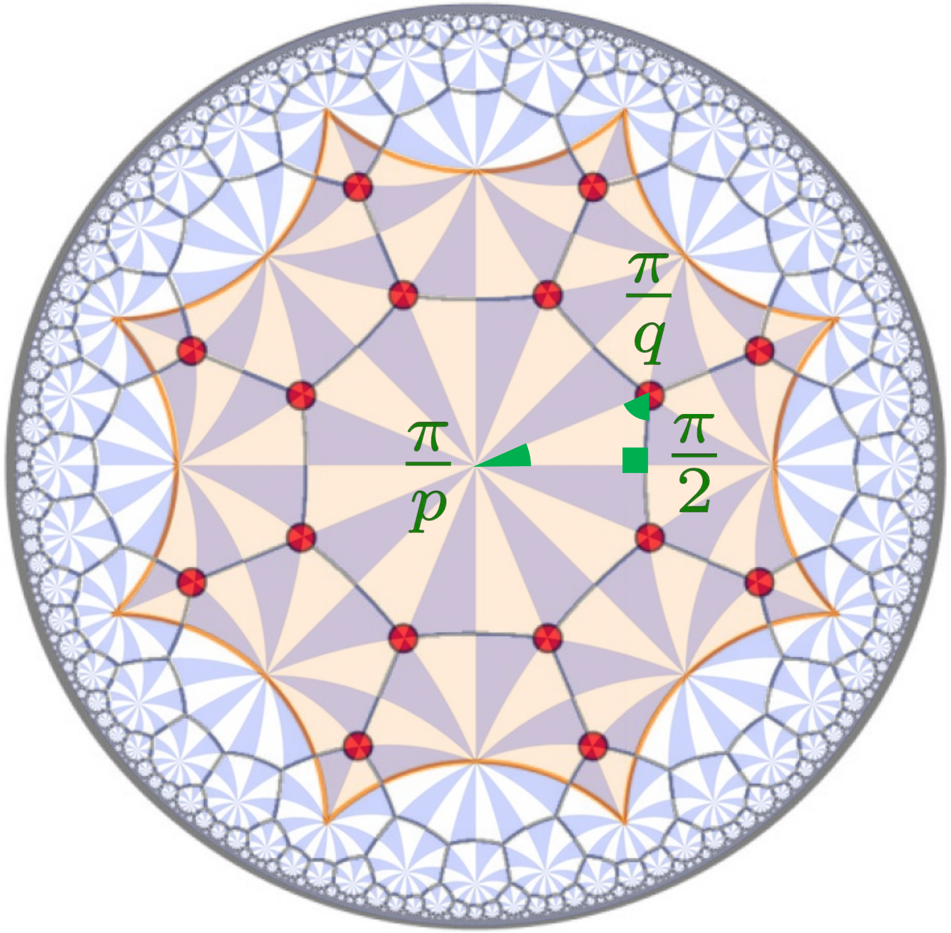
- Space group G = Euclidean triangle group



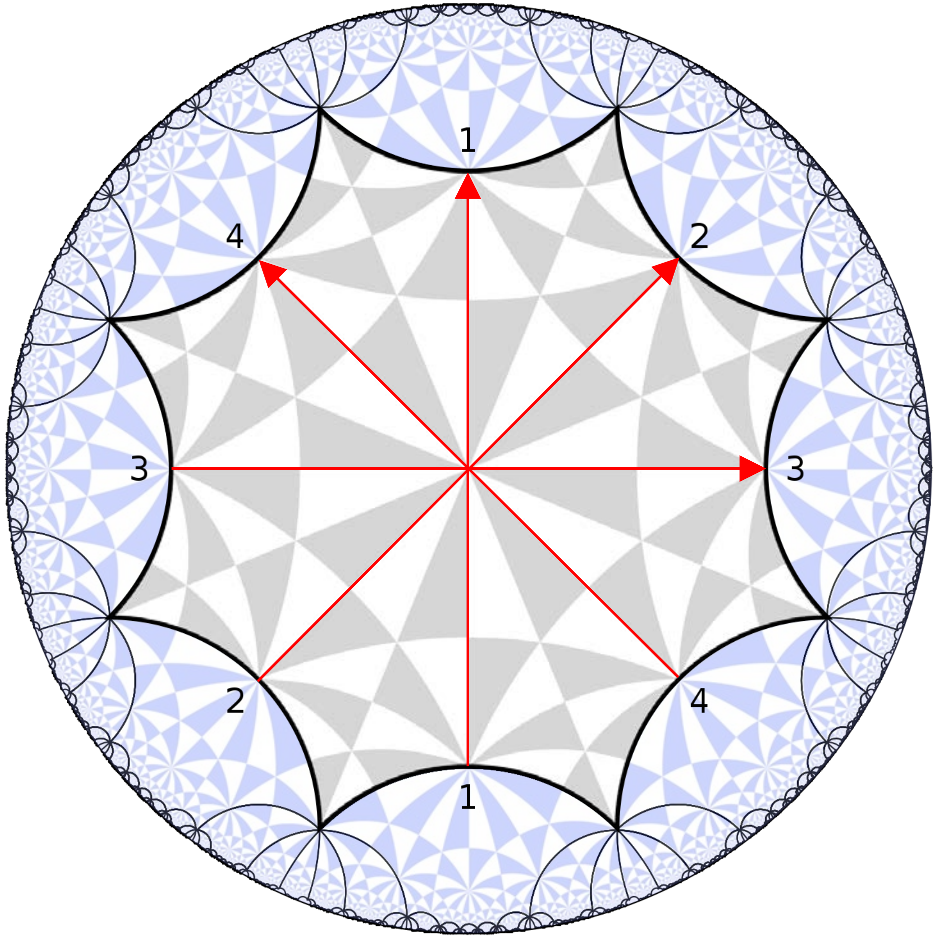
Hyperbolic crystallography



$$\mathcal{G} = \Delta(2, q, p)$$

- Space group G: hyperbolic triangle group, Fuchsian group = discrete subgroup of $PSL(2,\mathbb{R})$

Hyperbolic crystallography



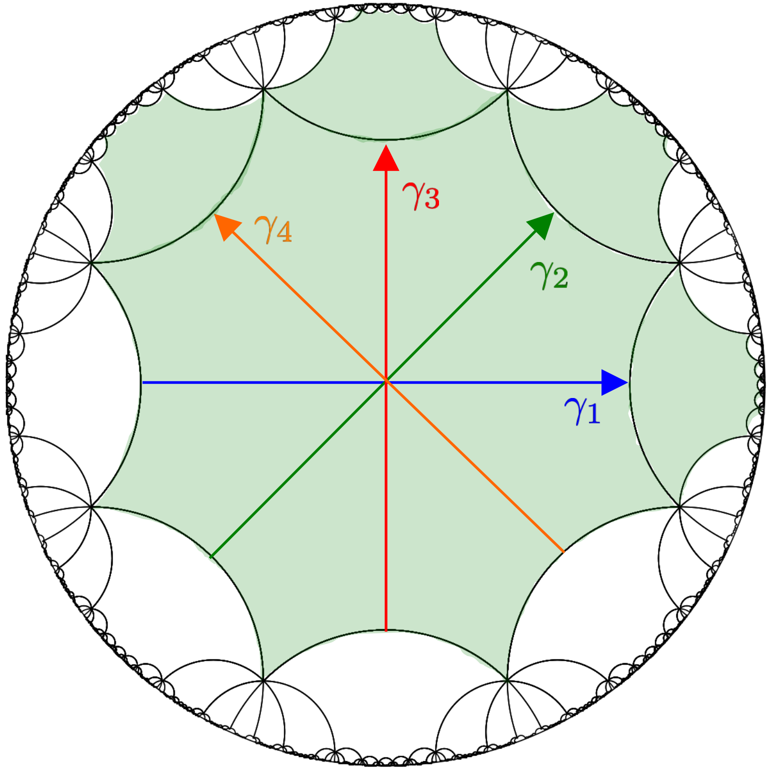
$$\mathcal{G} = \Delta(2, q, p)$$

$$\mathcal{T} = \Gamma \triangleleft \mathcal{G}$$

- Space group G: hyperbolic triangle group, Fuchsian group = discrete subgroup of $PSL(2,\mathbb{R})$
- **Translation group Γ**: torsion-free normal subgroup of G (Fenchel's conjecture)

JM & Rayan, Sci. Adv. '21, PNAS '22; Boettcher, Gorshkov, Kollár, JM, Rayan, Thomale, PRB '22

Hyperbolic translations



$$\Gamma = \langle \gamma_1, \gamma_2, \gamma_3, \gamma_4 : \gamma_1 \gamma_2^{-1} \gamma_3 \gamma_4^{-1} \gamma_1^{-1} \gamma_2 \gamma_3^{-1} \gamma_4 = 1 \rangle$$

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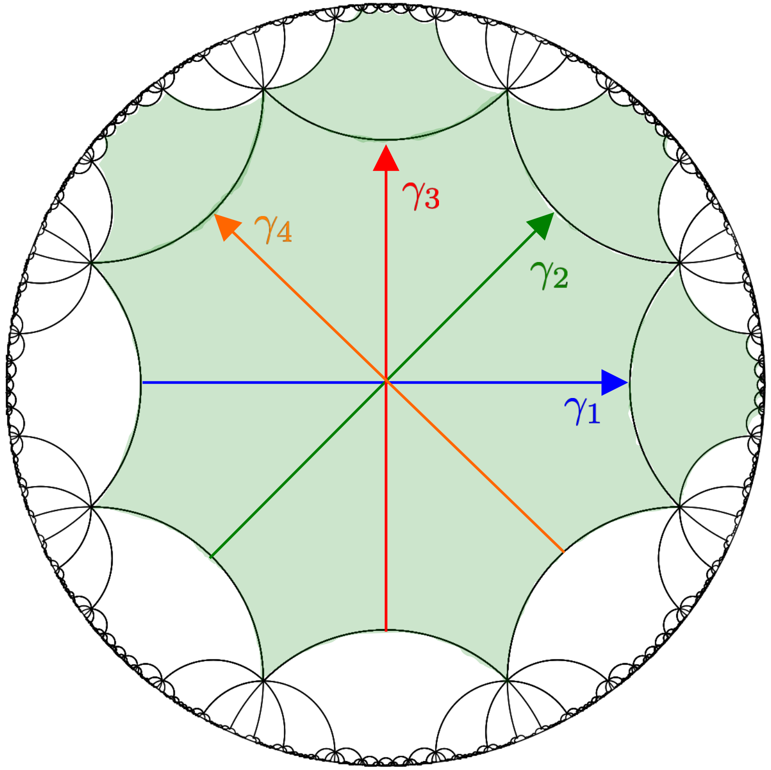
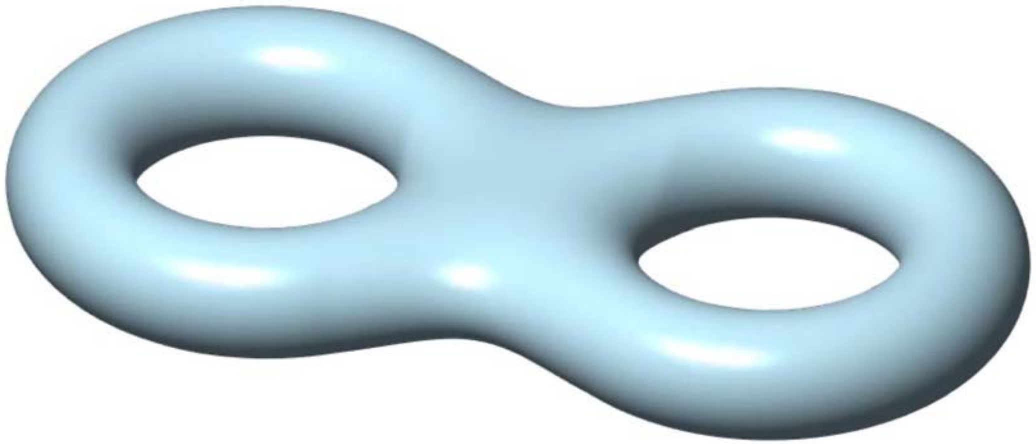


$\cong$



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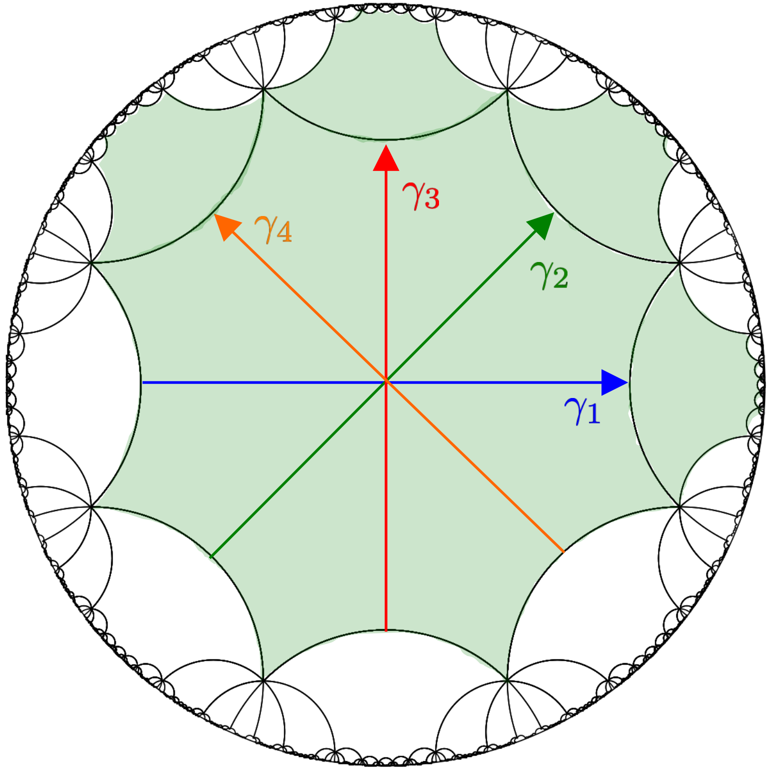
Hyperbolic crystal momentum

$$\psi(\gamma_j(z)) = e^{ik_j}\psi(z),$$

$$j = 1, 2, 3, 4$$



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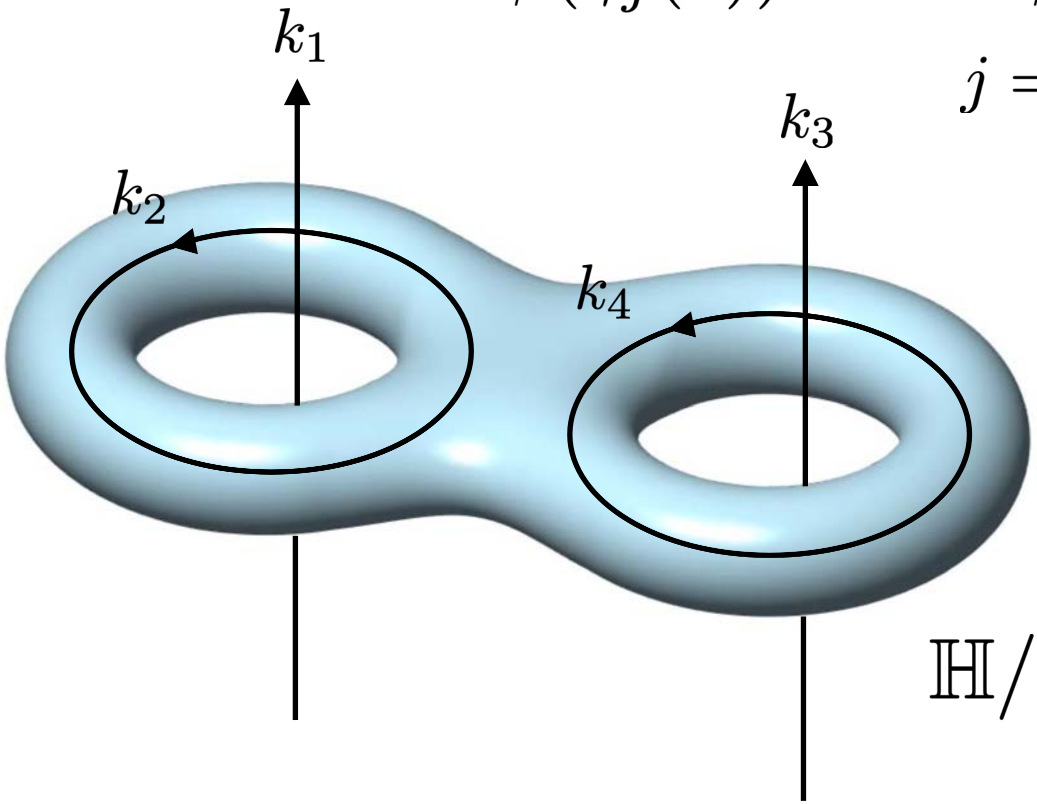

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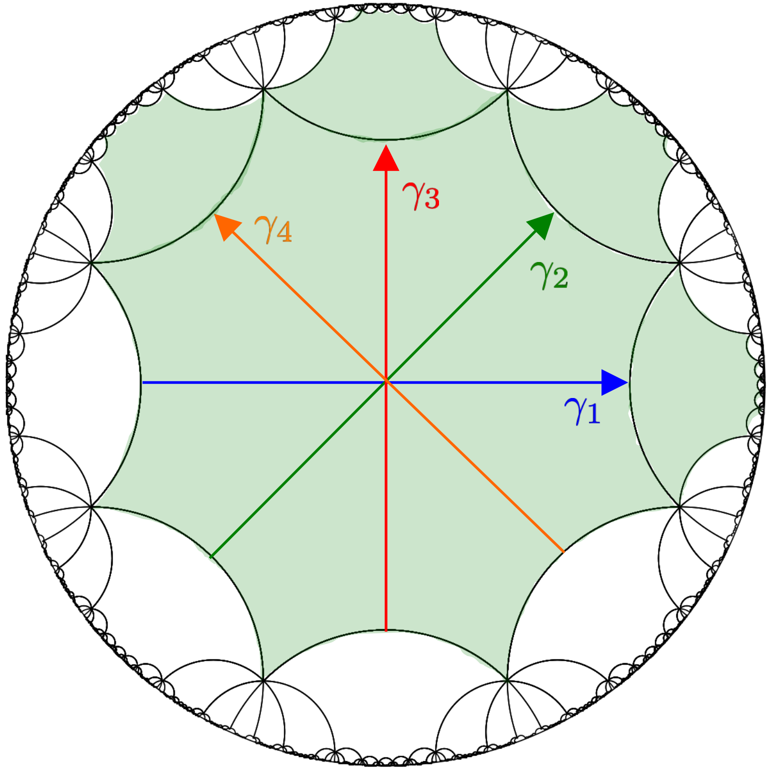
Many-body flux insertion

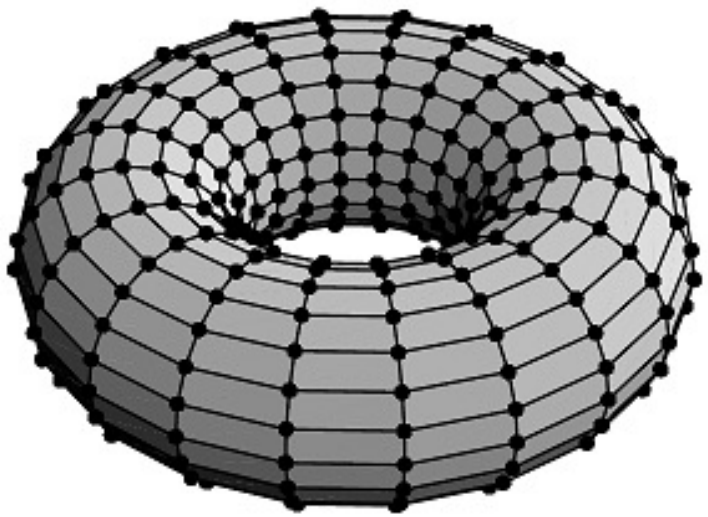


$$H = -t \sum_{\gamma \in \Gamma} \sum_{j=1}^{2g} e^{-iA_j} b_\gamma^\dagger b_{\gamma\gamma_j} + \mathrm{h.c.} + H_{\mathrm{int}}[n]$$

- How to define a finite periodic lattice (cylinder/torus) for momentum counting?

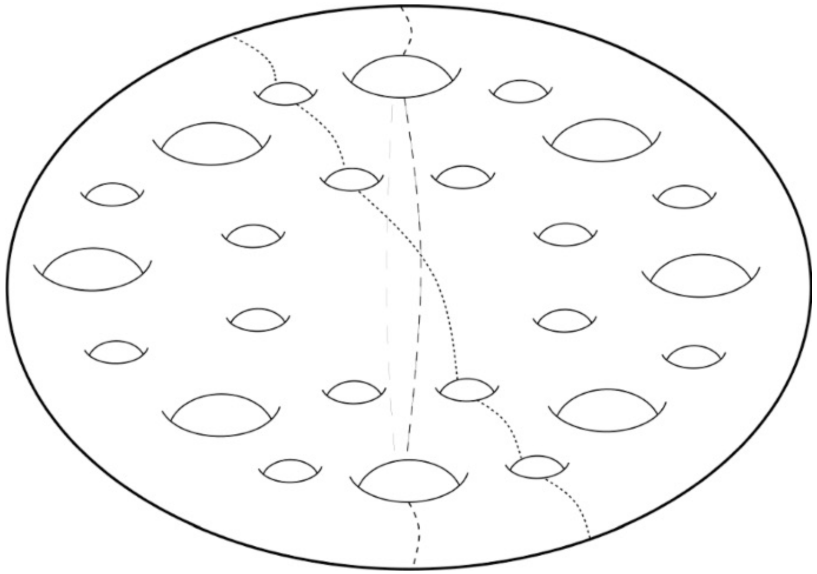
Euclidean vs hyperbolic compactifications

Euclidean



- Finite quotient $\mathbb{Z}^2/(L_1\mathbb{Z} \times L_2\mathbb{Z})$

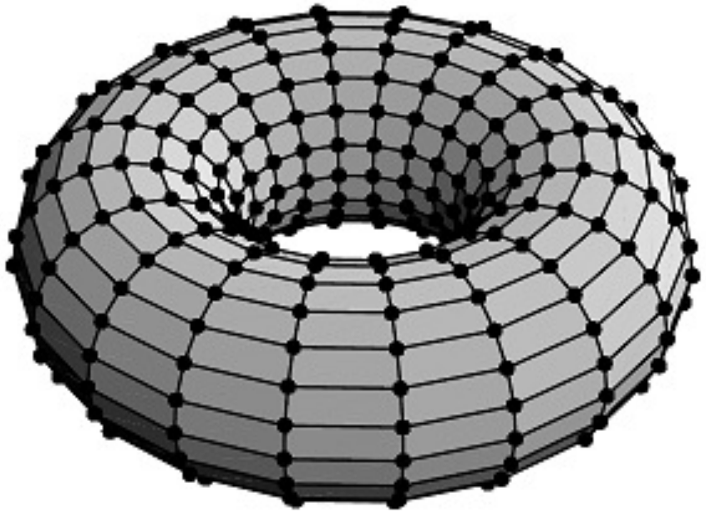
hyperbolic



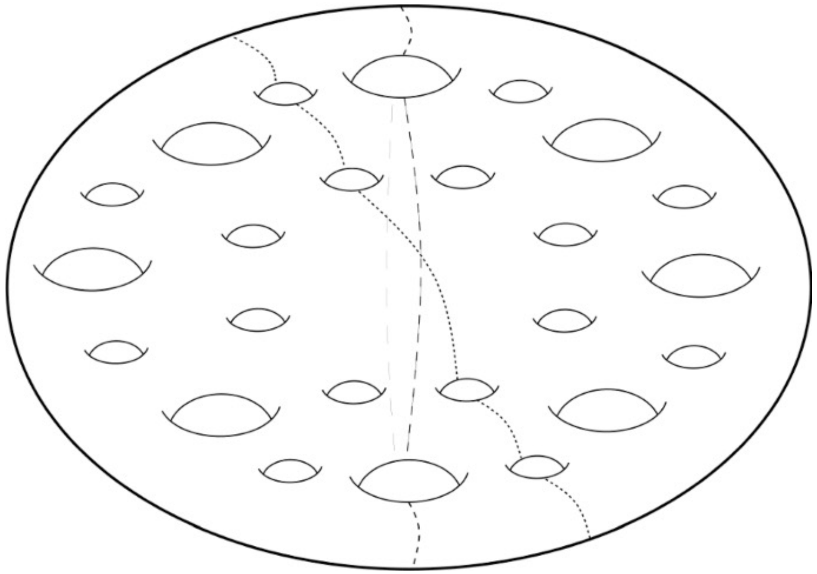
- Finite quotient $\Gamma/\Gamma_{\mathrm{PBC}}$

Euclidean vs hyperbolic compactifications

Euclidean



hyperbolic



- Finite quotient $\mathbb{Z}^2/(L_1\mathbb{Z} \times L_2\mathbb{Z})$
- 2 momenta P_1, P_2

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- $2g$ momenta $P_1, \ldots, P_{2g}$

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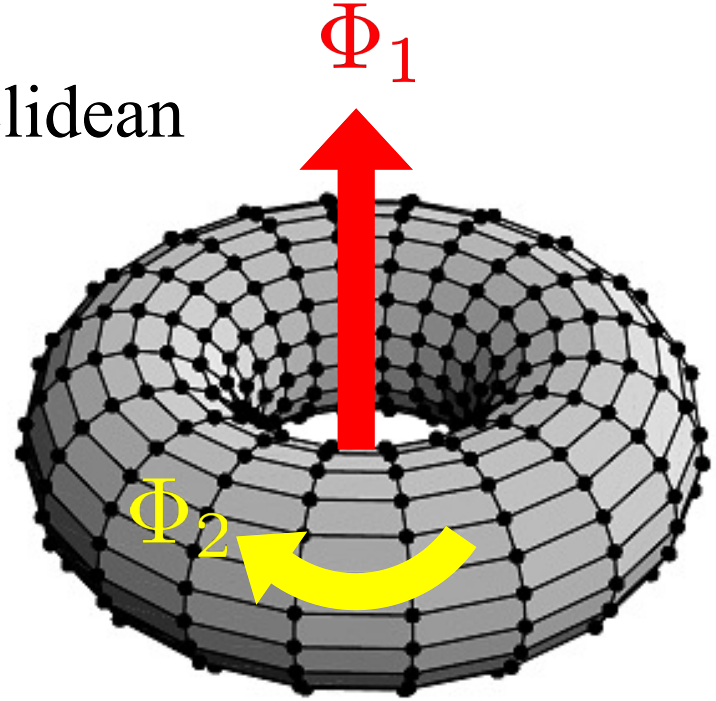
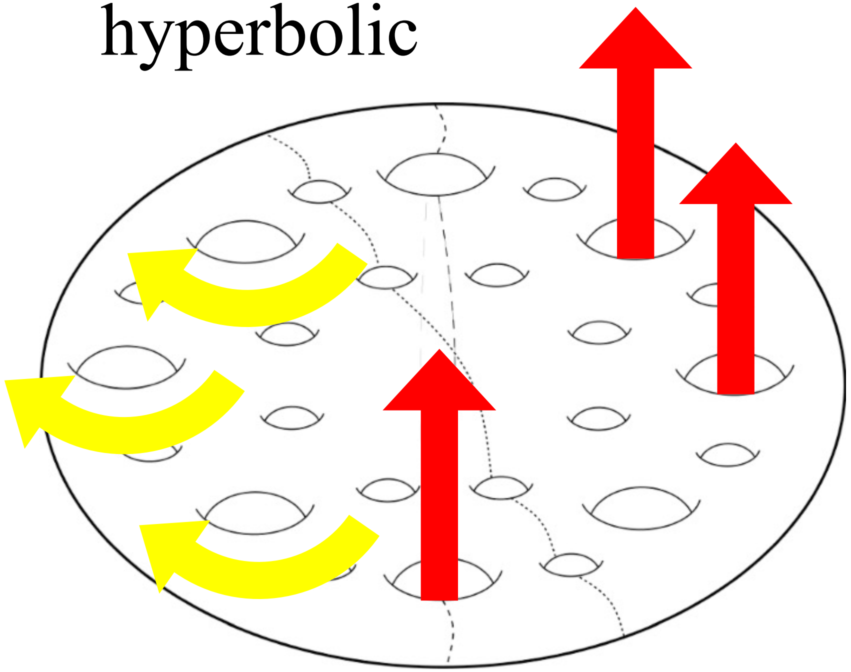


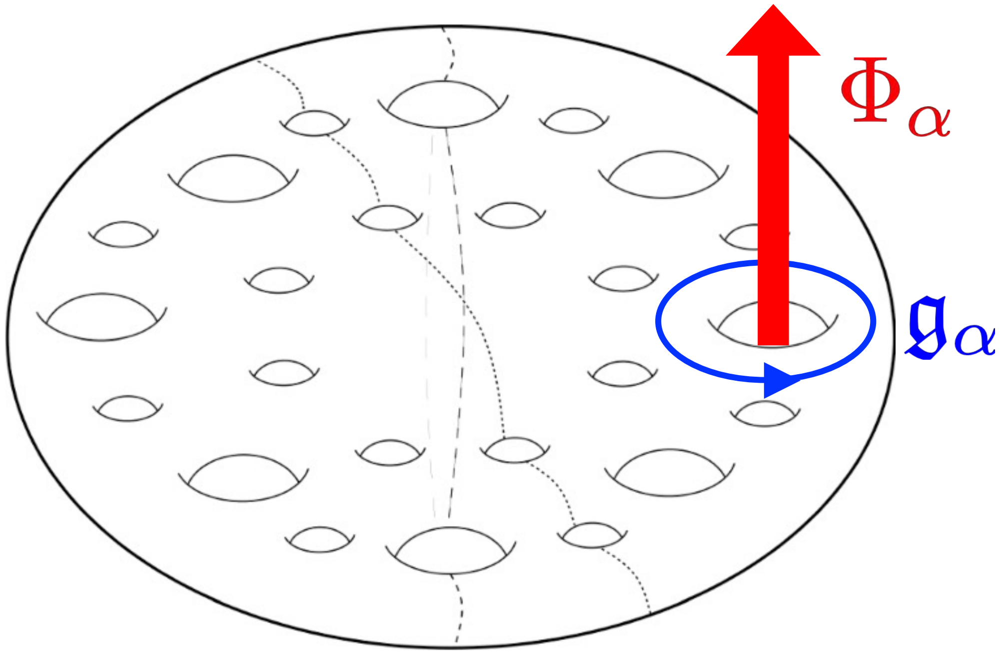
hyperbolic



- Finite quotient $\mathbb{Z}^2/(L_1\mathbb{Z} \times L_2\mathbb{Z})$
- 2 momenta P_1, P_2
- 2 fluxes Φ_1, Φ_2

- Finite quotient $\Gamma/\Gamma_{\text{PBC}}$
- $2g$ momenta $P_1, \ldots, P_{2g}$
- $2h$ fluxes $\Phi_1, \ldots, \Phi_{2h}$ with $h >> g$

The flux lattice



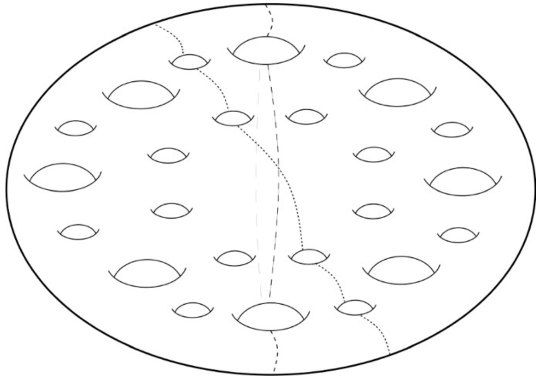
$$\Phi_\alpha = \sum_{j=1}^{2g} \Lambda_j(\mathfrak{g}_\alpha) A_j$$

- Abelianization map $\Lambda : \Gamma \to \Gamma/[\Gamma,\Gamma]$
- For N particles, allowed momentum differences $\Delta P_j/N = A_j$ live in **dual lattice** $\Lambda(\Gamma_{PBC})^*$
- $\mathbf{\Delta P} \neq 0 \pmod{2\pi}$ for certain compactifications implies trivial ground state is impossible

Elementary divisors

- $\Lambda(\Gamma_{\text{PBC}})^*$ can be determined by first "diagonalizing" $\Lambda(\Gamma_{\text{PBC}})$ over $\mathbb{Z}$ (**Smith normal form**), with "eigenvalues" $L_1, \ldots, L_{2g}$ (**elementary divisors**)

$$\mathbb{Z}^{2g}/\Lambda_{\text{PBC}} \cong \mathbb{Z}/L_1\mathbb{Z} \oplus \mathbb{Z}/L_2\mathbb{Z} \oplus \cdots \oplus \mathbb{Z}/L_{2g}\mathbb{Z}$$

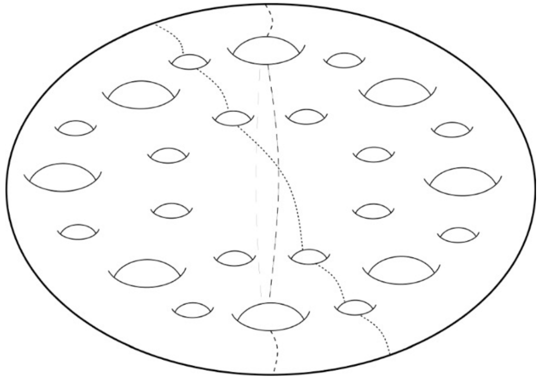


$L_j \sim$ "length" of periodic lattice along "direction" j

Ground-state degeneracy

$$\mathrm{GSD} \geq \prod_{j=1}^{2g} \frac{L_j}{\gcd(N, L_j)}$$

- Lower bound for ground-state degeneracy of N-particle system on periodic hyperbolic lattice



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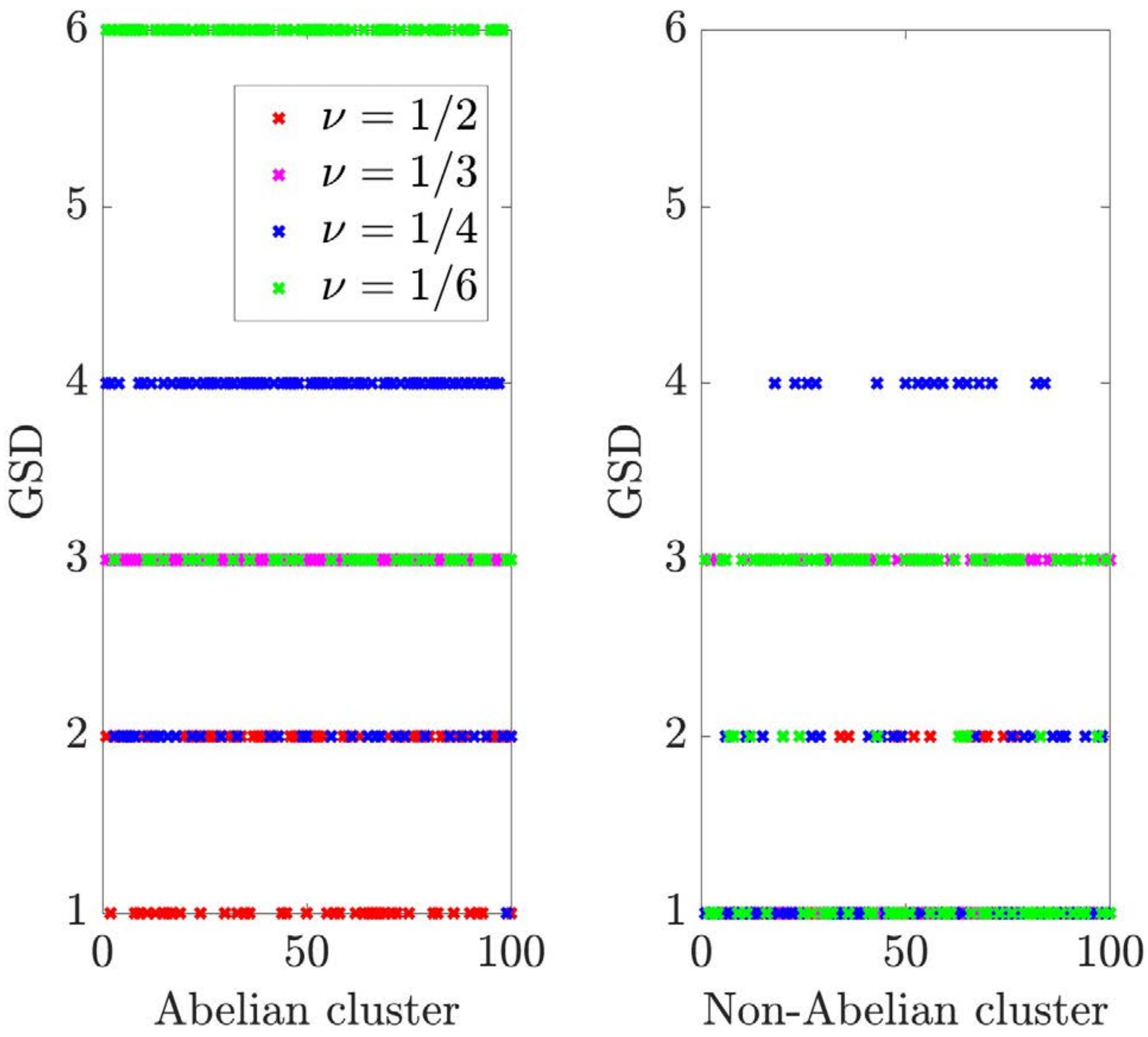

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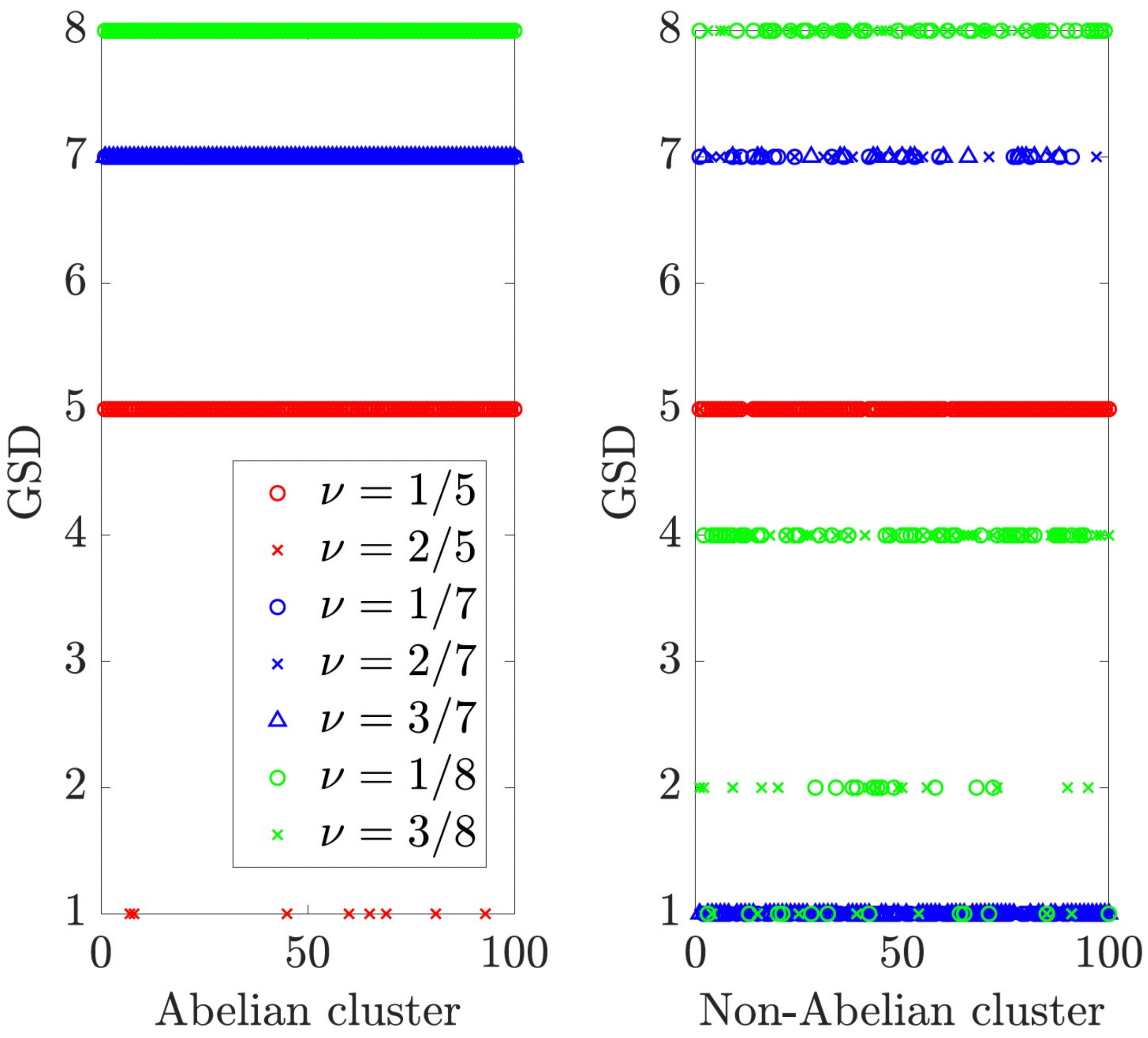
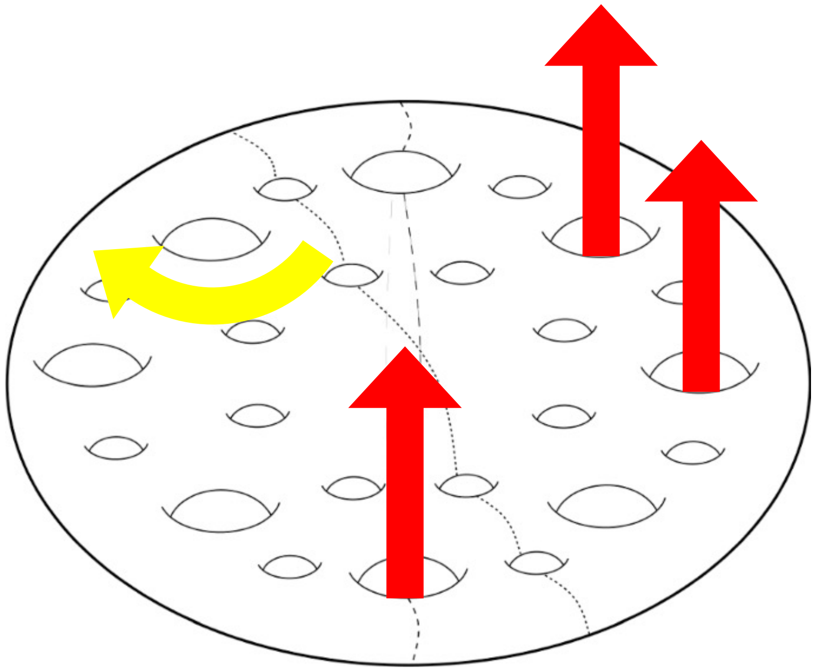


Summary [Part III]

- Oshikawa argument for Lieb-Schultz-Mattis theorem can be generalized to hyperbolic lattices
- Filling ν defined relative to unit cell of "hyperbolic translation group" Γ
- Example application: trivial ground state impossible at fractional fillings on $\{4g, 4g\}$ and $\{2g + 1, 4g + 2\}$ lattices, $g \geq 2$ (generalize square & triangular lattices, respectively)



arXiv:2605.15974

Open questions

- Hyperbolic Heisenberg antiferromagnet: nonperturbative determination of Néel temperature and zero-temperature correlation length (QMC?)
- Frustrated Heisenberg models: noncollinear orders / SO(3)-symmetric spin liquids?
- Exploit LSM constraints to diagnose topological order vs symmetry breaking in numerical exact diagonalization studies (momentum quantum numbers)
- Experimental realizations (?): frustrated spin models from photon-qubit interactions *(Bienias et al., PRL '22)*, Rydberg atom arrays with non-Euclidean geometries *(Periwal et al., Nature '21)*

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