# Overdamped by Design: Aperiodic Crystal Dynamics and Their Thermodynamic and Transport Signatures

**Matteo Baggioli**
**SJTU Shanghai**

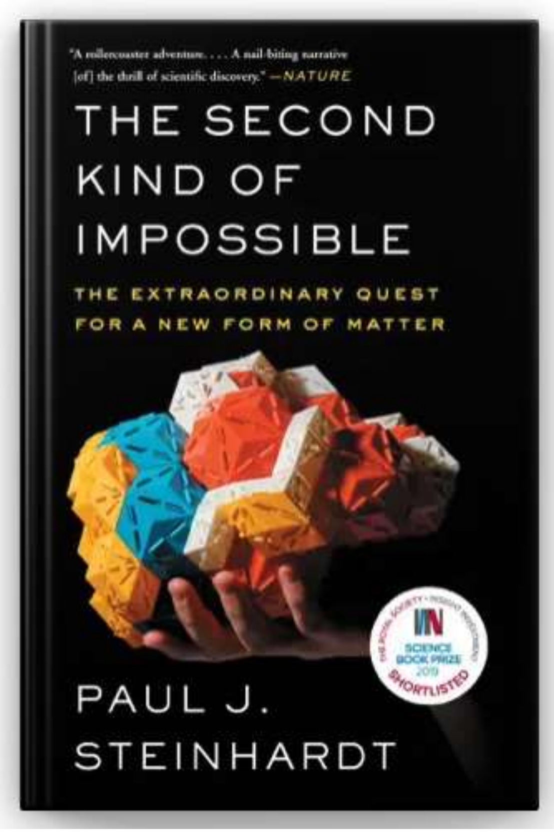
# Brief intro and why I am here

**Shanghai**

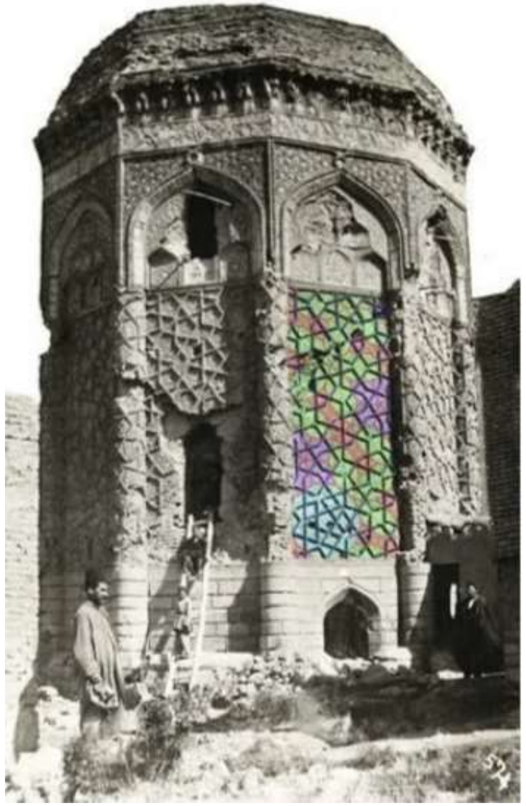
Amorphous solids
Liquids/Hydro
Defects
Topology
Holography

The reason is partially Paul, because he gave a fantastic colloquium in Madrid when I was a postdoc there, after which I immediately searched on Google for quasicrystals and wanted to know more!

# Quasicrystals




could end up in a James Bond’s movie



are beautiful

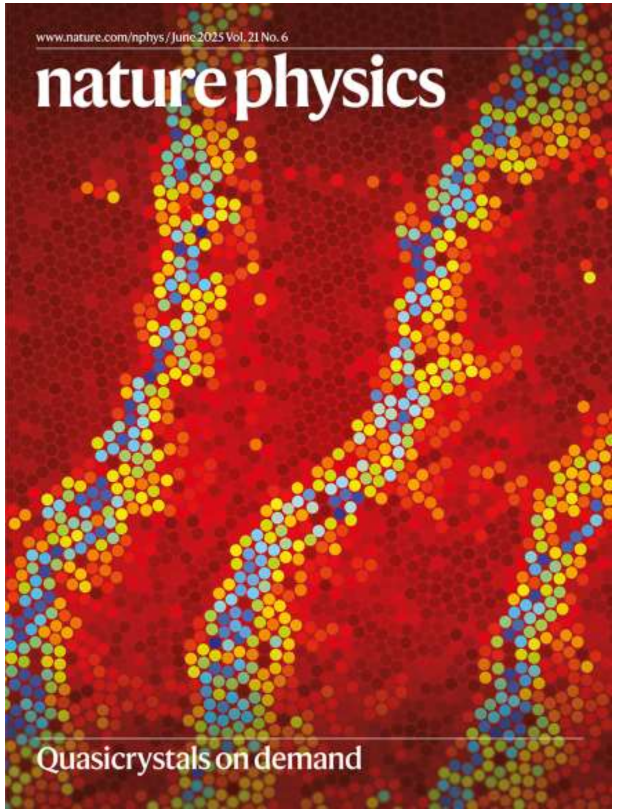
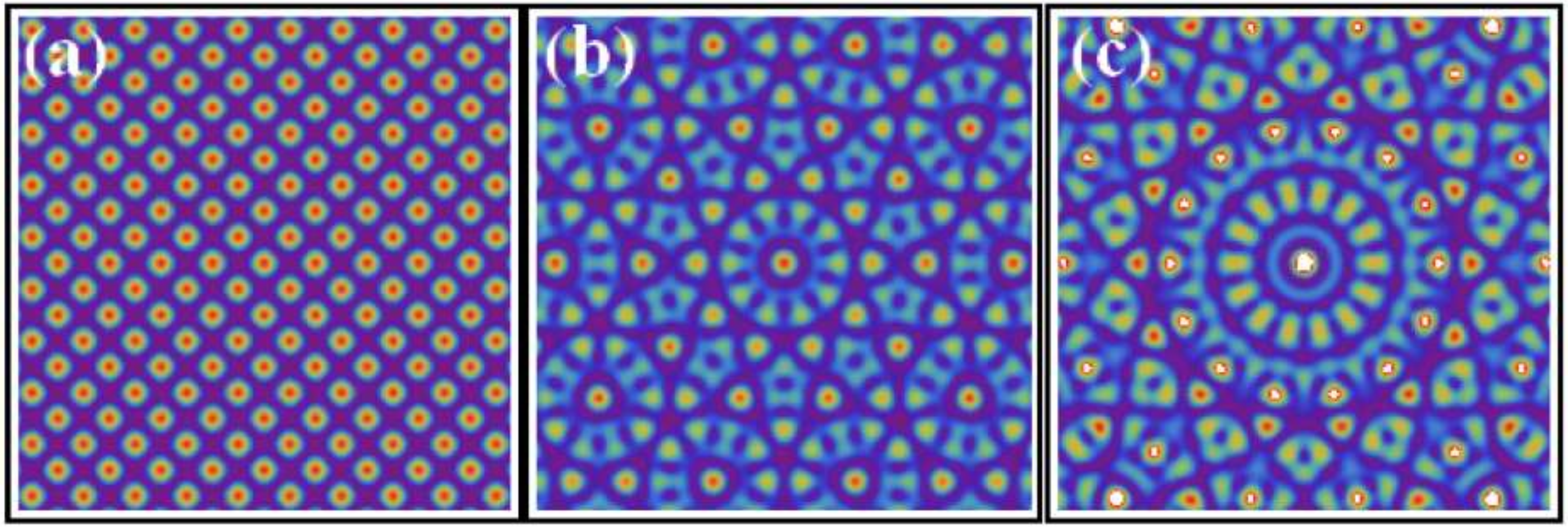



exhibit a lot of interesting physics

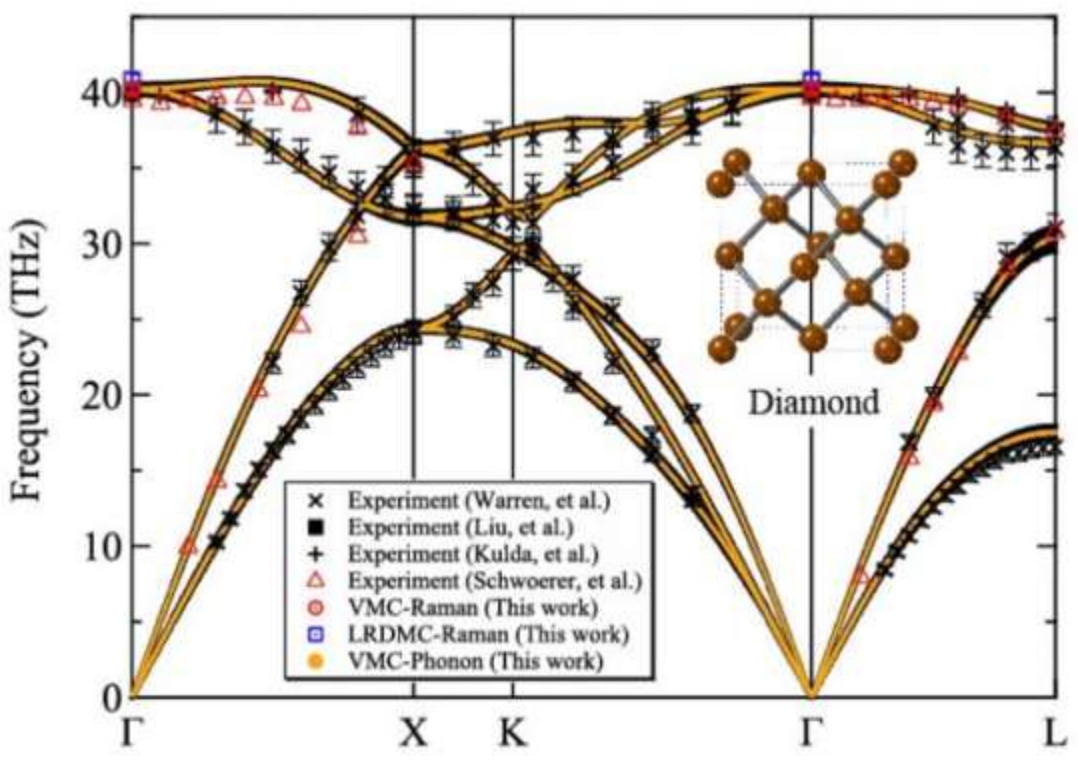
# Starting point

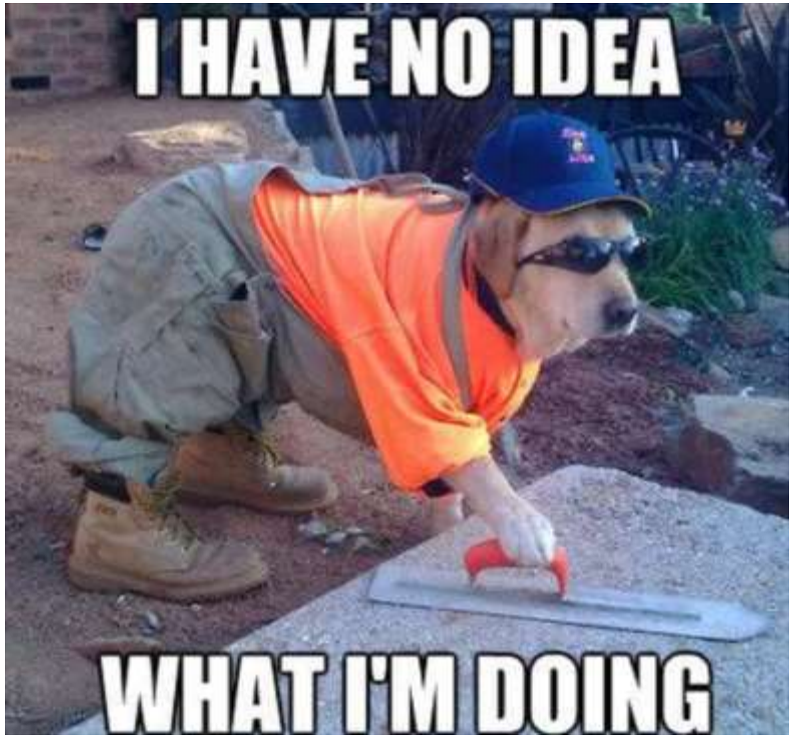



crystal ⟶ quasicrystal

No periodic lattice, discrete translational symmetry is broken, but long-range order is retained = aperiodic crystal

# [My] initial question




crystal → aperiodic crystal

low energy: acoustic phonons

low energy = ????

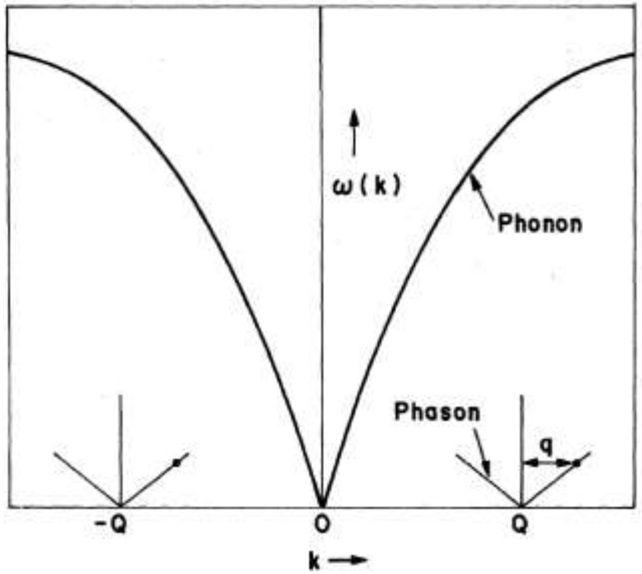
# Quick answer

PHYSICAL REVIEW B VOLUME 3, NUMBER 10 15 MAY 1971

**Observability of Charge-Density Waves by Neutron Diffraction**

A. W. Overhauser
*Scientific Research Staff, Ford Motor Company, Dearborn, Michigan* 48121
(Received 7 December 1970)

Aperiodic crystals exhibit additional low-energy degrees of freedom




Let us call it **PHASON**

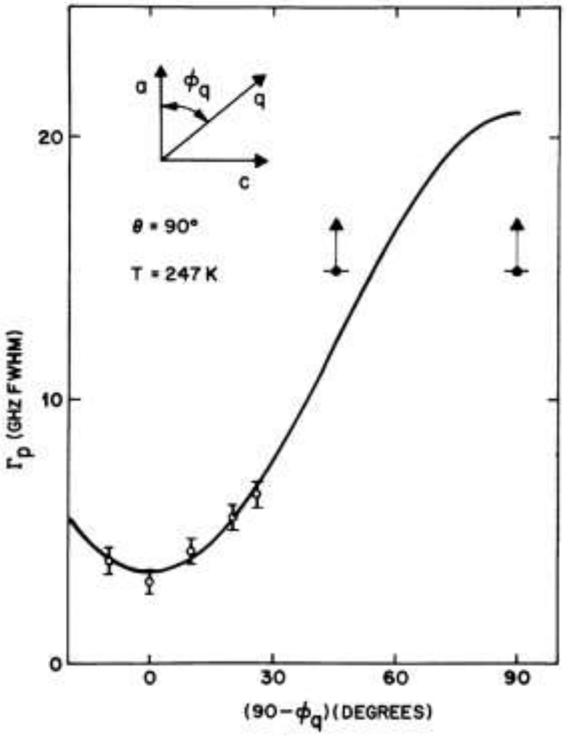
PHYSICAL REVIEW B VOLUME 25, NUMBER 3 1 FEBRUARY 1982

**Incommensurate structural phase transition in $BaMnF_4$: Light scattering from phasons**

K. B. Lyons, R. N. Bhatt, T. J. Negran, and H. J. Guggenheim
*Bell Laboratories, Murray Hill, New Jersey 07974*
(Received 5 March 1981; revised manuscript received 22 July 1981)

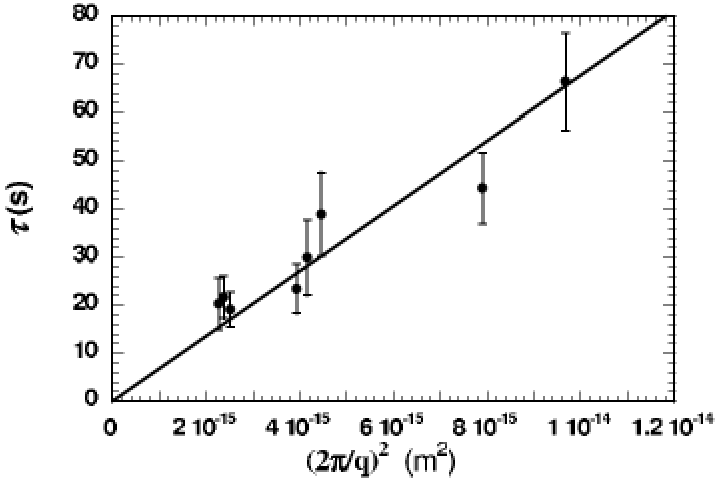
$$\omega = -iDq^2 + \ldots$$

The observed $\vec{q}$ dependence of the inelastic central-peak width is consistent with an interpretation based on phason scattering and provides evidence for a strongly anisotropic, overdamped phason response in the incommensurate phase.




# Light-scattering experiments in BaMnF4 found evidence that phase fluctuations are purely diffusive

X-rays: i-AlPdMn Quasicrystal →
[*Francoual et al, PRL 2003*]



Solid State Communications, Vol.33, pp.407 - 411.
Pergamon Press Ltd. 1980. Printed in Great Britain.

OBSERVATION OF EXCITATIONS IN THE INCOMMENSURABLE PHASES OF BIPHENYL BY INELASTIC NEUTRON SCATTERING

H. Cailleau

Groupe de Physique Cristalline ERA au CNRS n° 015
Université de Rennes Campus de Beaulieu, 35042 RENNES CEDEX, France

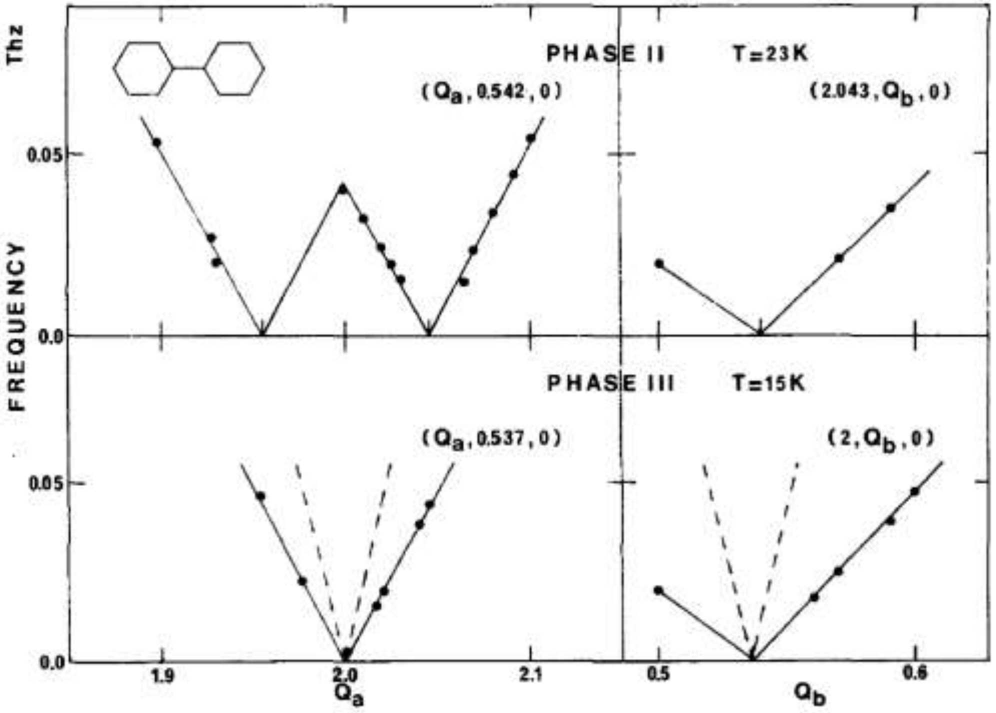
F. Moussa

Laboratoire Léon Brillouin CEN Saclay, BP n° 2, 91190 GIF SUR YVETTE, France

C.M.E. Zeyen and J. Bouillot

Institut Max Von Laue - Paul Langevin, 156X, 38042 GRENOBLE CEDEX, France

(received 20 November 1979 by E.F. Bertaut)




Inelastic neutron-scattering experiments in biphenyl showed the existence of propagating phase fluctuations.

$$\omega = vq + \dots$$

VOLUME 49, NUMBER 25 PHYSICAL REVIEW LETTERS 20 DECEMBER 1982

# Phason Dynamics of Incommensurate Crystals

R. Zeyher and W. Finger
*Max-Planck-Institut für Festkörperforschung, D-7000 Stuttgart 80, Federal Republic of Germany*
(Received 26 July 1982)

Dynamic structure factor:

$$S(\vec{q}, \omega) = \frac{2T\gamma\Delta^2}{(\omega^2 - \sum_\alpha c_\alpha^2 q_\alpha^2)^2 + \omega^2\gamma^2}.$$

The transition between diffusive and oscillating response occurs for $q \sim 1/\Lambda$. If we take for $c_\alpha$ typical values for sound velocities and the above estimate for $\gamma$, $\Lambda$ is of the order of 100 Å. Thus we would expect that light scattering experiments would probe mainly the diffusive regime whereas inelastic neutron scattering experiments would probe mainly the propagating regime.

Low q: diffusive

High q: propagating

PHYSICAL REVIEW B VOLUME 32, NUMBER 11 1 DECEMBER 1985

Hydrodynamics of icosahedral quasicrystals

T. C. Lubensky and Sriram Ramaswamy
*Department of Physics, University of Pennsylvania, Philadelphia, Pennsylvania 19104*

John Toner
*IBM Thomas J. Watson Research Center, Yorktown Heights, New York 10598*

(Received 20 June 1985)

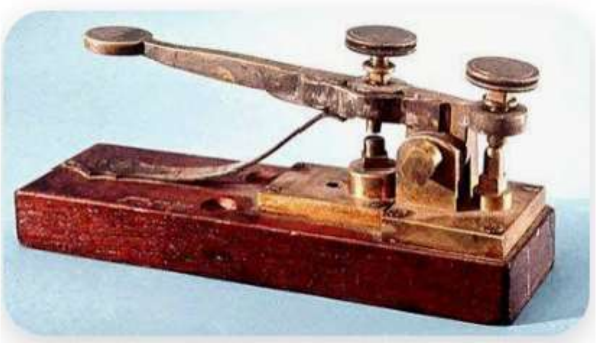
In the previous sections, we found that the modes associated with the phason field **w** are diffusive rather than propagating. This is a result of friction that opposes relative motion of the incommensurate mass-density waves. If this friction were turned off, then the modes associated with **w** would be propagating. Alternatively, if the friction coefficient coupling the incommensurate waves is small, there may exist a regime of wave numbers where the **w** modes are effectively propagating modes as has been observed in mercury chain salts.[14–17]

modes with anisotropic velocities. Does this regime exist? Probably not, since the physics of the situation suggests that $\gamma$ is much too large. The argument is analogous to

A priori, the continuous description of quasicrystals admits the same crossover but the wave-vector at which this happens is too large, hence very unlikely.

To my knowledge, nobody saw this in experiments [is there a deeper reason?]

# One equation to bind them all



Heaviside (1876)

$$\frac{\partial^2 V}{\partial x^2} = LC\,\frac{\partial^2 V}{\partial t^2} + RC\,\frac{\partial V}{\partial t}$$

Telegraph equation

$$\omega^2 + i\omega/\tau = v^2 q^2$$

This equation emerges in many physical systems

ELSEVIER

Physics Reports

Volume 865, 15 June 2020, Pages 1-44

Gapped momentum states

Matteo Baggioli [a], Mikhail Vasin [b], Vadim Brazhkin [b], Kostya Trachenko [c]

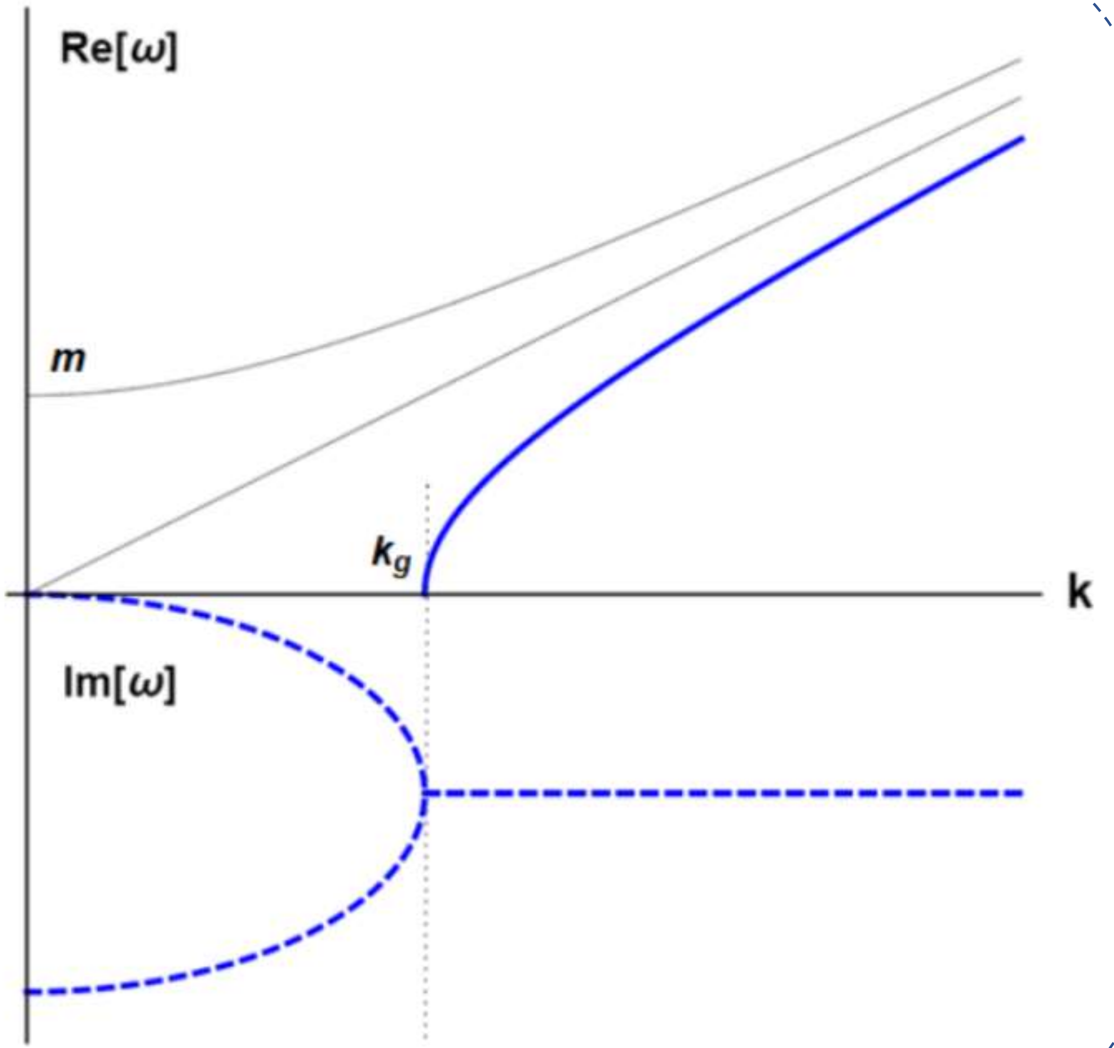
# Wave-vector gap

$$\omega = -\frac{i}{2\tau} \pm \sqrt{c^2k^2 - \frac{1}{4\tau^2}}$$

Real w emerges above a critical k

$$k_g = \frac{1}{2c\tau}$$

Note: you need slightly higher k to become underdamped, roughly w~1/tau

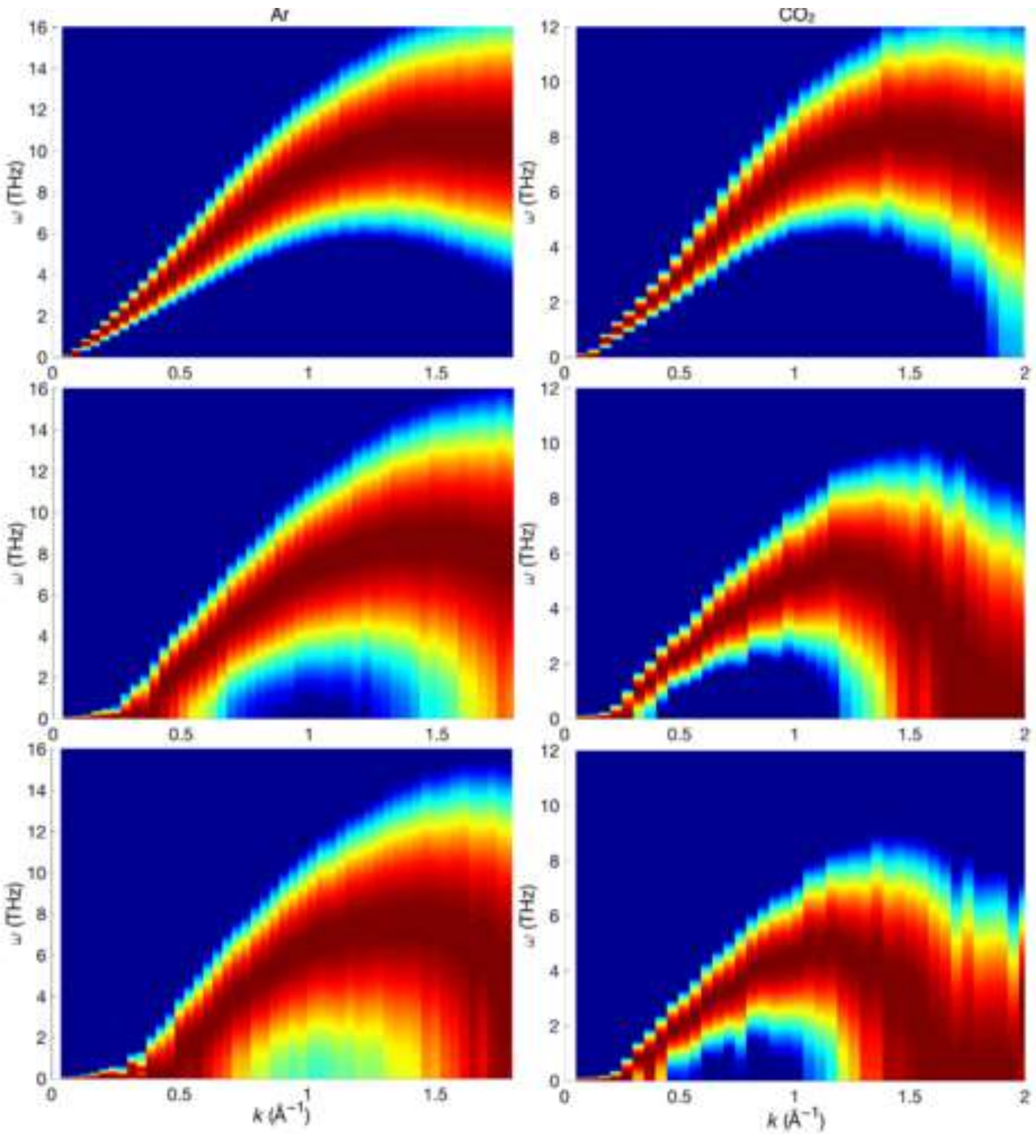


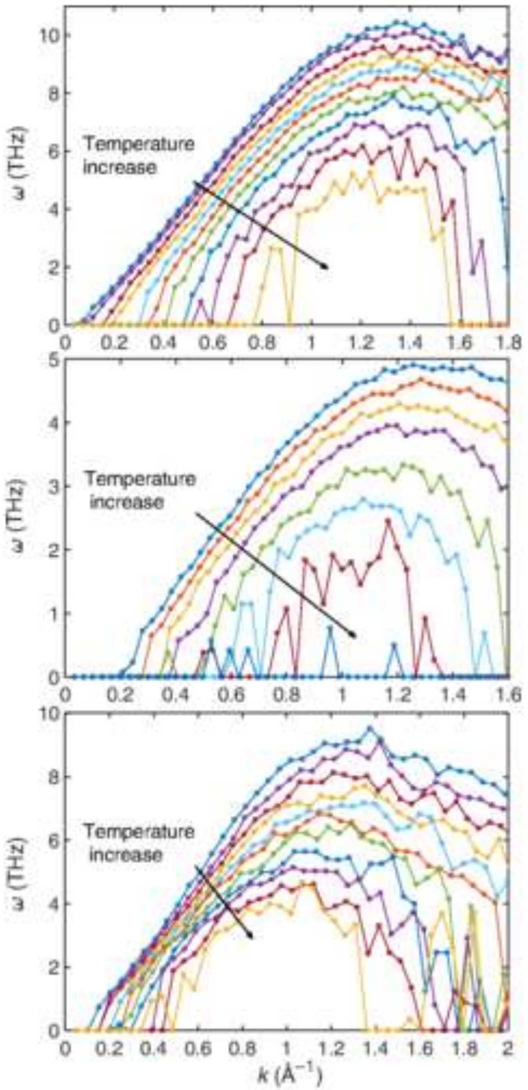
Emergence and Evolution of the $k$ Gap in Spectra of Liquid and Supercritical States

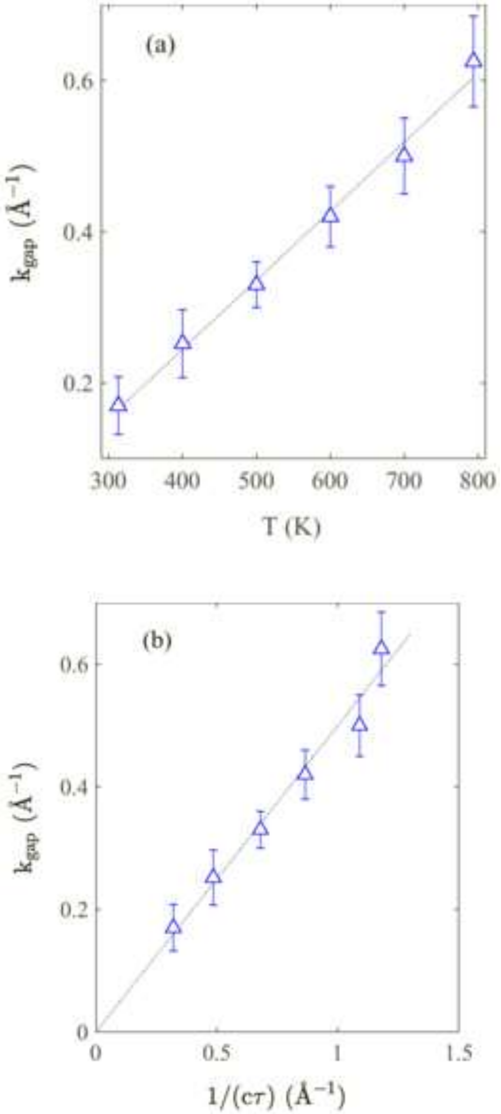
C. Yang, M. T. Dove, V. V. Brazhkin, and K. Trachenko
Phys. Rev. Lett. **118**, 215502 – Published 26 May 2017

Collective modes and gapped momentum states in liquid Ga: Experiment, theory, and simulation

R. M. Khusnutdinoff, C. Cockrell, O. A. Dicks, A. C. S. Jensen, M. D. Le, L. Wang, M. T. Dove, A. V. Mokshin, V. V. Brazhkin, and K. Trachenko
Phys. Rev. B **101**, 214312 – Published 29 June 2020; Erratum Phys. Rev. B **103**, 099901 (2021)



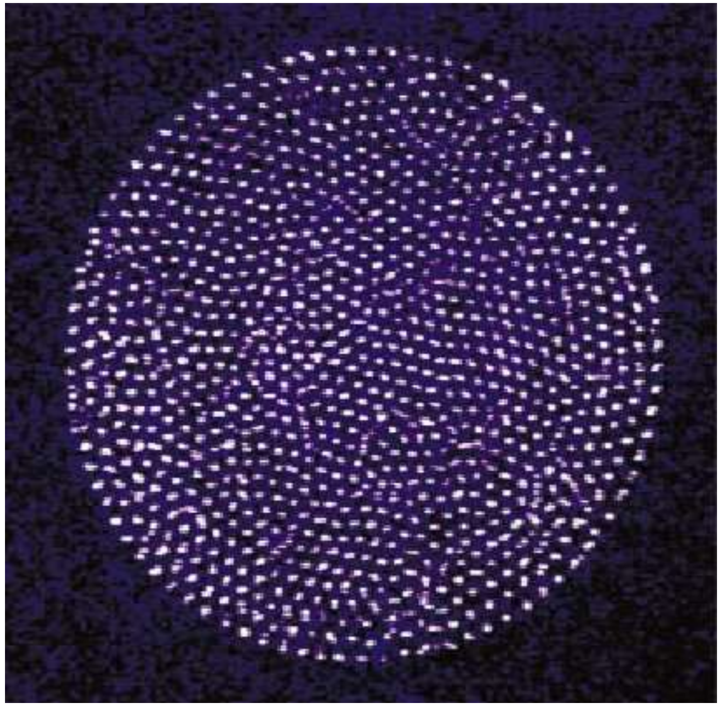



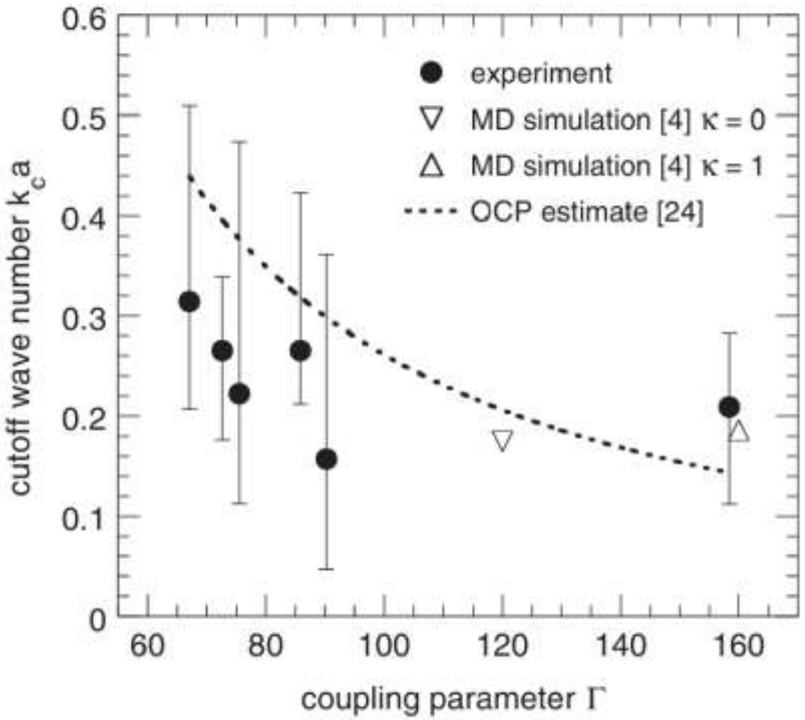
# Experimental observation in dusty plasmas

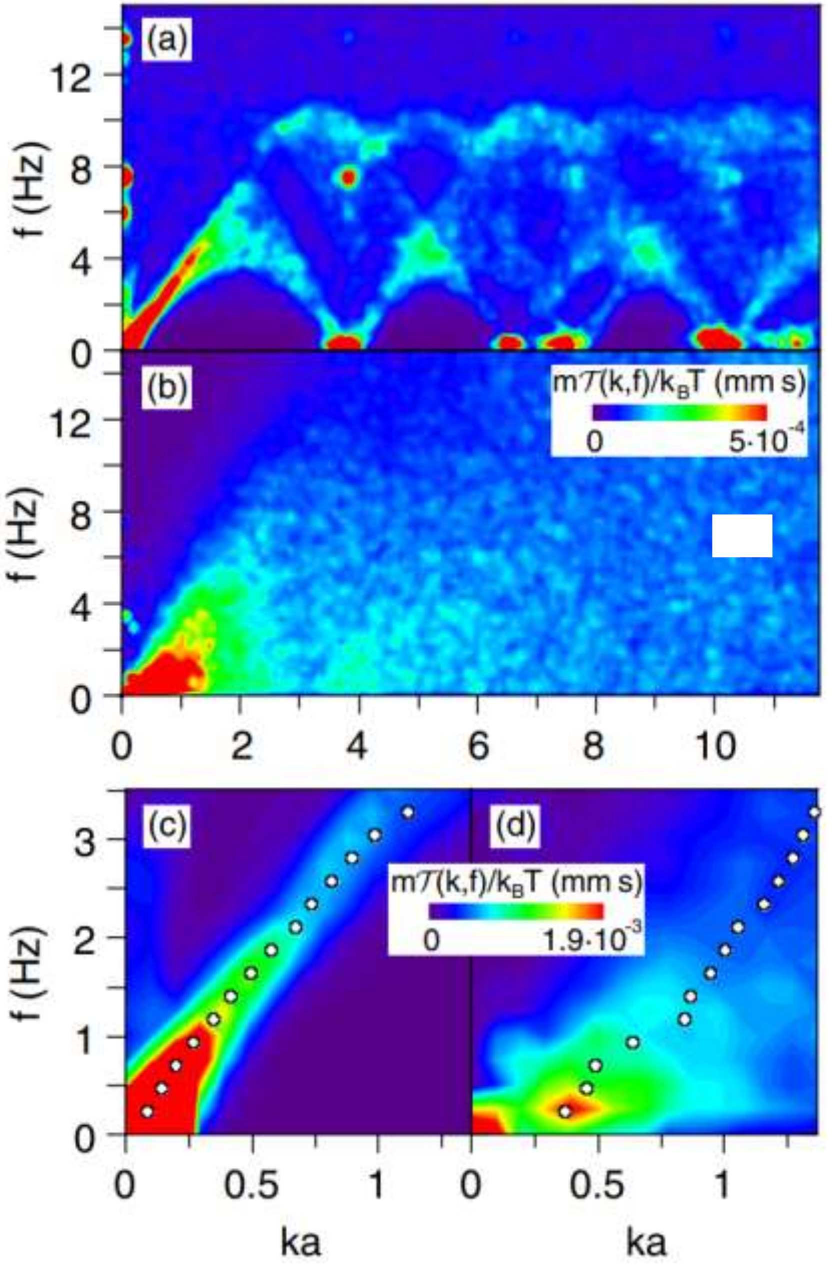
## First experimental observation

Cutoff Wave Number for Shear Waves in a Two-Dimensional Yukawa System (Dusty Plasma)

V. Nosenko, J. Goree, and A. Piel
Phys. Rev. Lett. **97**, 115001 – Published 12 September 2006





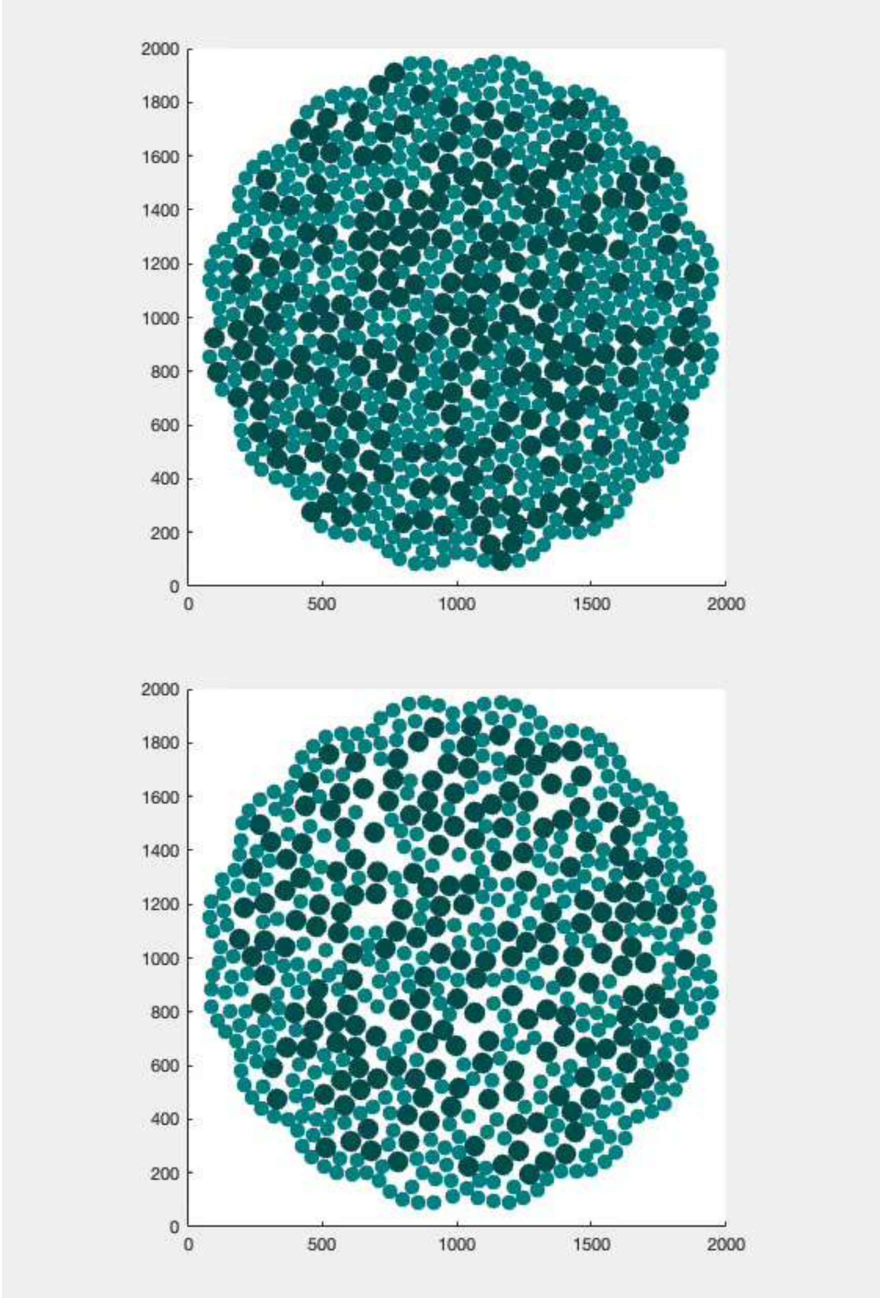



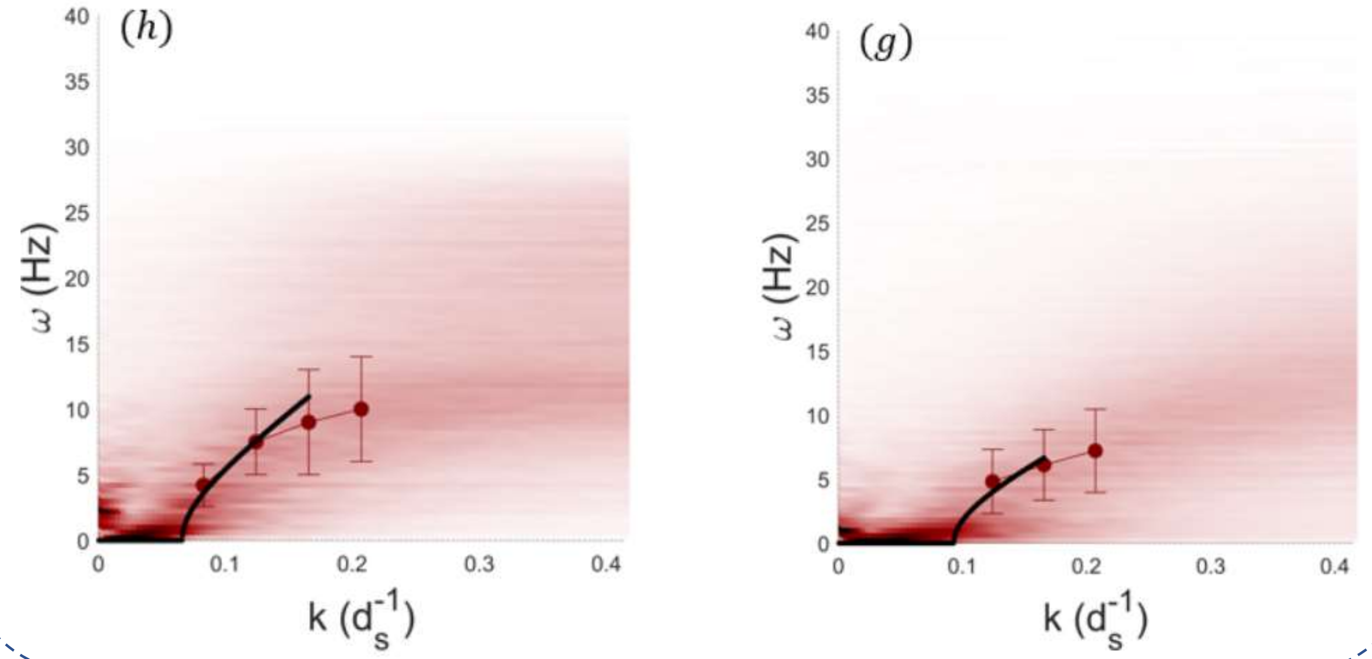
# Granular fluids

[M.B., J. Zhang, Z. Zheng, C. Jiang, Communications Physics 2025]

Experimental observation of k-gap in vibrated granular matter

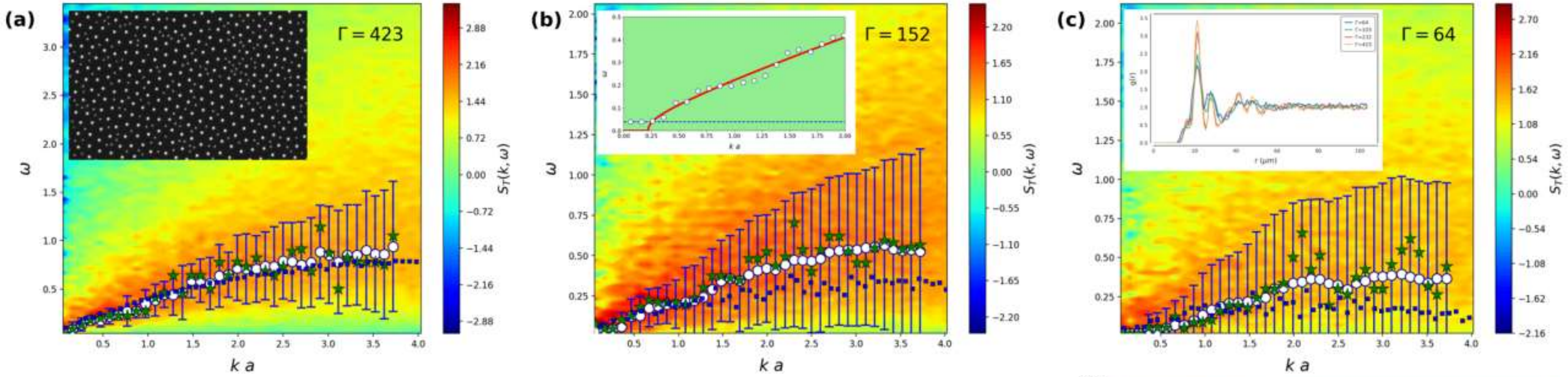
Temperature is played by inverse packing fraction

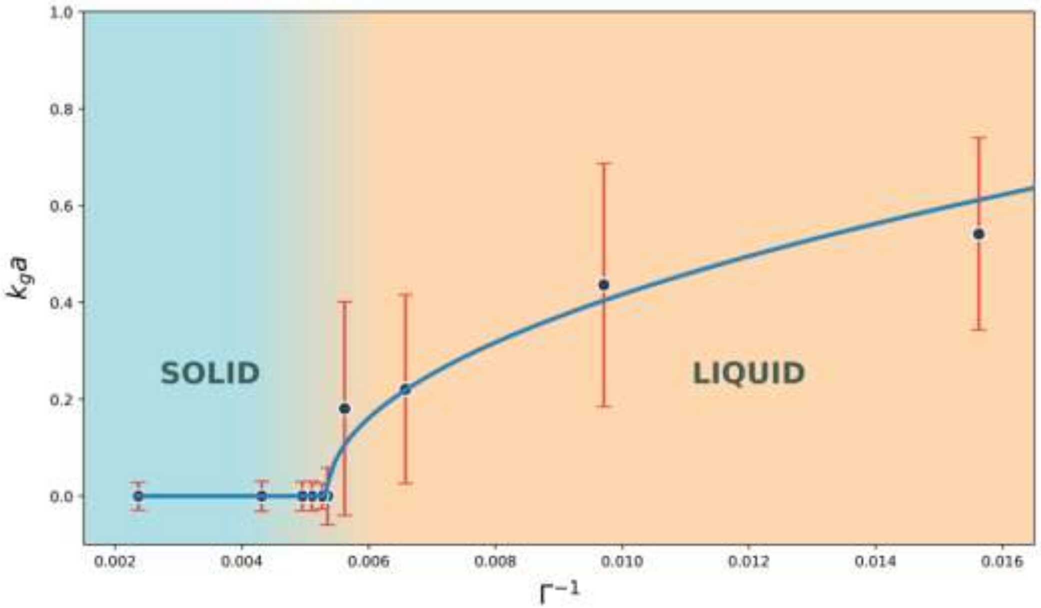


# Colloidal fluids

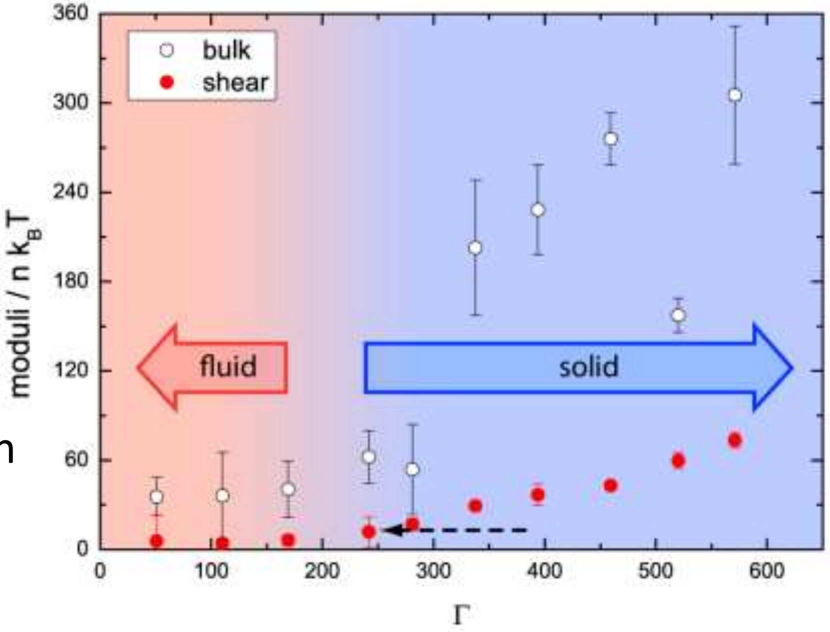
[Bai, Keim, MB, PRR 2025]




k-gap observed in colloidal fluids

It vanishes at the glass transition where static shear modulus goes to 0 (Keim et al. PRL 2012)



# Basic electrodynamics

$$\nabla^2 \mathbf{E} = \mu\,\epsilon\,\frac{\partial^2 \mathbf{E}}{\partial t^2} + \mu\,\sigma\,\frac{\partial \mathbf{E}}{\partial t}$$

photon in a medium with nonvanishing conductivity

$$\boxed{k_g = \frac{1}{2c\tau} = \frac{\sigma}{2c\epsilon}}$$

Not commonly discussed because we usually take k complex and w real

[shear]

phonon propagation in liquids ←→ propagation of EM waves in conductors

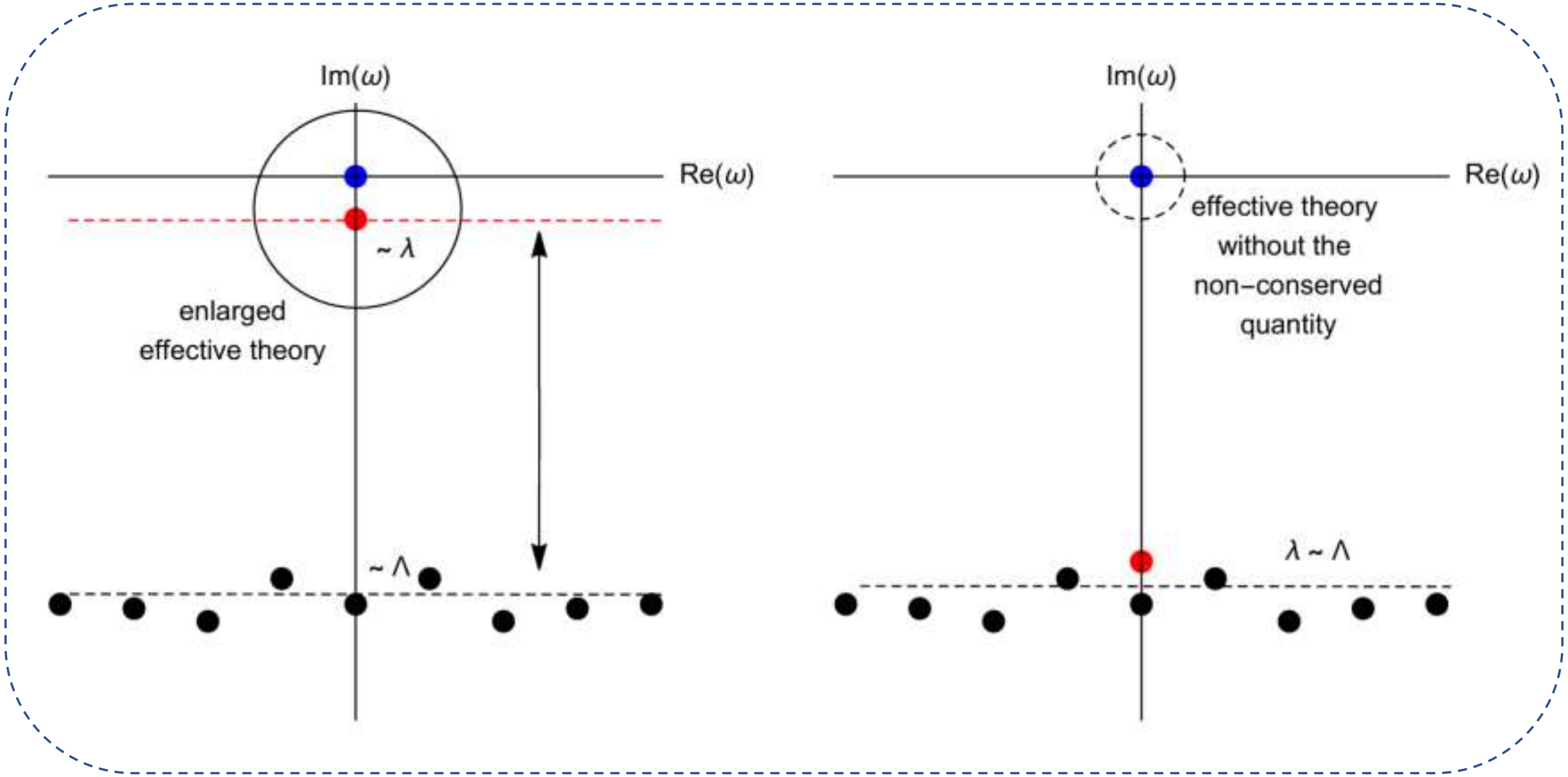
# Breaking symmetries gently

$$\partial_t \langle \rho_a \rangle + \partial_i \mathcal{J}^i_a = 0\,, \quad \partial_t \langle \mathcal{O} \rangle + \partial_i \mathcal{J}^i_{\mathcal{O}} = -\frac{\langle \mathcal{O} \rangle}{\tau}$$

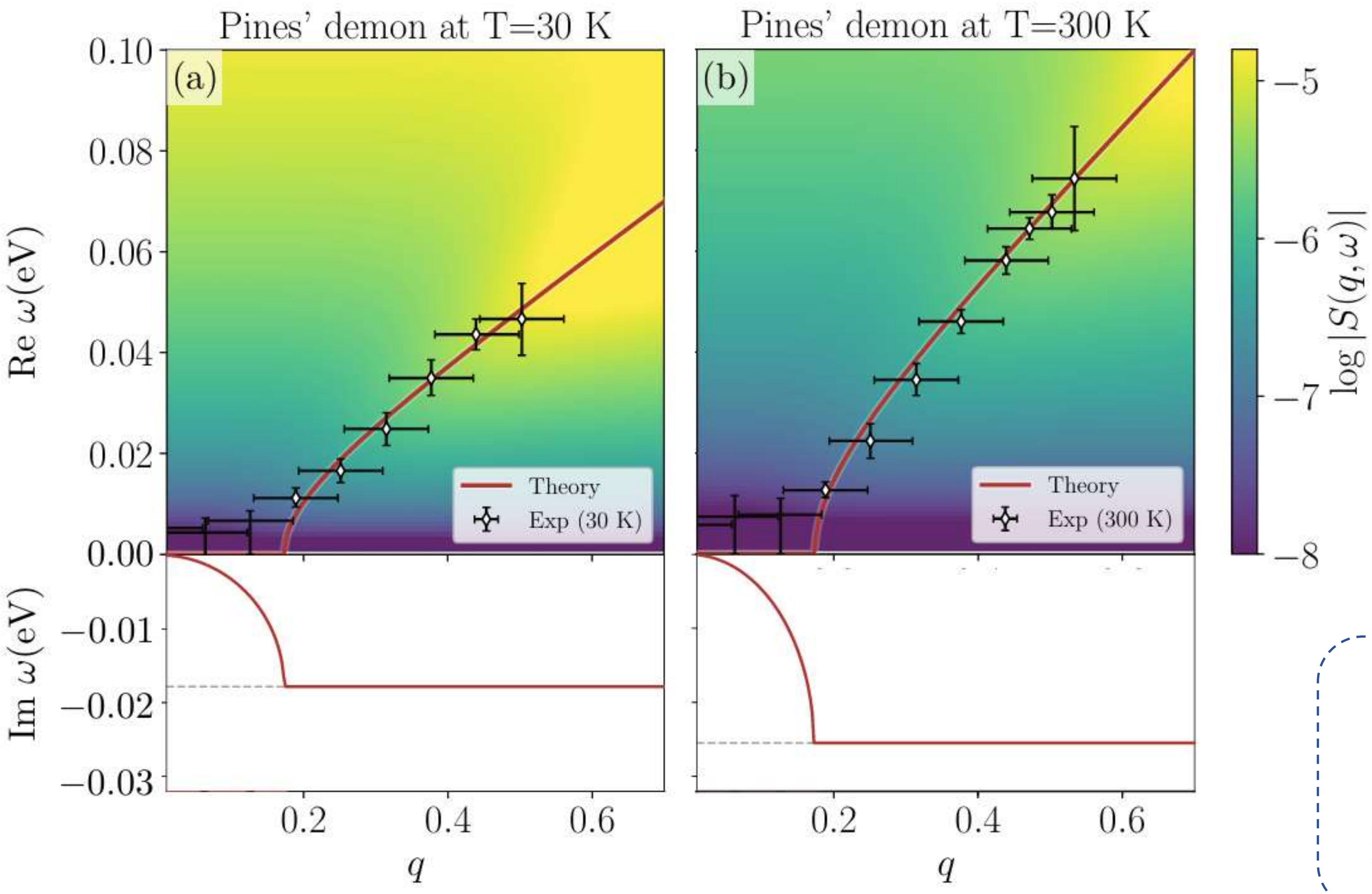
Under the assumption of one non-hydro pole separated from the others and close to the origin

quasi-hydrodynamics

[Grozdanov, Lucas, Poovuttikul, PRD 2019]



# An example in the quantum world




Simplest example of quasi-hydrodynamics (Galilean fluid with momentum relaxation, aka Drude model)

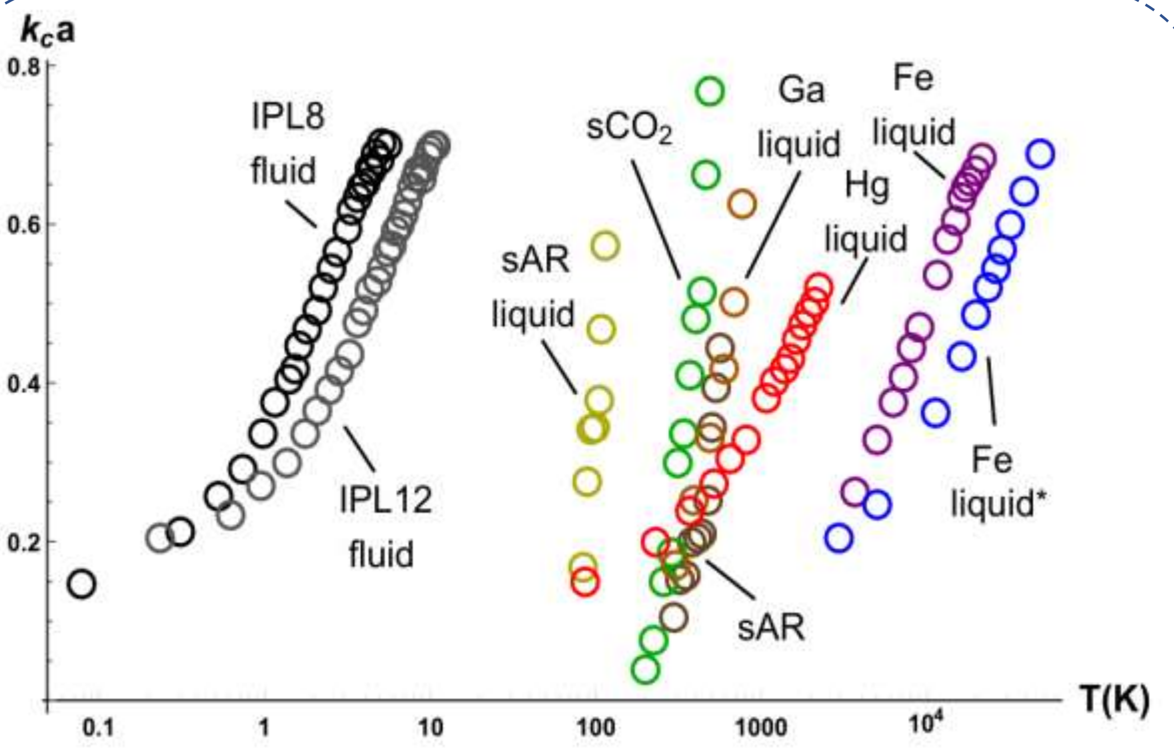
$$\partial_t \pi^i + n\partial^i \mu = -\Gamma n v^i - e\frac{n}{q}E^i,$$

$$\partial_t n + \partial_i (n v^i) = 0.$$

[Sanchez-Martınez,Gouteraux, Flicker, ArXiv 2026]

# Universality and radius of convergence

Wigner-Seitz radius ``a``



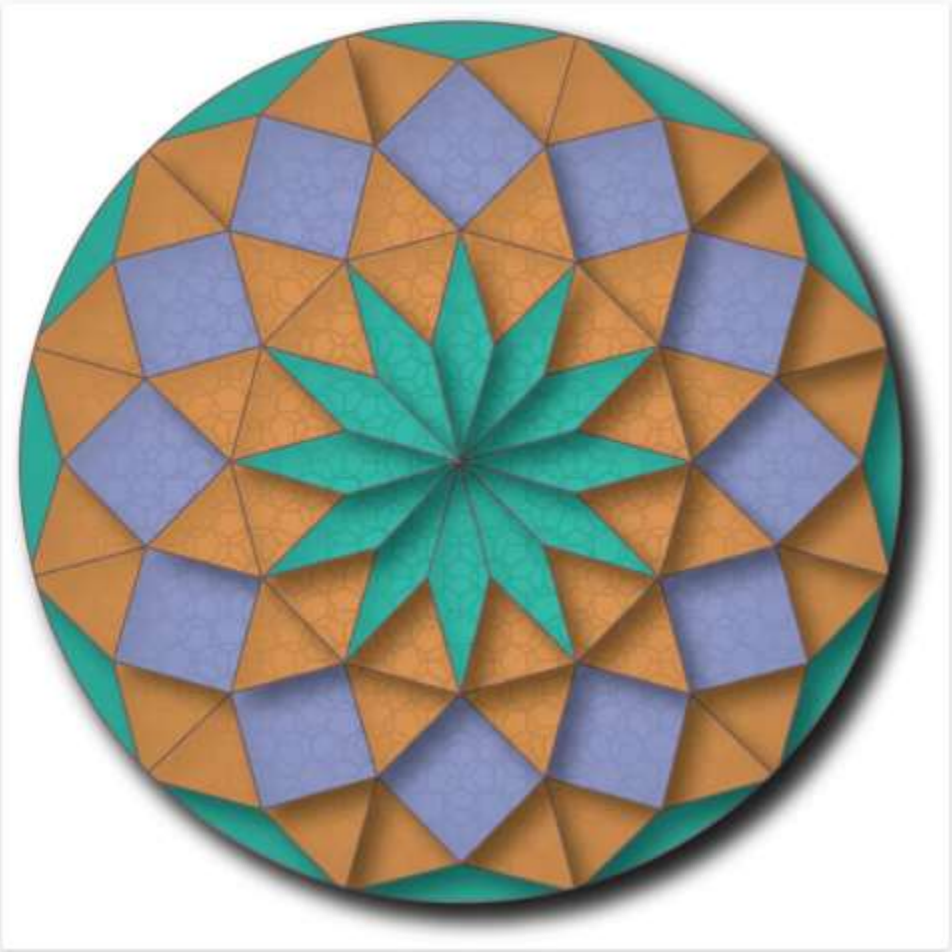

radius of convergence
of linear hydrodynamic series

$$\omega^{(i)}(k) = \sum_{n=1}^{\infty} \alpha_n^{(i)} k^n ,$$

| Liquid | $k_c\, a$ |
|---|---|
| 2D Yukawa (MD) [20] | 0.25 |
| dusty plasma (MD) [21, 22] | 0.3-1.2 |
| 2D Yukawa (EXP) [23] | 0.16-0.31 |
| Liquid Fe (EXP) [24, 25] | 0.3 |
| Liquid Cu (EXP) [24, 25] | 0.4 |
| Liquid Zn (EXP) [24, 25] | 0.3 |
| 3D LJ fluid (MD) [26] | 0.2-0.7 |
| Liquid Fe (MD) [26] | 0.2-0.7 |
| IPL8-IPL12 fluid (MD) [26] | 0.2-0.7 |
| Liquid Hg (MD) [26] | 0.15-0.55 |
| Supercritical Ar (MD) [27] | 0.05-0.8 |
| Subcritical liquid Ar (MD) [27] | 0.2-0.7 |
| Supercritical $CO_2$ [27] | 0.1-0.5 |
| Liquid Ga (EXP, MD) [28] | 0.25-0.6 |
| 2D Coulomb classical fluids (MD) [29] | 0.3-2 |
| Quark Gluon Plasma [30] | 3.3 |

[MB, PRD (2021)]

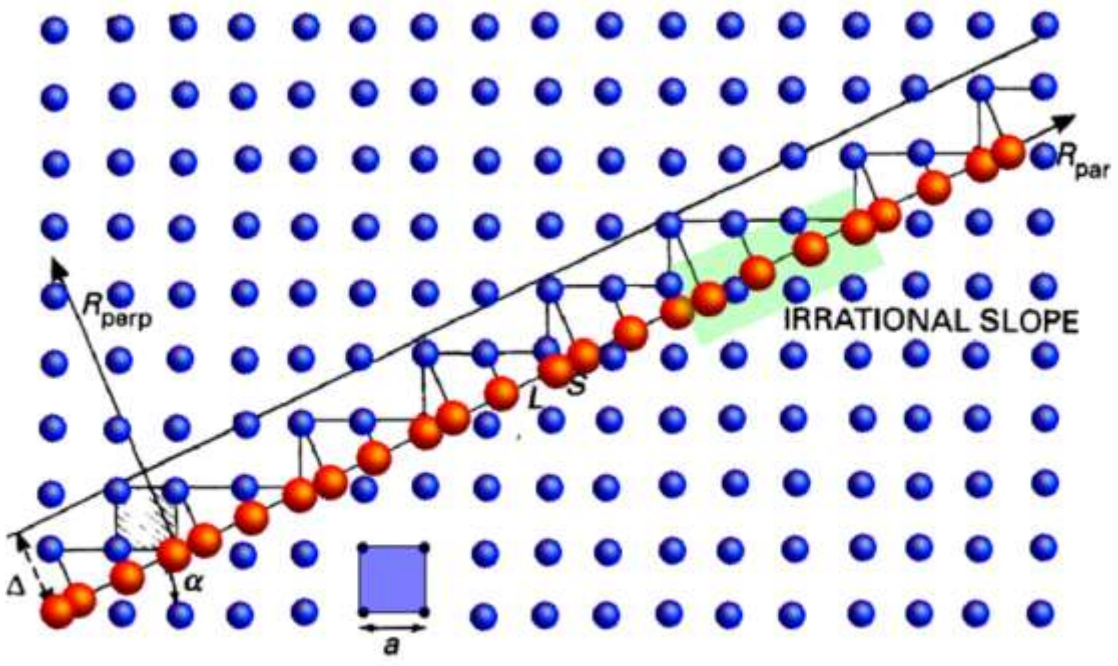
# Back to quasicrystals



# Superspace approach

$$\rho(\vec{x}) = \frac{1}{V}\sum_{\vec{G}} \rho(\vec{G})\, e^{i\,\vec{G}\cdot\vec{r}}, \qquad \vec{G} = \underbrace{n_1\,\hat{x}_1 + n_2\,\hat{x}_2 + n_3\,\hat{x}_3}_{d\,\text{physical dimensions}} + \underbrace{\sum_{i=d}^{D-d} n_i\,\hat{x}_i}_{D-d}$$

quasicrystal = periodic crystal in higher-dimensional space




$$\ldots SLLSLLLSLLS \ldots$$

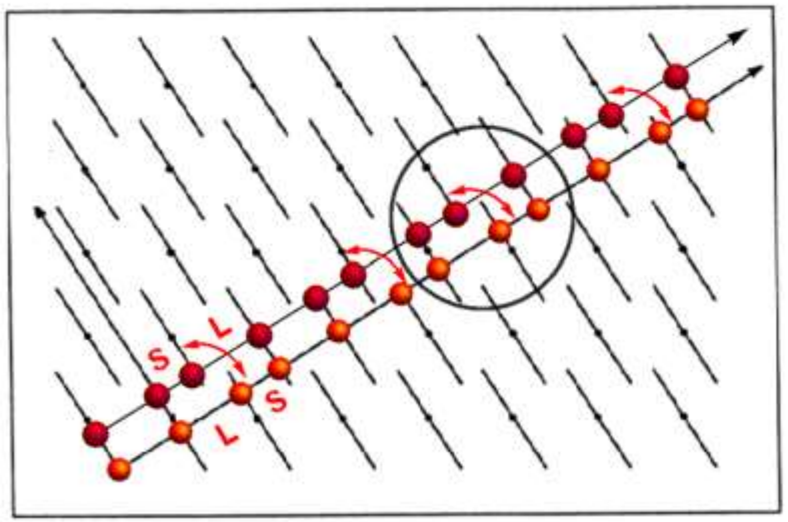
projection at irrational angle

# Phason mode

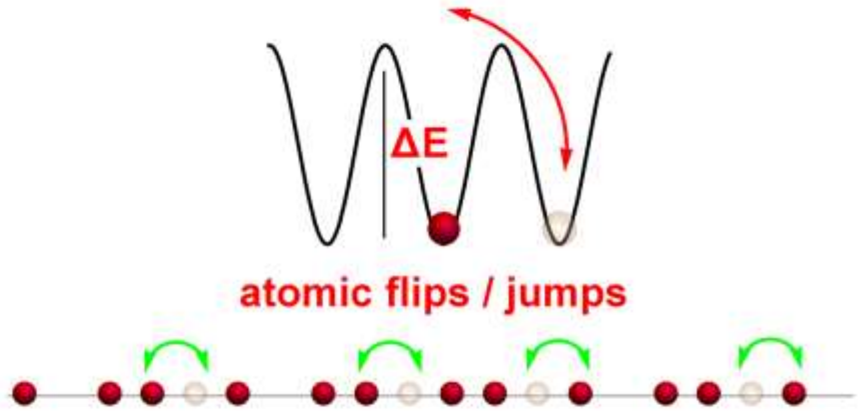
## Dynamics in the perpendicular direction give rise to the so-called phason mode

Free energy is invariant under a rigid shift along the transverse directions and therefore a gapless hydrodynamic mode will be associated with this process.

These displacements do not corresponds to infinitesimal shifts of the atomic positions. On the contrary, they correspond to atomic flips or atomic jumps.




$$\ldots LS \ldots \quad \rightarrow \quad \ldots SL \ldots$$

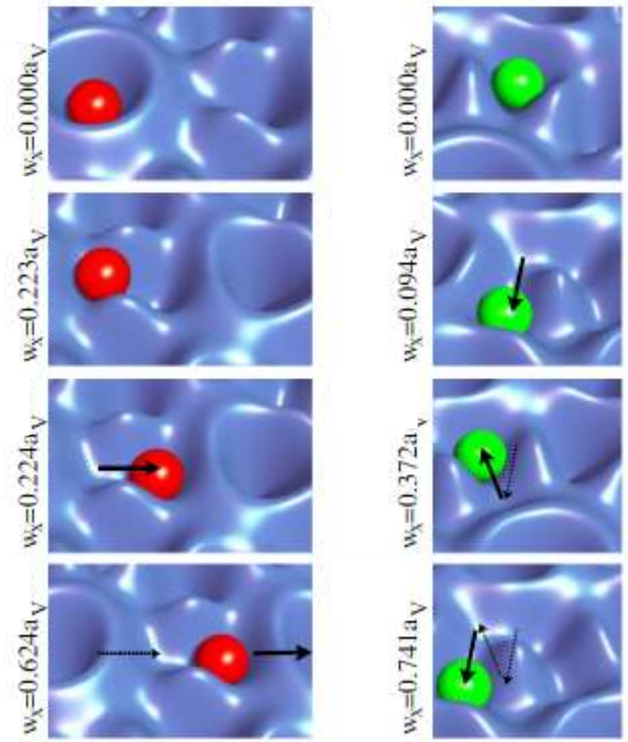


# Observation of phason jumps

PRL **108**, 218301 (2012) PHYSICAL REVIEW LETTERS week ending 25 MAY 2012

**What Phasons Look Like: Particle Trajectories in a Quasicrystalline Potential**

Justus A. Kromer,[1] Michael Schmiedeberg,[2,*] Johannes Roth,[3] and Holger Stark[1]
[1]*Institut für Theoretische Physik, Technische Universität Berlin, D-10623 Berlin, Germany*
[2]*Institut für Theoretische Physik 2: Weiche Materie, Heinrich-Heine-Universität Düsseldorf, D-40204 Düsseldorf, Germany*
[3]*Institut für Theoretische und Angewandte Physik, Universität Stuttgart, D-70550 Stuttgart, Germany*
(Received 27 February 2012; published 24 May 2012)



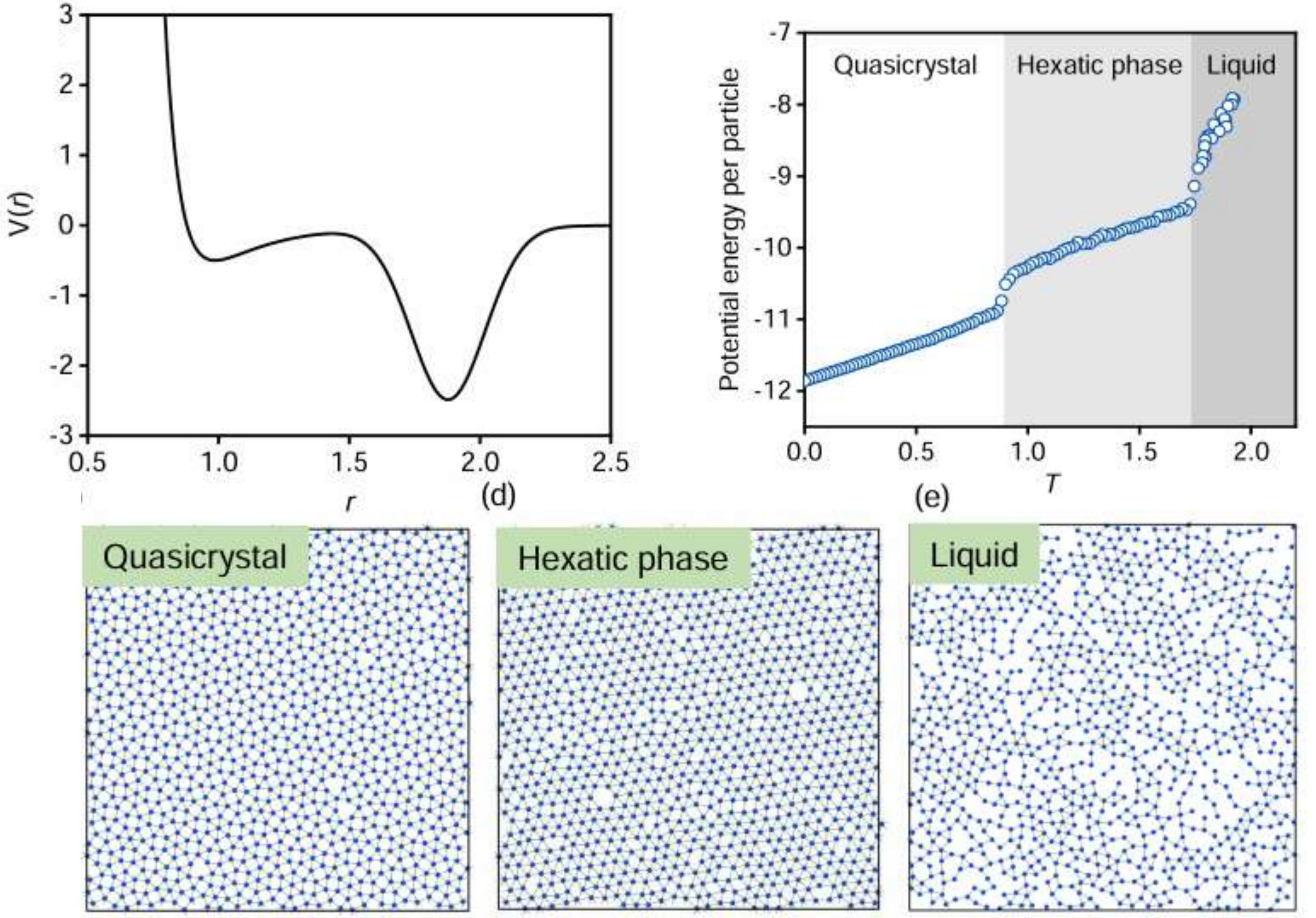

Phason jumps observed in colloidal systems

Sometimes, dynamics more cooperative with
carousel-like successive flipping events

# Glassy dynamics in quasicrystals

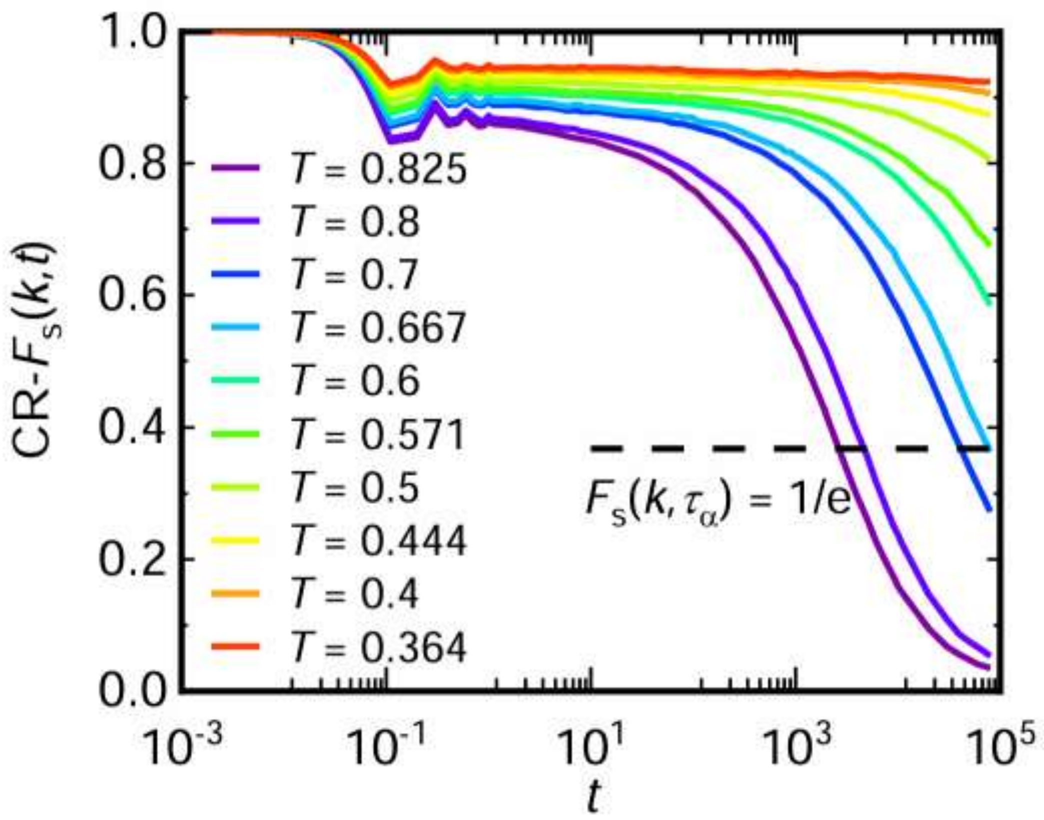
[Zhao, MB, Xu, Douglas, Wang, J. Chem. Phys. 162, 234503 (2025)]

Target: clarify the idea of "*quasicrystals as intermediate phase between crystals and glasses*"

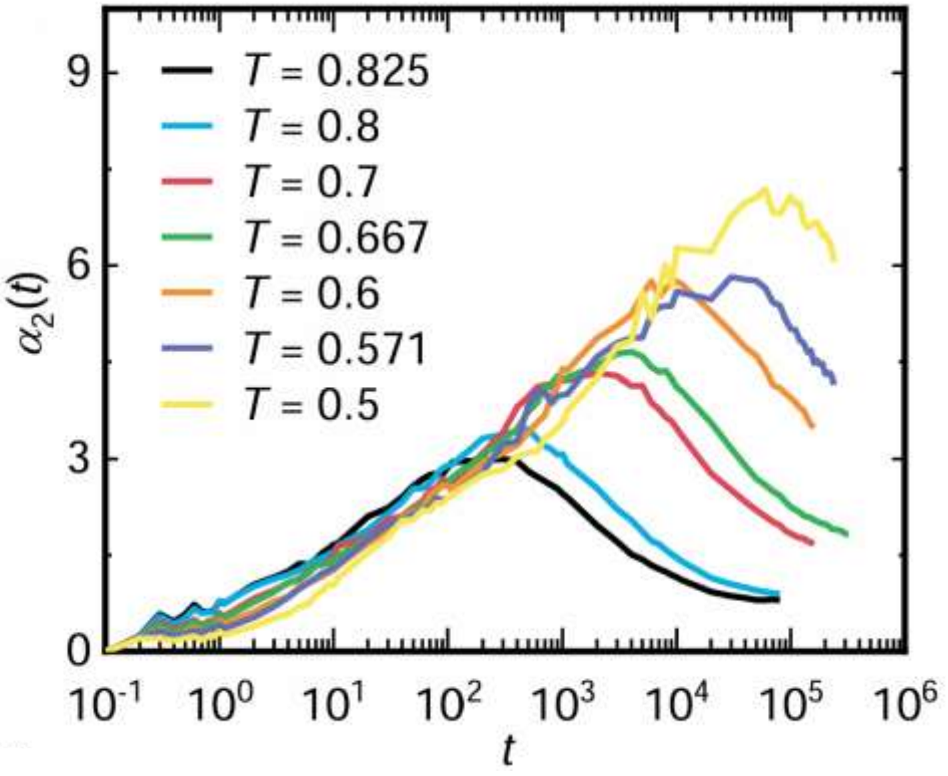


# Glassy dynamics in quasicrystals

Two typical glassy features



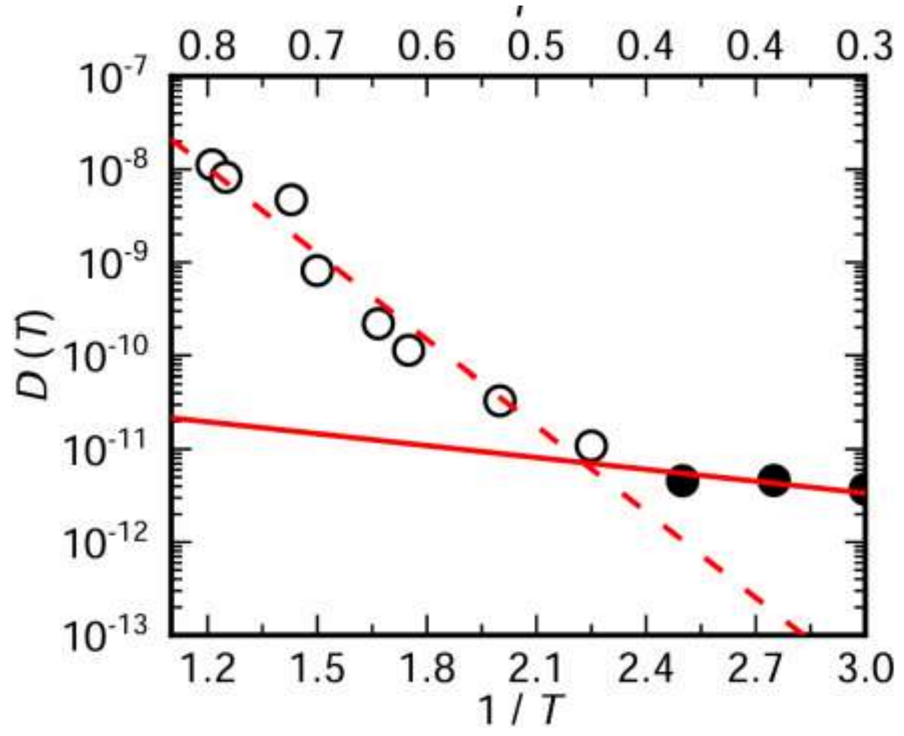

Two-step structural relaxation



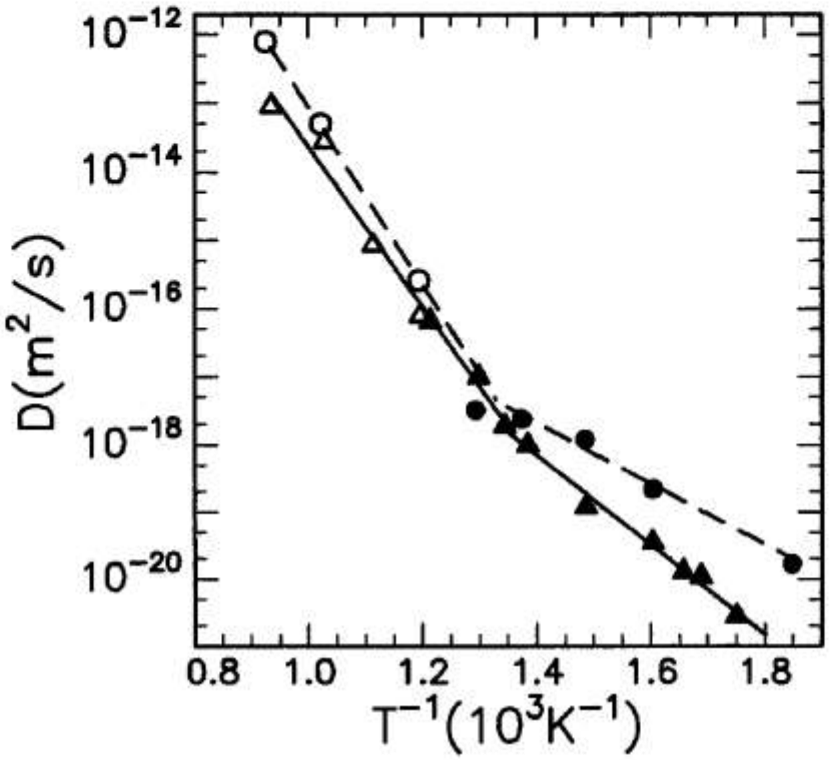

Dynamical heterogeneities

# Signatures of phason dynamics?

Our simulations




Experiments: Phys. Rev. Lett. 80, 1014 (1998)




A Mechanism for Self-Diffusion in Quasi-Crystals

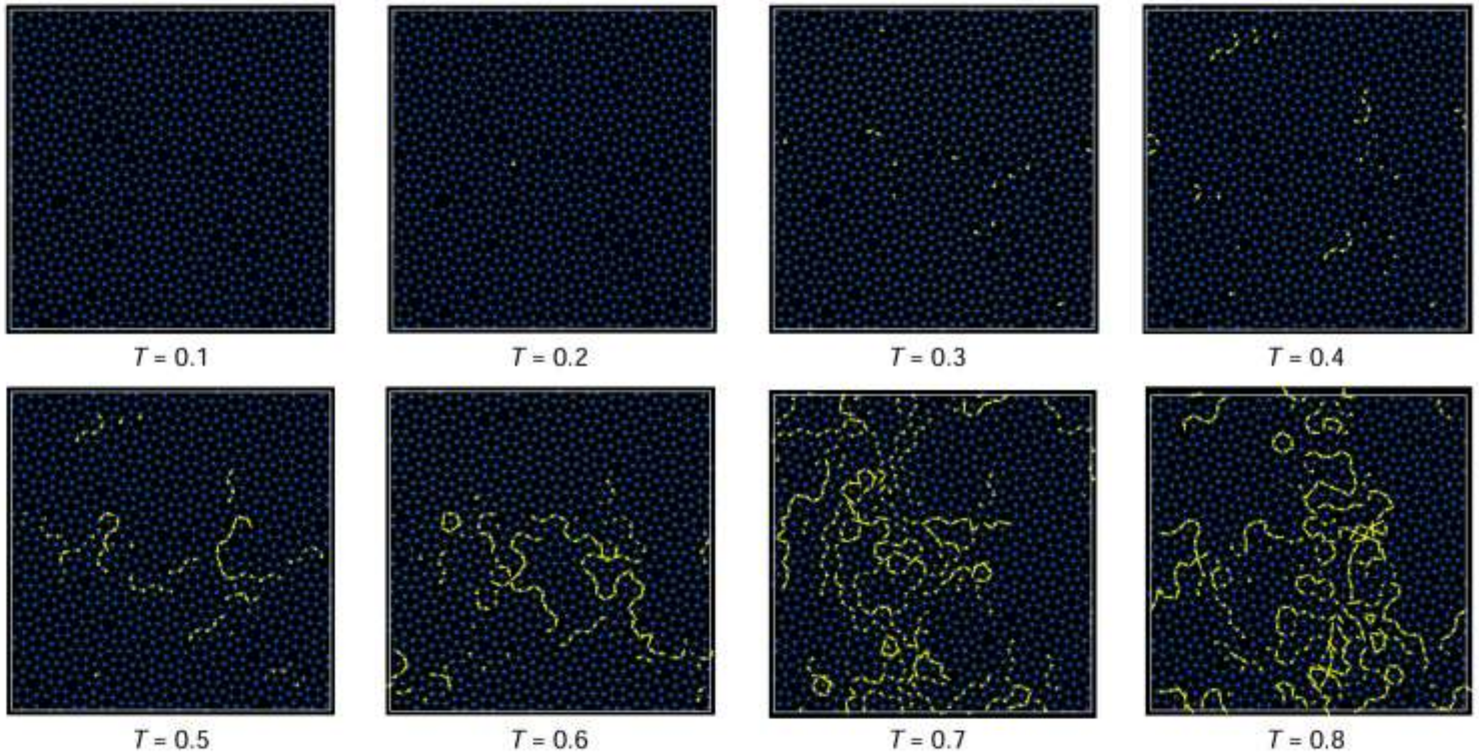
P. A. Kalugin and A. Katz

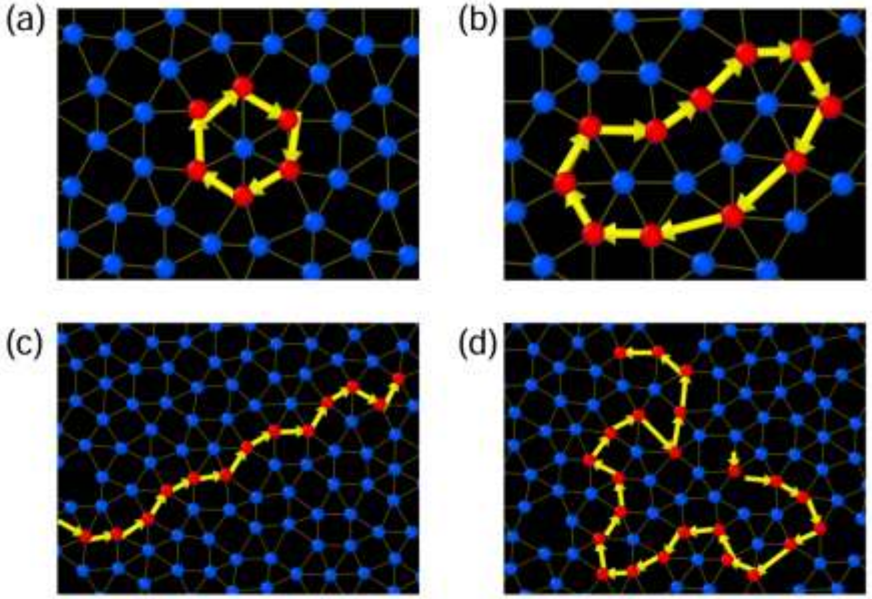
Published under licence by IOP Publishing Ltd

Europhysics Letters, Volume 21, Number 9

Anomalous low-T diffusion: phason assisted

# Cooperative phason dynamics = strings?




Cooperative phason flips?

At high temperature, string-like cooperative rearrangements in the form of loops or open strings

Discussed before as carousel motion of phasons

# An effective field theory approach

[MB, Landry, SciPost Phys. 9, 062 (2020)]

Diffusive phason does not come from a conserved charge or continuity equation

From a symmetry point of view, the diffusive nature of the phason at long wavelengths is due to the fact that the internal translations, or phason shifts, are symmetries of the system with no associated Noether currents.

The latter feature is compatible with the EFT description only because of the presence of dissipation (finite temperature) and the lack of periodic order.

# An effective field theory approach

For details (and other examples) see

M. J. Landry, *Dynamical chemistry: Non-equilibrium effective actions for reactive fluids* (2020), arXiv:2006.13220.

M. J. Landry, *Second sound and non-equilibrium effective field theory* (2020), arXiv:2008.11725.

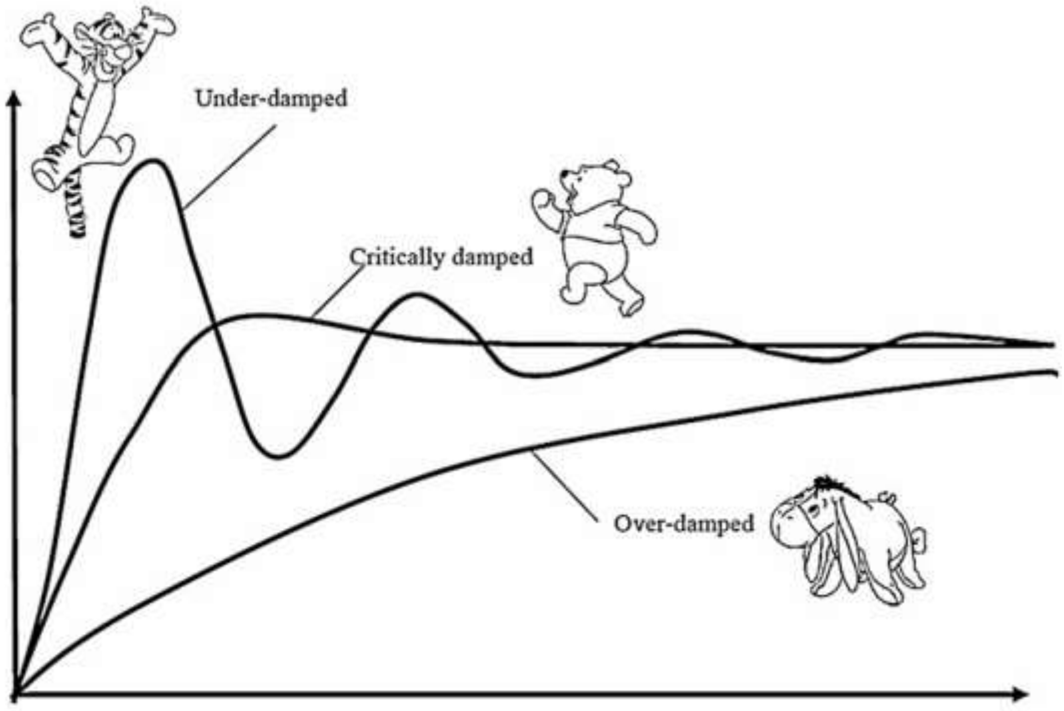
$$H^{44}\,\partial_\mu\partial^\mu\psi_r^4 + \frac{\partial F^4}{\partial Z^4}\,\partial_0^2\psi_r^4 - M^{44}\,\partial_0\psi_r^4 = 0\,.$$

An equivalent statement often mentioned in the literature, is that the phason shifts leave the free energy unchanged but they do not commute with the hamiltonian of the system [62]. In particular, letting $\mathcal{P}_4$ be the generator of the quasicrystal shift symmetry, we expect that

$$\langle [H,\mathcal{P}_4]\rangle \sim \langle H^\dagger\mathcal{P}_4 - \mathcal{P}_4 H\rangle \sim \frac{d\langle\mathcal{P}_4\rangle}{dt} \sim M^{44}\,,$$

= potential energy barrier

# Summary so far:

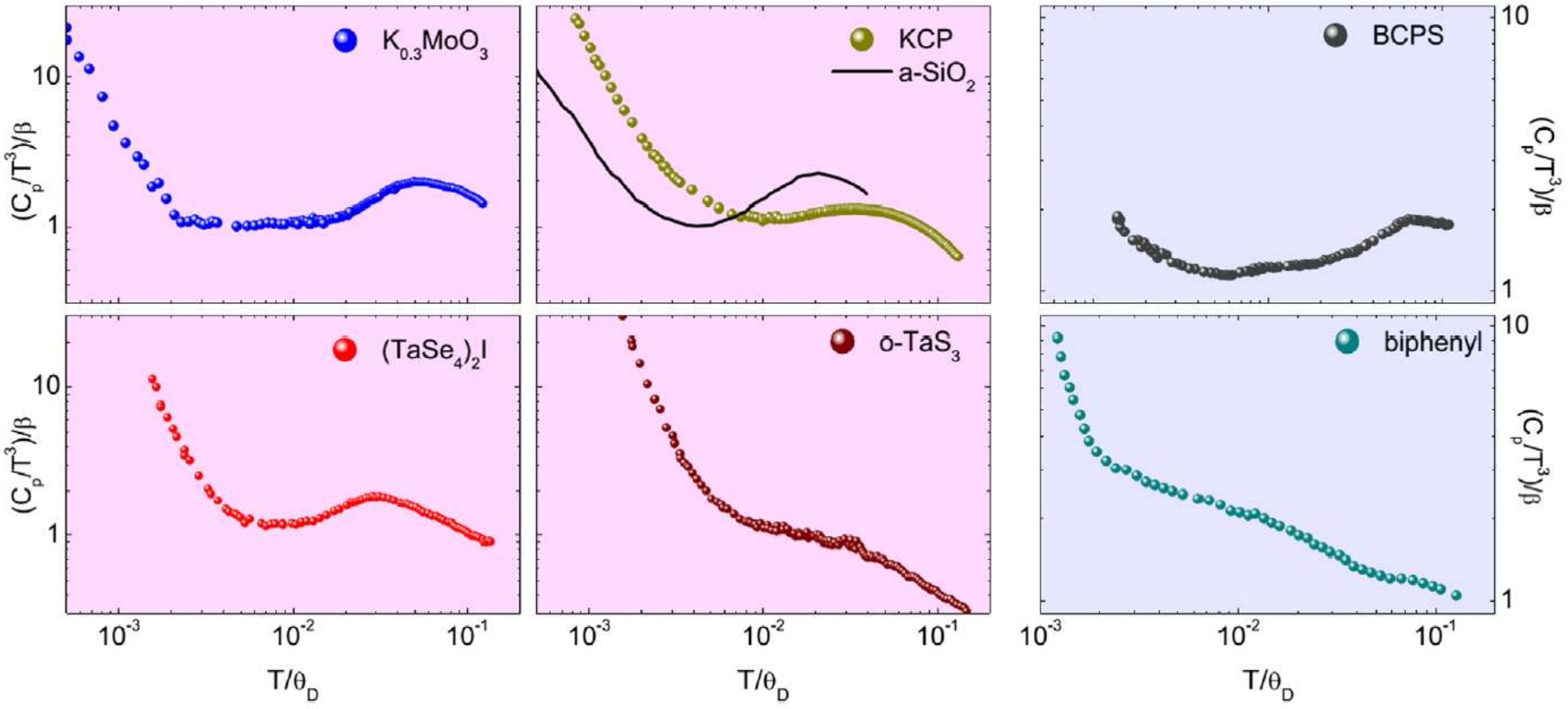



The phason is an overdamped bosonic mode

Next: consequences

# Glassy heat capacity

Phys. Rev. Lett. 114, 195502 (2015)




(1) Linear in T at low temperature, (2) anomalous peak deviation over Debye

PRL **93,** 245902 (2004) PHYSICAL REVIEW LETTERS week ending 10 DECEMBER 2004

**Explanation of the Glasslike Anomaly in the Low-Temperature Specific Heat of Incommensurate Phases**

A. Cano* and A. P. Levanyuk†

- Linear in T from phason (not Sommerfeld)
- Bump from amplitude mode

Linear in T is a generic feature of overdamped modes

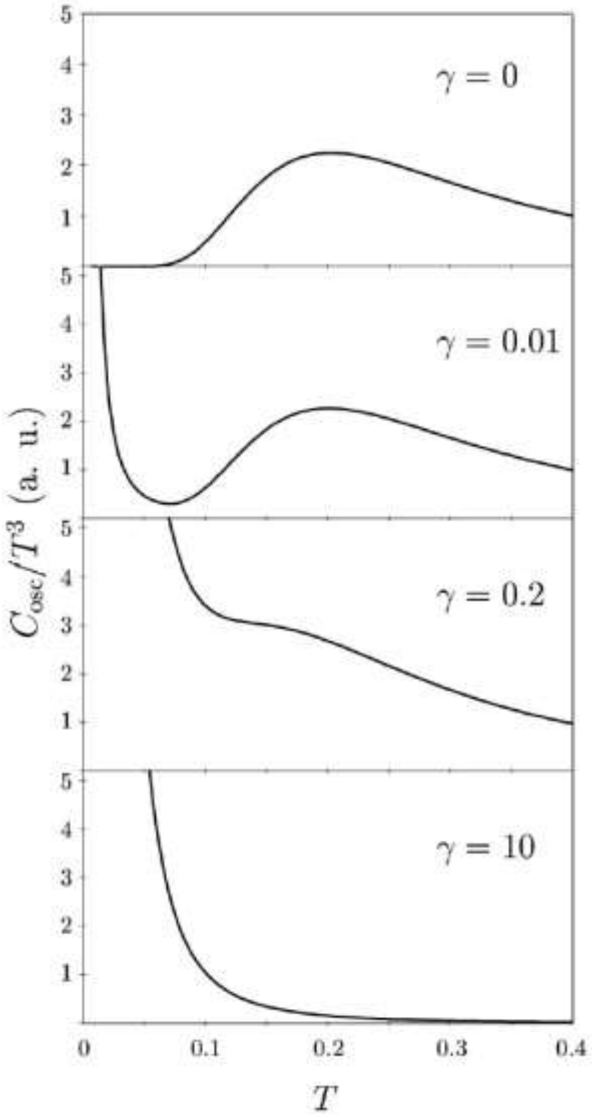
LETTERS AND COMMENTS

The entropy of a damped harmonic oscillator

A B Pippard

Published under licence by IOP Publishing Ltd
European Journal of Physics, Volume 8, Number 1

1987




[MB, Zaccone, IJMP B, Vol. 35, No. 04, 2130002 (2021)]
[Jiang, Zaccone, Setty, MB, PRB 108, 054203 (2023)]

# Strange metal physics from overdamped bosons

Article | Open access | Published: 10 January 2022

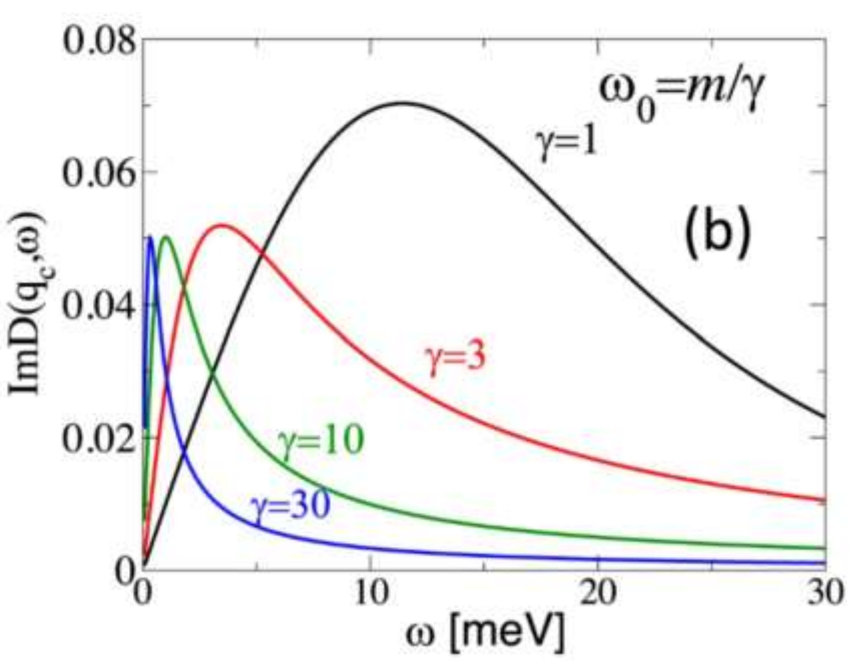
## Dissipation-driven strange metal behavior

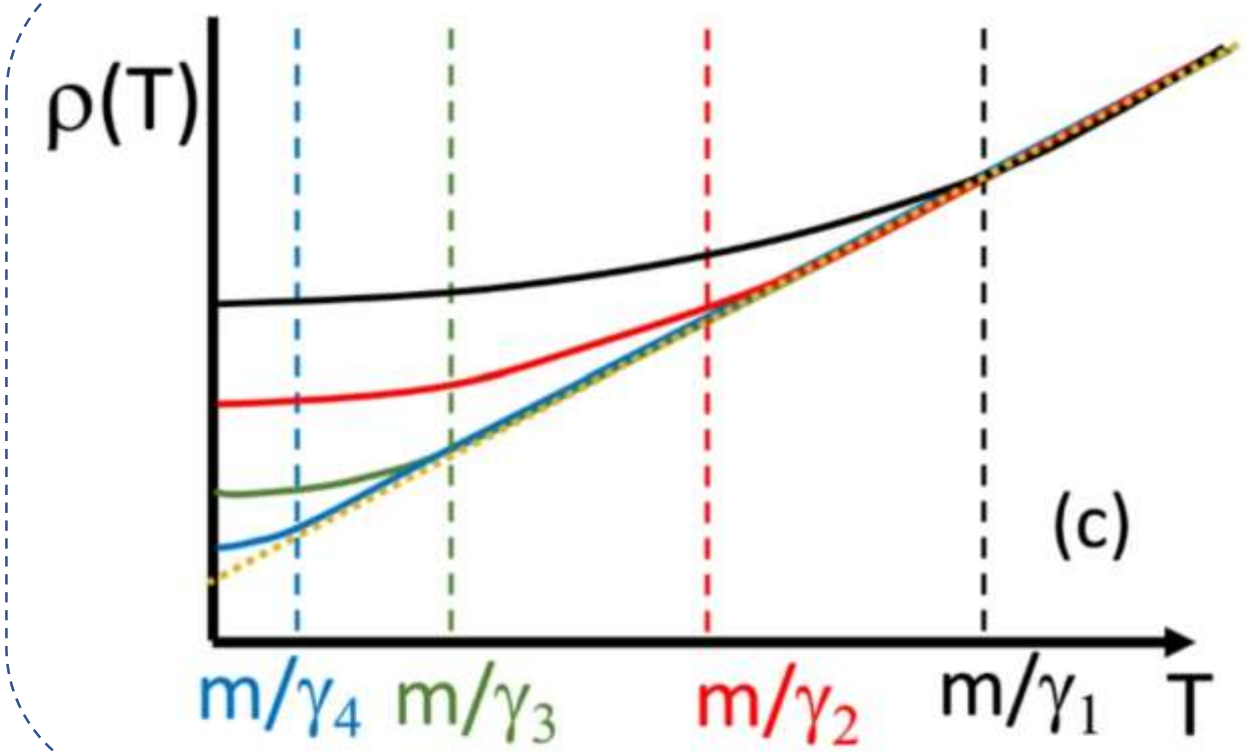
Sergio Caprara, Carlo Di Castro, Giovanni Mirarchi, Götz Seibold & Marco Grilli

Communications Physics 5, Article number: 10 (2022) | Cite this article



$$\mathrm{Im}\, D(\mathbf{q},\omega) = \frac{\gamma\omega}{\left(m + \bar{\nu}|\mathbf{q}-\mathbf{q}_c|^2 - \omega^2/\overline{\Omega}\right)^2 + \gamma^2\omega^2},$$



# Strange metal physics from phasons

**Extended linear-in-$T$ resistivity due to electron-phason scattering in moiré superlattices**

Héctor Ochoa[1,2,3] and Rafael M. Fernandes[4]

Show more

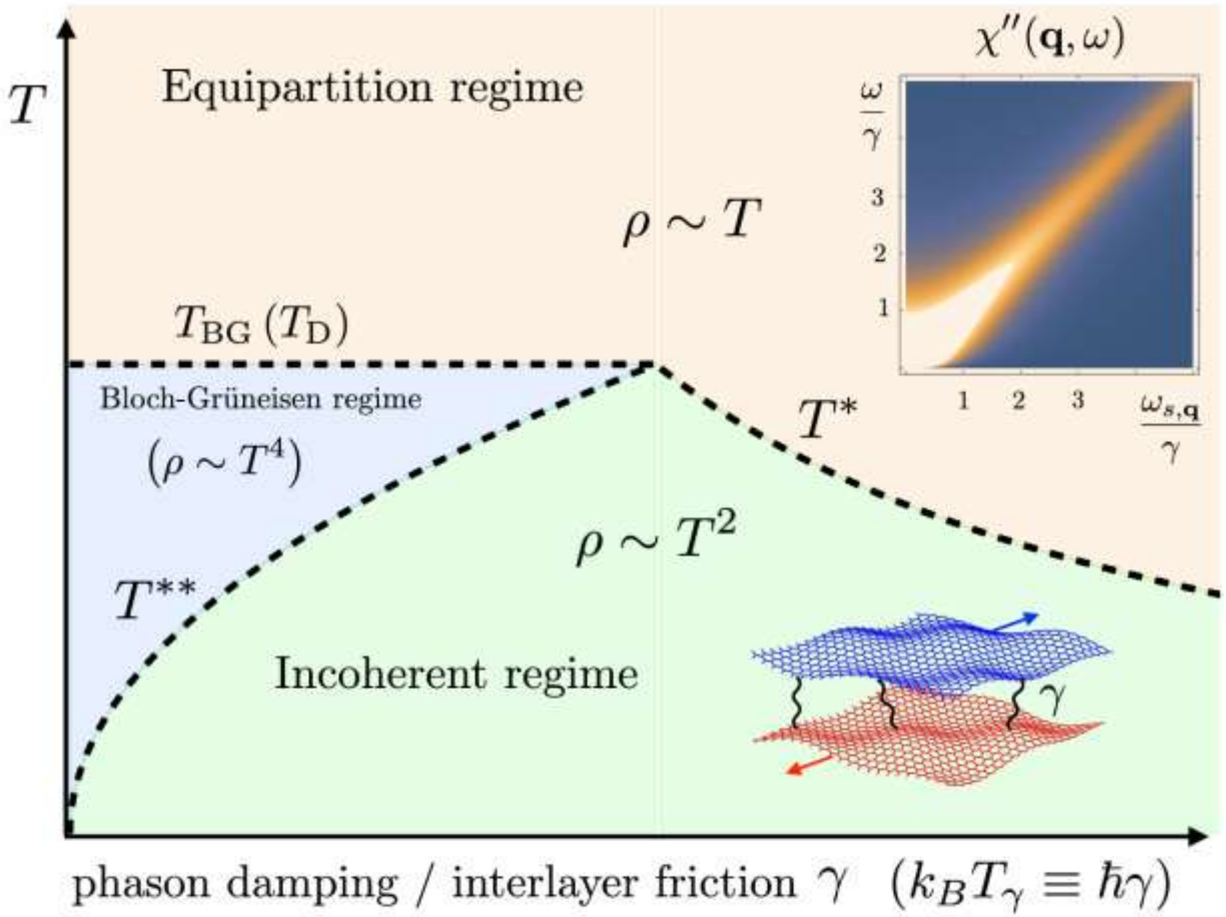
Phys. Rev. B **108**, 075168 – **Published 31 August, 2023**

DOI: https://doi.org/10.1103/PhysRevB.108.075168

$$\chi_s\left(\mathbf{q},\omega\right)=\frac{\varrho^{-1}}{\omega_{s,\mathbf{q}}^2-\omega^2-i\gamma\omega}.$$

$$\omega_{s,\mathbf{q}}=c_s|\mathbf{q}|$$

Same dispersion



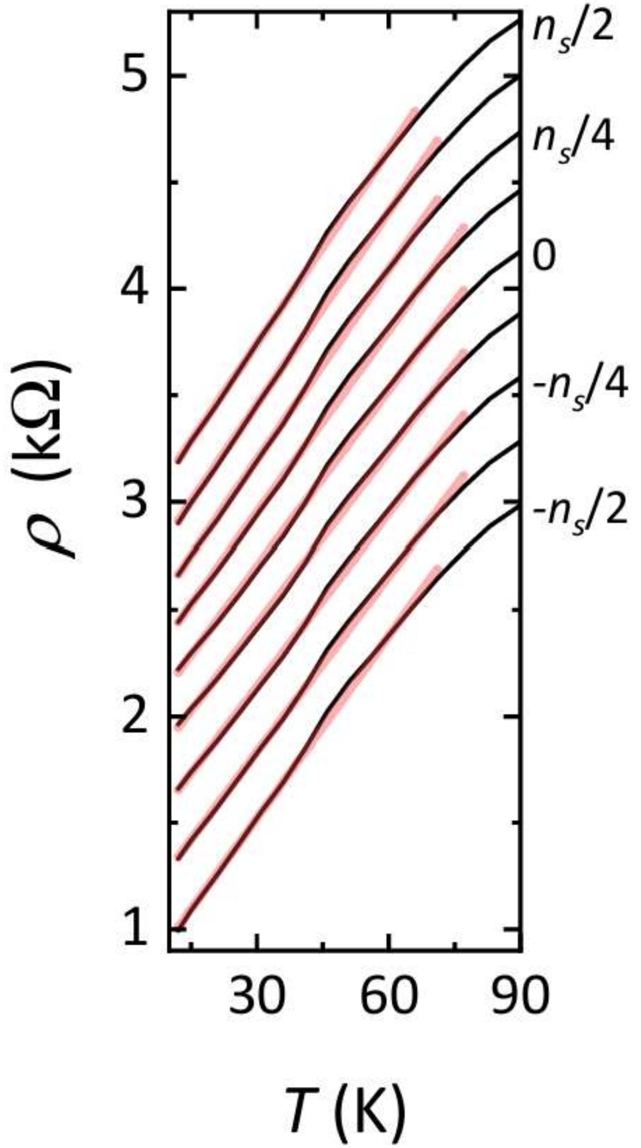
# Experimental signatures

arXiv:2511.01691 [pdf, ps, other] cond-mat.mes-hall

**Phason-driven temperature-dependent transport in moiré graphene**

**Authors:** Alex Boschi, Alejandro Ramos-Alonso, Vaidotas Mišeikis, Kenji Watanabe, Takashi Taniguchi, Fabio Beltram, Stiven Forti, Antonio Rossi, Camilla Coletti, Rafael M. Fernandes, Héctor Ochoa, Sergio Pezzini

*‘’We show that the experimental trends are compatible with scattering processes mediated by longitudinal phasons, which dominate the resistivity over the contribution from conventional acoustic phonons of the monolayer.’’*

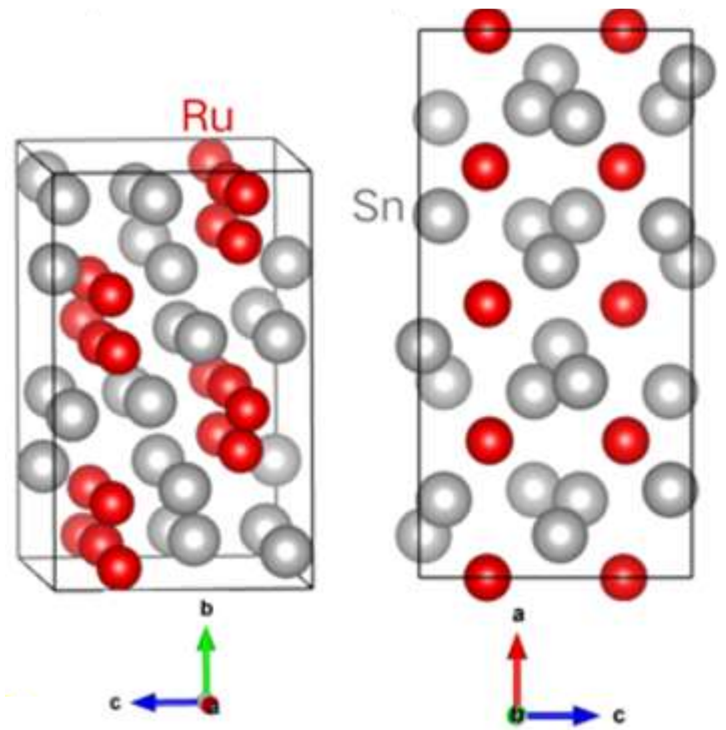


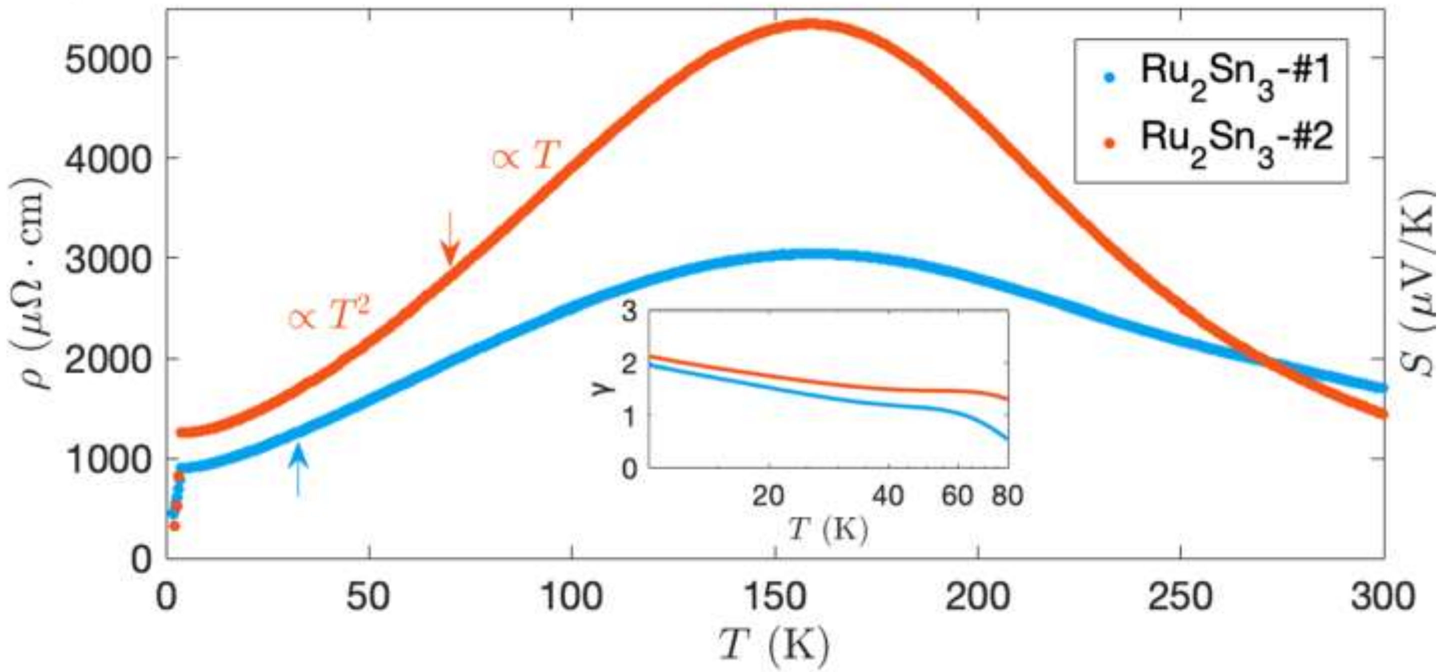
# Similar results in chimney ladder crystals

arXiv:2503.03155 [pdf, ps, other] cond-mat.mtrl-sci cond-mat.soft

**Glass-like anomalies and unconventional thermoelectric transport in chimney ladder crystals**

**Authors:** Srinivas V. Mandyam, Weicen Dong, Xiaoxian Yan, Binru Zhao, Yasong Wu, Chunhao Su, Elen Duverger-Nedellec, Junfa Lin, Tianlong Xia, Zhiying Zhao, Xi Chen, Jiong Yang, Jie Ma, Hui Xing, F. Malte Grosche, Matteo Baggioli



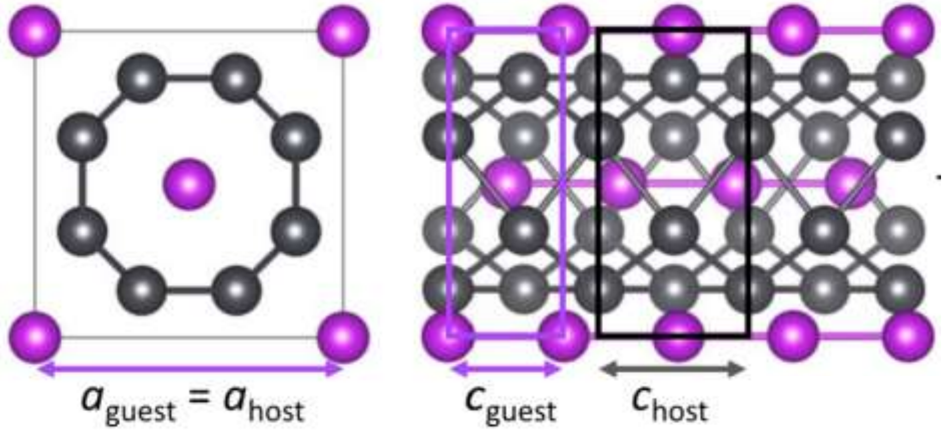

$$\rho(T) = \rho_0 + \alpha T^\gamma$$

# Phason enhanced superconductivity?

**Strong coupling superconductivity in a quasiperiodic host-guest structure**

PHILIP BROWN, KONSTANTIN SEMENIUK, DIANDIAN WANG, BARTOMEU MONSERRAT, CHRIS J. PICKARD, AND F. MALTE GROSCHE Authors Info & Affiliations

SCIENCE ADVANCES • 13 Apr 2018 • Vol 4, Issue 4 • DOI: 10.1126/sciadv.aao4793




*``These properties may be attributed to the peculiar phonon spectrum of incommensurate host-guest structures, which exhibit additional quasi-acoustic sliding modes, suggesting a pathway toward strong coupling superconductivity with the potential for enhanced transition temperatures and high critical fields.''*

In Phys. Rev. B 108, 054203 (2023) we proposed a simple explanation using the Eliashberg formalism with the overdamped phason propagator

# More signatures and consequences?

FEATURED IN PHYSICS | EDITORS' SUGGESTION

**Phason-Dominated Thermal Transport in Fresnoite**

M. E. Manley[1,*], A. F. May[1], B. L. Winn[2], D. L. Abernathy[2], R. Sahul[3], and R. P. Hermann[1]

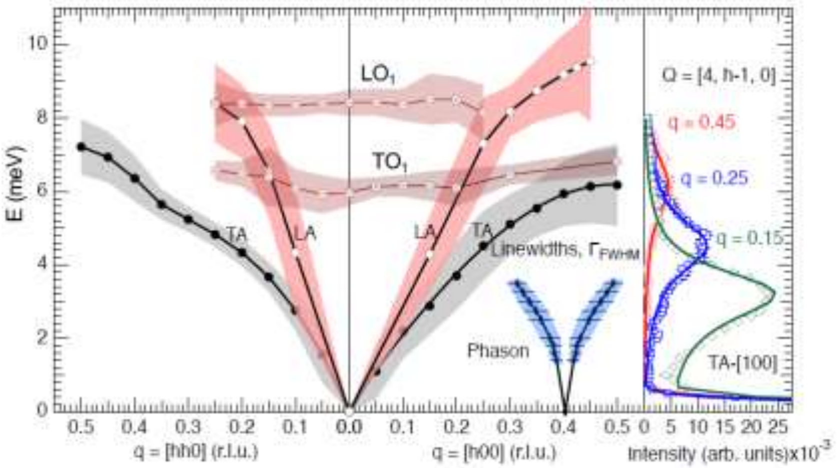
ARTICLE

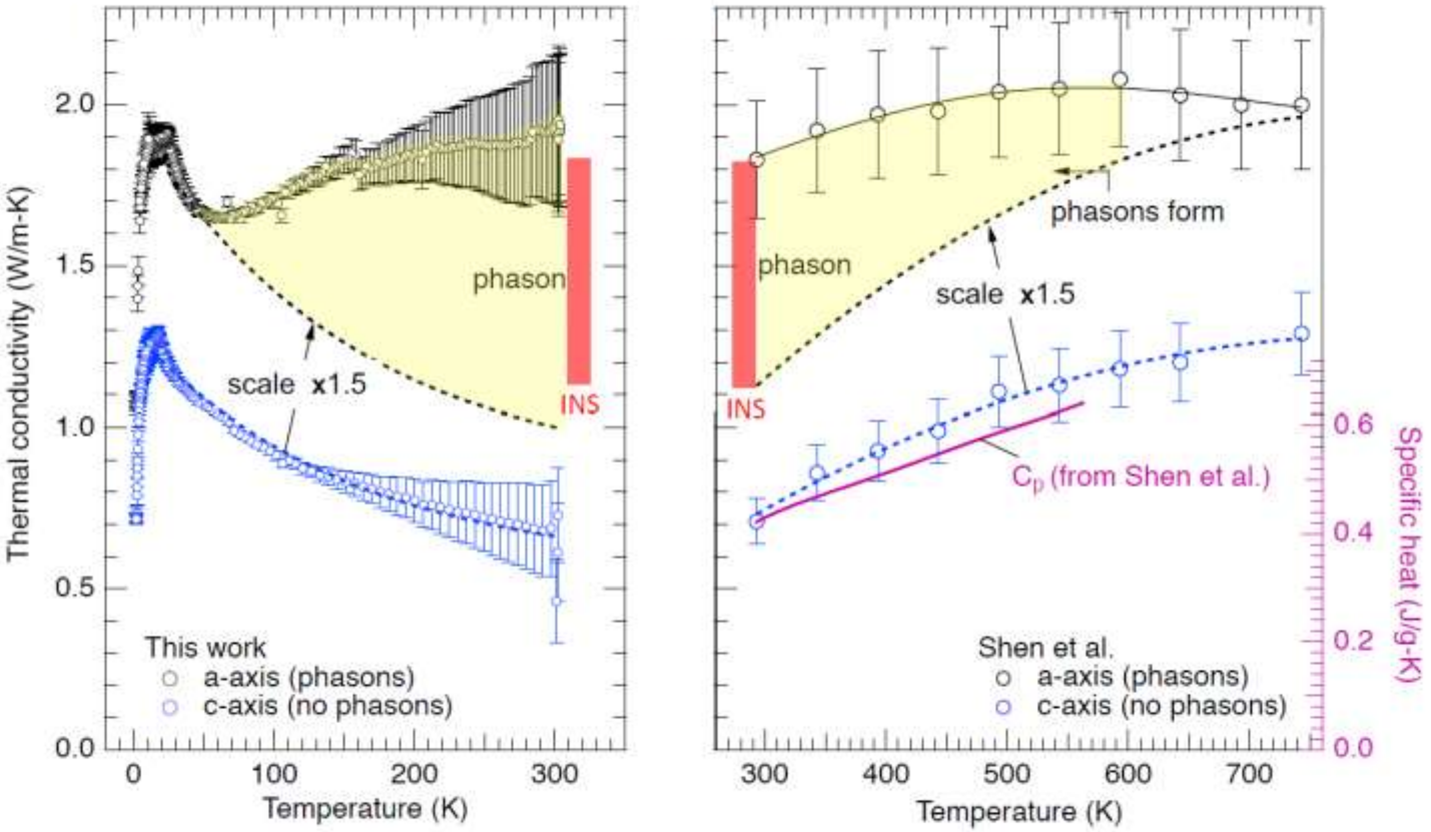
DOI: 10.1038/s41467-018-04229-1 OPEN

Supersonic propagation of lattice energy by phasons in fresnoite

M.E. Manley[1], P.J. Stonaha[1], D.L. Abernathy[2], S. Chi[2], R. Sahul[3], R.P. Hermann[1] & J.D. Budai[1]

Example:
enhancement of lattice thermal conductivity



# Aside comment [from chat with Latham, 1 hr ago]

RESEARCH ARTICLE | PHYSICS

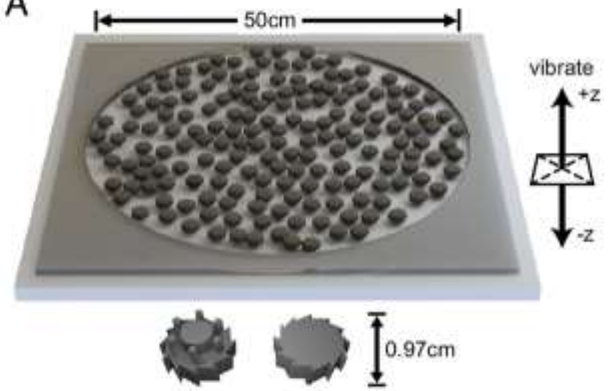
## Three-stage melting of a macroscopic continuous spacetime crystal

Guoqing Liu, Jimin Bai, Matteo Baggioli, and Jie Zhang   Authors Info & Affiliations

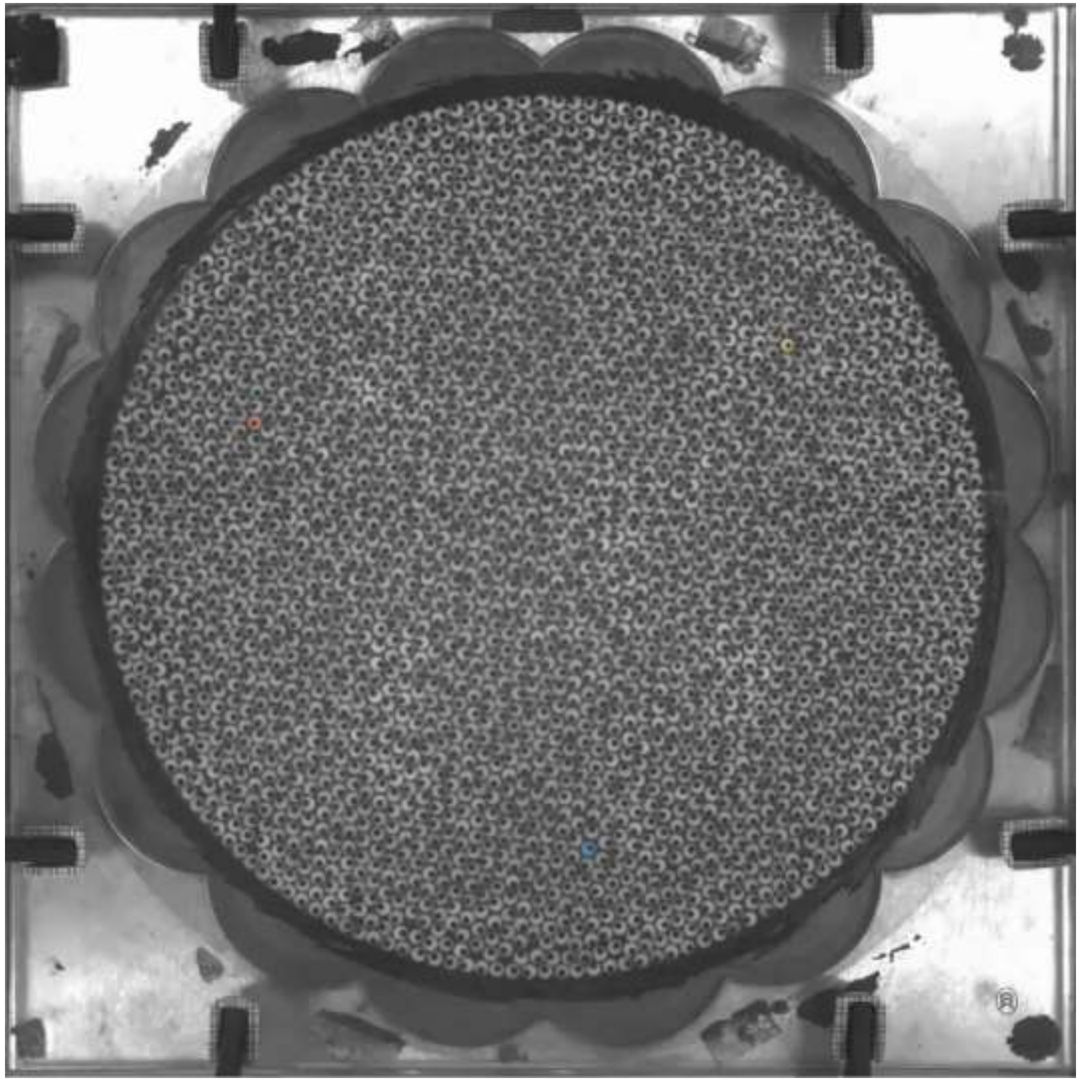
Edited by Ivan I. Smalyukh, University of Colorado Boulder, Boulder, CO; received April 17, 2026; accepted June 2, 2026 by Editorial Board Member Paul Chaikin

**June 30, 2026** | 123 (27) e2613063123 | https://doi.org/10.1073/pnas.2613063123

A




Can we make a spacetime quasicrystal?



Article | Published: 19 January 2024

## Quasi-crystalline order in vibrating granular matter

A. Plati, R. Maire, E. Fayen, F. Boulogne, F. Restagno, F. Smallenburg & G. Foffi

*Nature Physics* **20**, 465–471 (2024) | Cite this article

# Outlook

- Quasicrystals should not exist by they do!

- Their dynamics are highly non-trivial with interesting intermediate behaviors and unexpected consequences

- Many aspects to explore…

# Thank you for listening!

Matteo Baggioli
b.matteo@sjtu.edu.cn
SJTU Shanghai