HYPERBOLIC LATTICES

Quantum computation and fabrication

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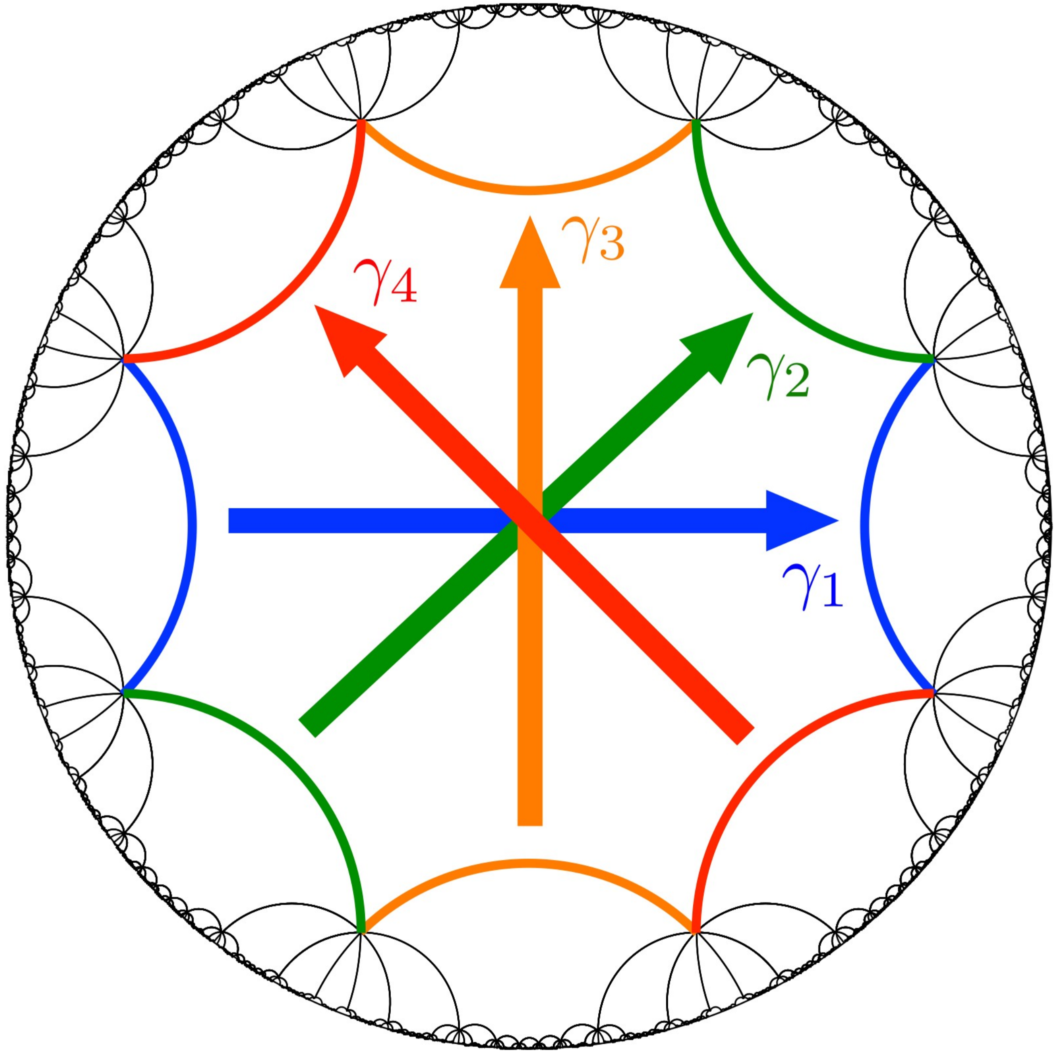
ICMS · Edinburgh · 21 July 2026



Hyperbolicity, three ways

1

BAND THEORY

Γ

translation group

$\Sigma_g = \mathbb{H}^2/\Gamma$

compact genus-g quotient

COMPACT / PERIODIC

2

QUANTUM CODES

$H_1(\Sigma_g; \mathbb{Z}_2)$

logical Z operators (cycles)

$H^1(\Sigma_g; \mathbb{Z}_2)$

logical X operators (cocycles)

COMPACT / PERIODIC

3

RESONATOR DEVICES

$\Lambda/\Gamma \hookrightarrow \Sigma_g$

compact quotient graph underlying the design

$A = (A_{ij})$

weighted adjacency of an open patch of Λ; $A_{ij} = 0$ if i, j are uncoupled

FINITE / OPEN AT PRESENT

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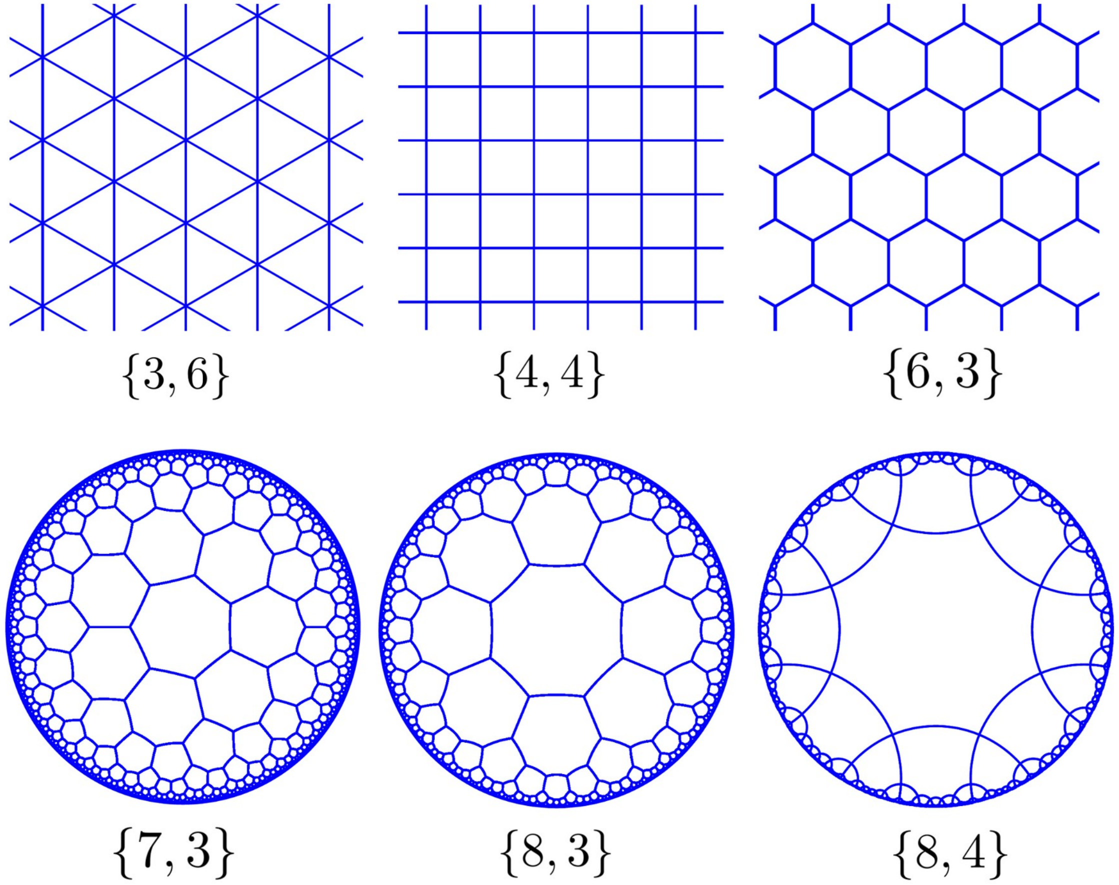
FINITE / OPEN AT PRESENT

Finite hyperbolic patches do not suppress their boundary



Two consequences

Γ nonabelian

TRANSLATIONS

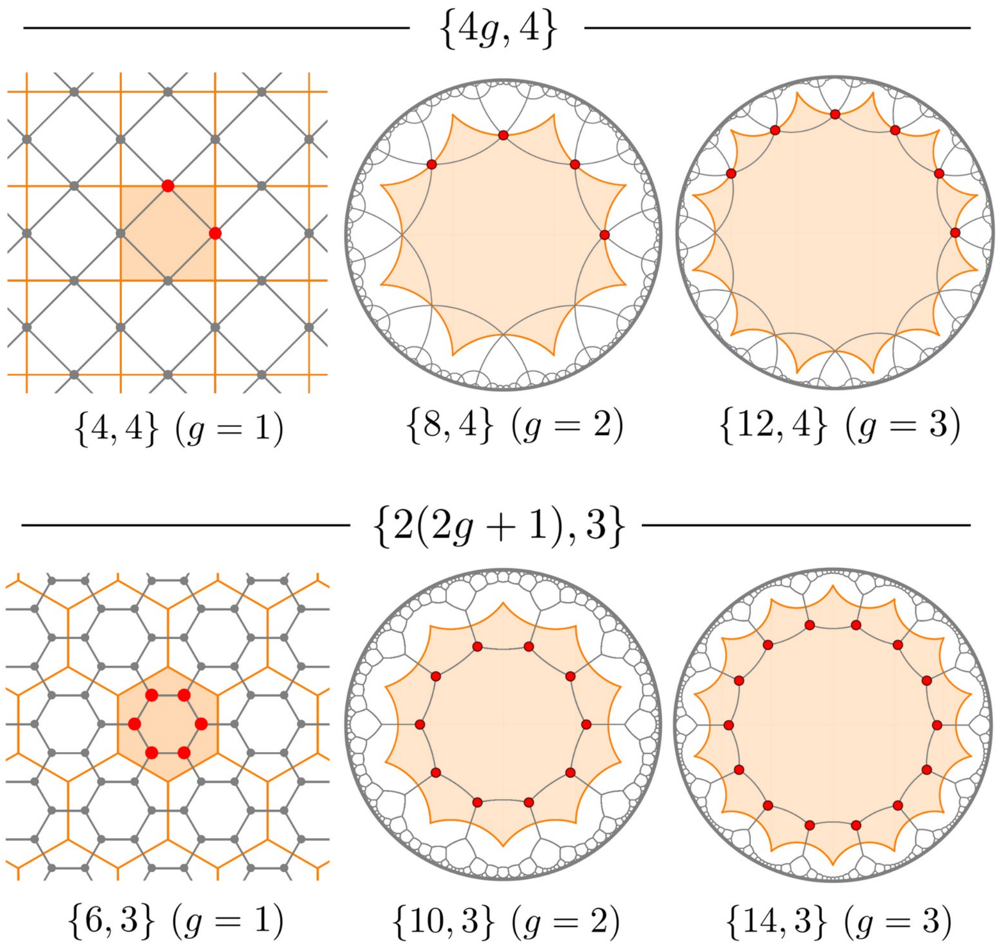
Two primitive hyperbolic translations need not commute.

$|\partial\Lambda| \,/\, |\Lambda| \nrightarrow 0$

BOUNDARY

Boundary degrees of freedom remain a macroscopic fraction of a finite patch.

Crystallography supplies finite quotient data

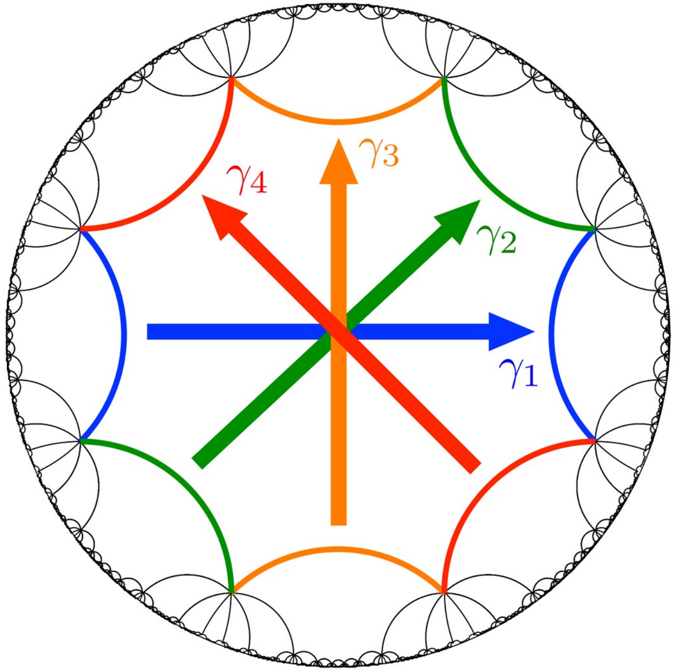


Euclidean-Hyperbolic Dictionary

Lattice: regular {p,q} tessellation
Translations: Fuchsian group Γ
Unit cell: fundamental polygon D
Compactification: a compact quotient of genus g
Hamiltonian: finite Bloch matrix H(k)

Boettcher et al., Crystallography of Hyperbolic Lattices, PRB 105 (2022).

Automorphic Bloch states and rank-one characters

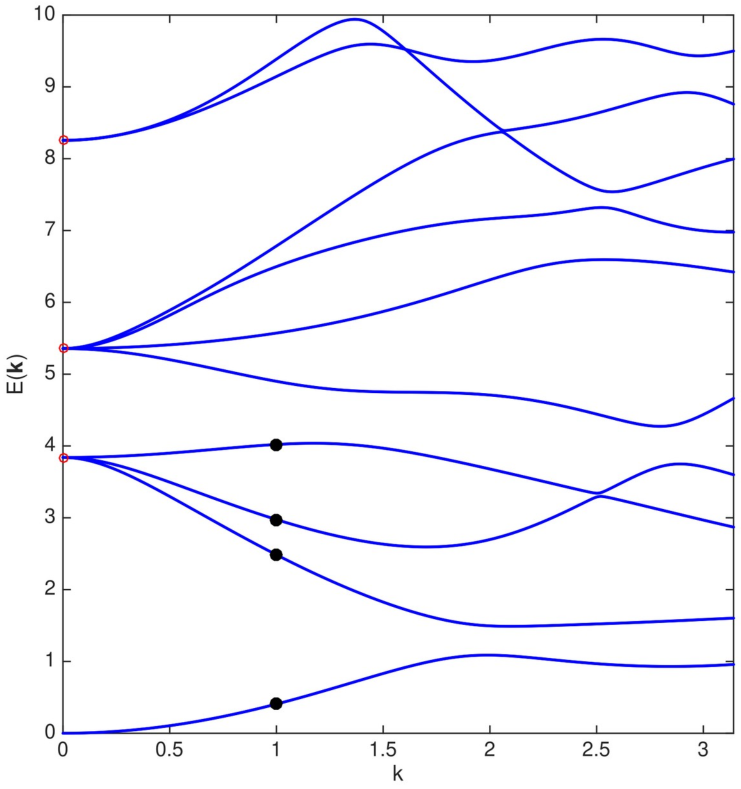


$$\psi(\gamma^{-1}z) = \chi_k(\gamma)\,\psi(z)$$

$\gamma \in \Gamma$ hyperbolic translation
$\chi_k : \Gamma \to U(1)$ rank-one character
$H = -\Delta + V$ is local
k: global automorphy data, 2g phases

k labels flat holonomies that are akin to crystal momenta

Automorphic Bloch states and rank-one characters



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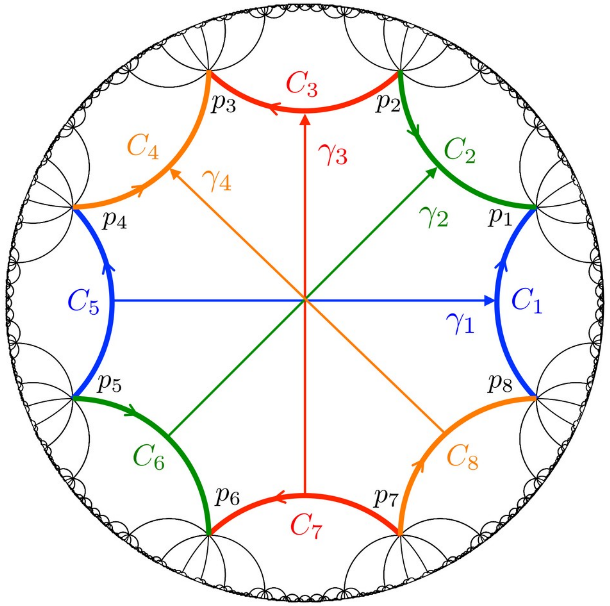
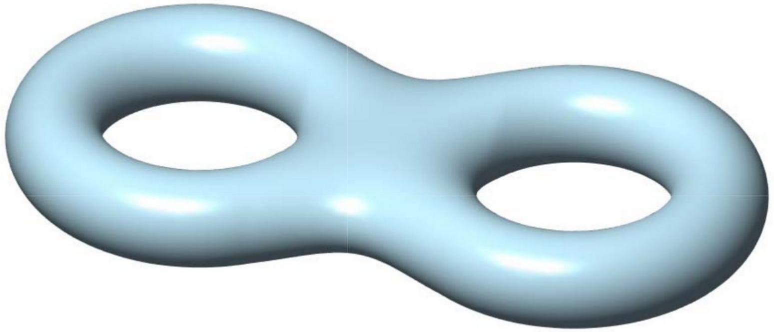
Side pairings create compact higher-genus unit cells

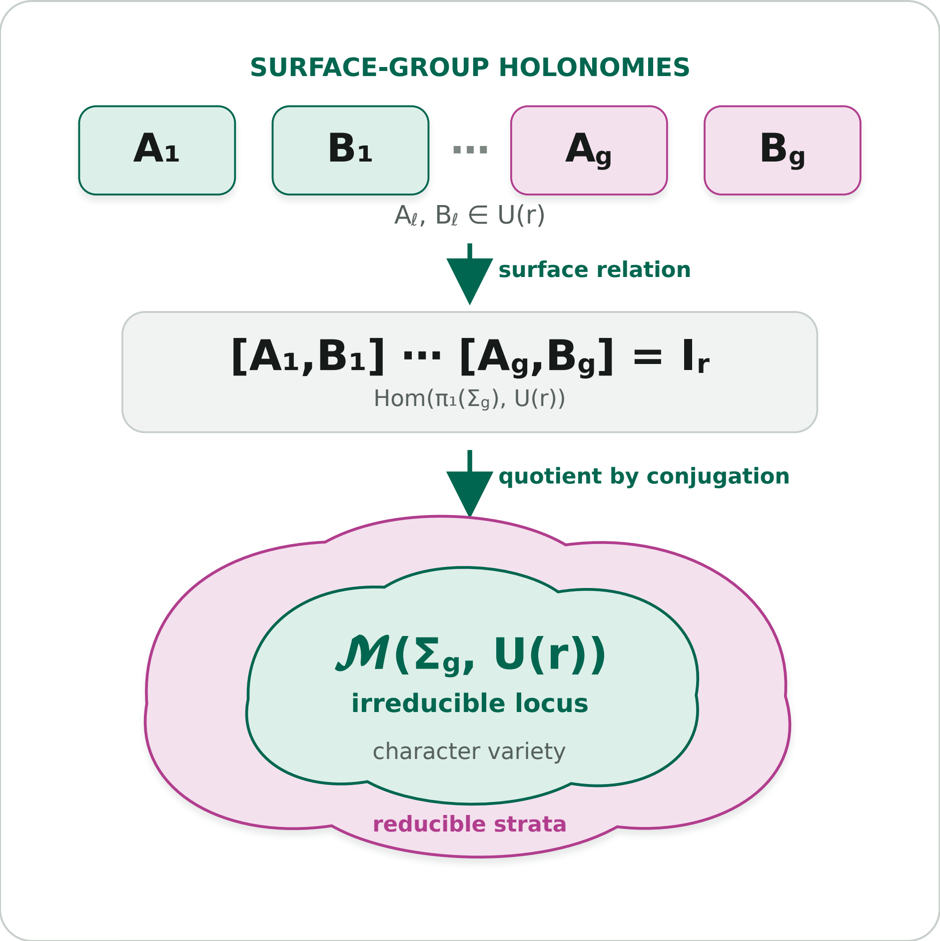


→



$$\Sigma_g = \mathbb{H}^2 / \Gamma$$

Nonabelian sectors require a larger momentum space

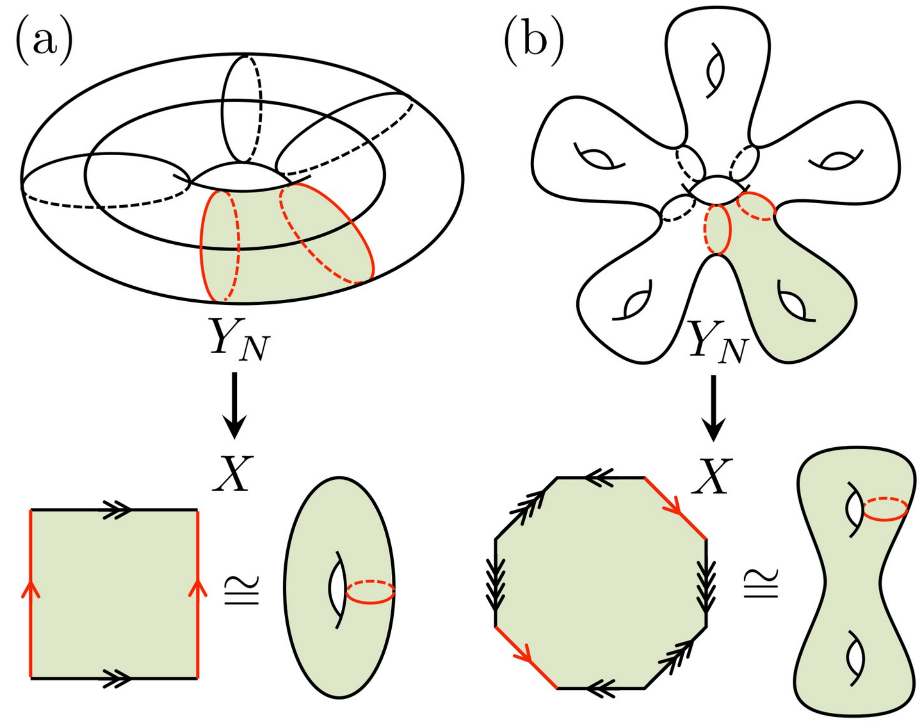


rank 1		rank r > 1
$\mathrm{Jac}(\Sigma_g) \cong T^{2g}$	→	$\mathcal{M}(\Sigma_g, U(r))$

What changes for r > 1

Wavefunction: r components
Holonomy: an irreducible map $\Gamma \to U(r)$
Moduli space: dimension $\propto r^2$ at fixed genus
Strata: reducible loci now, may be singular

Normal finite covers implement periodic closure



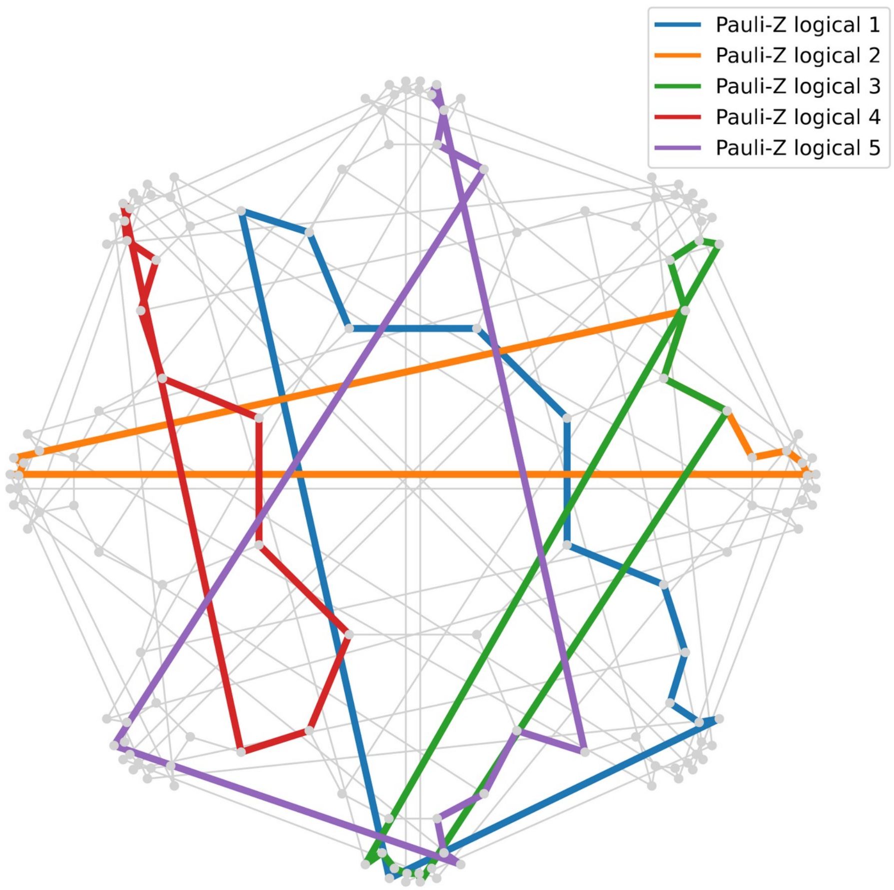
$\Gamma_0 \triangleleft \Gamma \quad |\Gamma / \Gamma_0| = N$

$g(Y_n) = N[g(X) - 1] + 1$

PBC: paired boundary sites become neighbours
Cover: $Y_n \to X = \mathbb{H}^2/\Gamma$
Spectrum: irreducible representations of Γ/Γ_0

QUANTUM COMPUTATION

Hyperbolic quantum error-correcting codes



arXiv:2504.07800 · arXiv:2603.27004

Hyperbolic codes: constant rate and logarithmic distance

$k / n \to c > 0$

ENCODING RATE

Logical qubits grow linearly with physical qubits.

$d = \Theta(\log n)$

DISTANCE

Shortest nontrivial cycles grow only logarithmically.

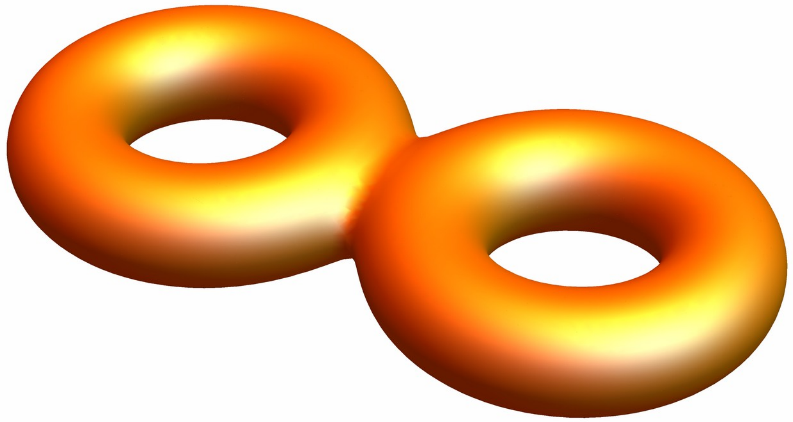
$O(1)$

CHECK WEIGHT

A fixed $\{p,q\}$ lattice gives bounded-degree LDPC checks.

Euclidean toric code: vanishing rate, but $d = \Theta(\sqrt{n})$

A genus-g cellulation defines a CSS surface code



CELLULATION	QEC OBJECT
edge e	physical qubit
face f	Z-type parity check
vertex v	X-type parity check
$H_1(\Sigma; \mathbb{Z}_2)$	logical Z operators
$H^1(\Sigma; \mathbb{Z}_2)$	logical X operators

$$n = |E| \qquad k = 2g$$

Mahmoud, Ali, and Rayan, Phys. Rev. A 113 (2026), 042426.

CSS commutation is the identity $\partial_1\partial_2 = 0$

$$C_2 \xrightarrow{\partial_2} C_1 \xrightarrow{\partial_1} C_0$$

C_2	C_1	C_0
faces / Z checks	edges / physical qubits	vertices / X checks

Z-check matrix = ∂_2^T

X-check matrix = ∂_1

commutation: $\partial_1\partial_2 = 0$

$H_1 = \ker \partial_1 / \operatorname{im} \partial_2$

logical Z from homology · logical X from dual cohomology

Euler characteristic fixes the asymptotic encoding rate

$$pF = 2E \qquad qV = 2E \qquad n = E$$

incidence counting

$$\chi(\Sigma_g) = V - E + F = 2 - 2g$$

topology of the closed quotient

$$k = 2g = E - V - F + 2$$

$$k / n = 1 - 2/p - 2/q + 2/n$$

hyperbolic condition: $1/p + 1/q < 1/2 \quad \Rightarrow \quad \lim k/n = 1 - 2/p - 2/q > 0$

For a closed regular {p,q} cellulation with qubits on edges.

Distance is a primal–dual systolic quantity

Z distance · primal systole

$$\min\{|c| : 0 \neq [c] \in H_1(\Sigma_g; \mathbb{Z}_2)\}$$

shortest noncontractible cycle in the primal graph

X distance · dual systole

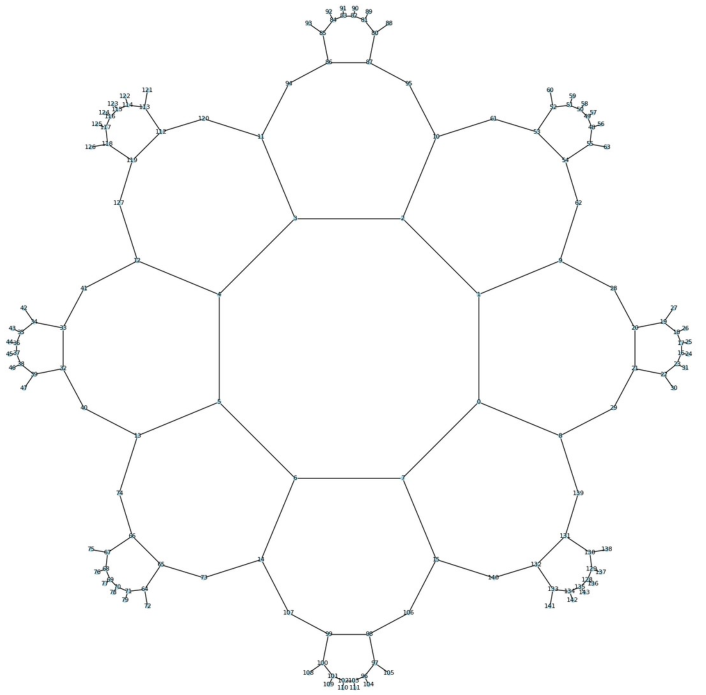
$$\min\{|\alpha| : 0 \neq [\alpha] \in H^1(\Sigma_g; \mathbb{Z}_2)\}$$

shortest nontrivial cocycle, or dual cycle

code distance = min{primal systole, dual systole} = $\Theta(\log n)$

Boundary identifications create logical homology

OPEN PATCH



side pairings + boundary edges →

PERIODIC GRAPH

Boundary identifications create logical homology

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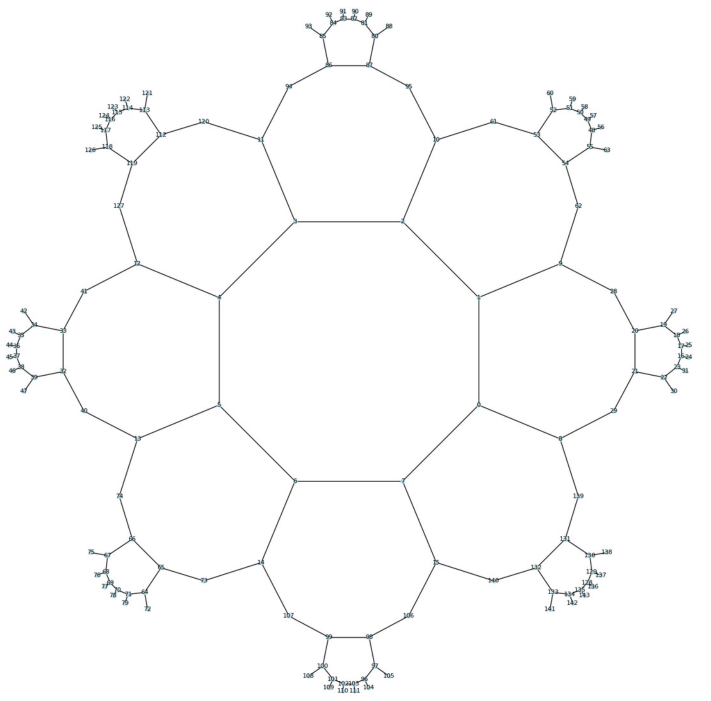

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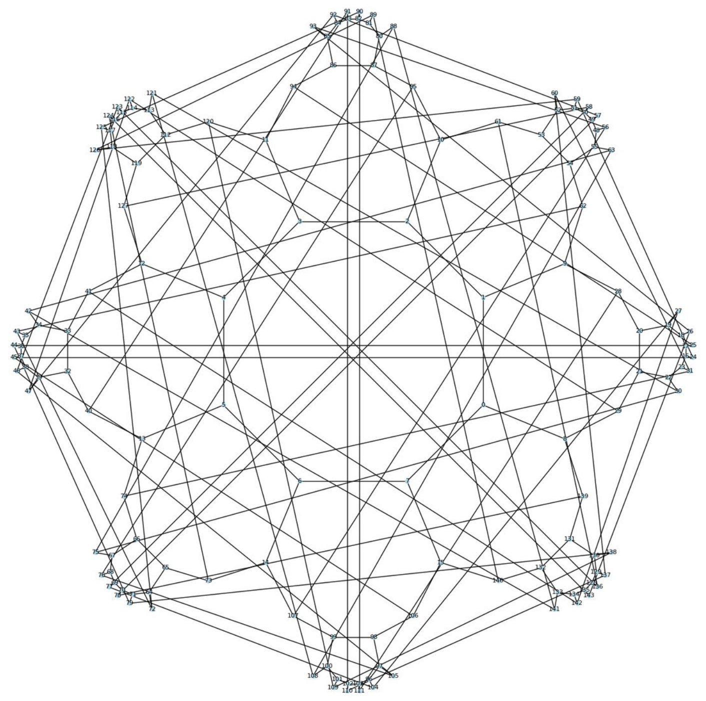

Mahmoud, Ali, and Rayan, Phys. Rev. A 113 (2026), Fig. 2.

Tilings determine stabilizers and logical operators

01

Choose {p,q}

Select a tiling and compatible Bravais lattice.

→

02

Impose PBC

Construct the finite periodic quotient graph.

→

03

Find plaquettes

Build the parity-check supports.

→

04

Find H_1 and H^1

Extract cycles and dual cocycles.

→

05

Benchmark

Compute d, k/n, and logical error rates.

Output: quotient graph, stabilizers, logical basis, and benchmarks

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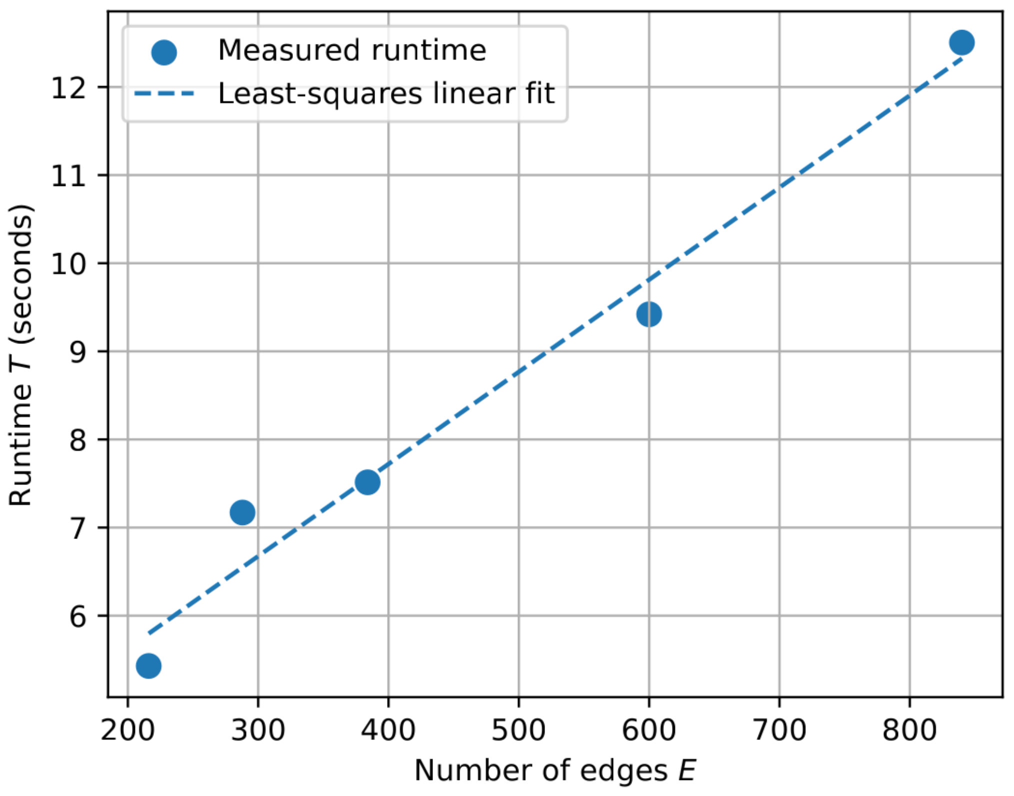
HCB runtime is approximately linear over the tested sizes



What the algorithm returns

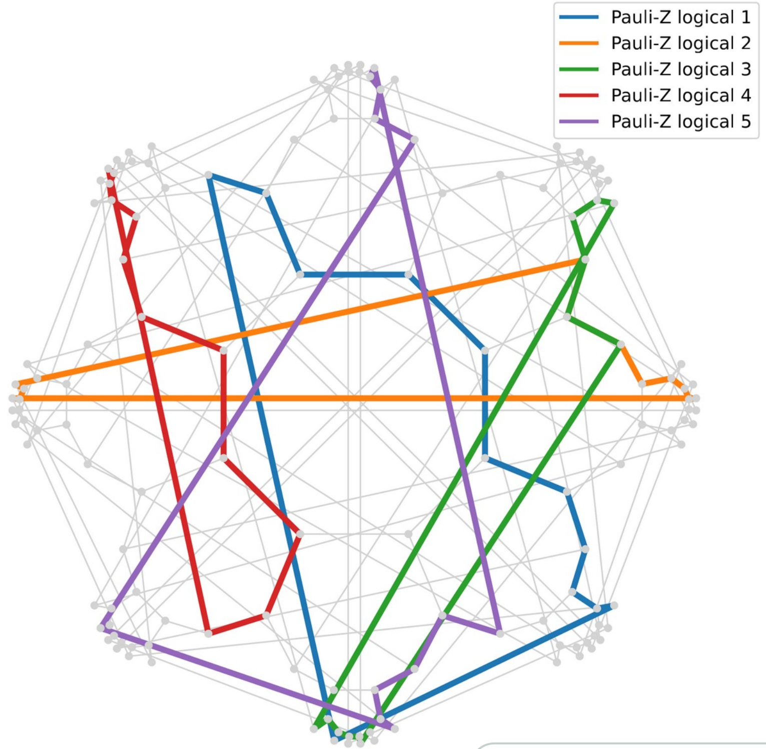
Stabilizers: plaquette boundaries and their span
Logical basis: primal cycles and dual cocycles
Canonical pairing: odd intersection for each X_i,Z_i pair
Distance: shortest nontrivial representative

Observed: 5.5–12.5 s for 216–840 edges

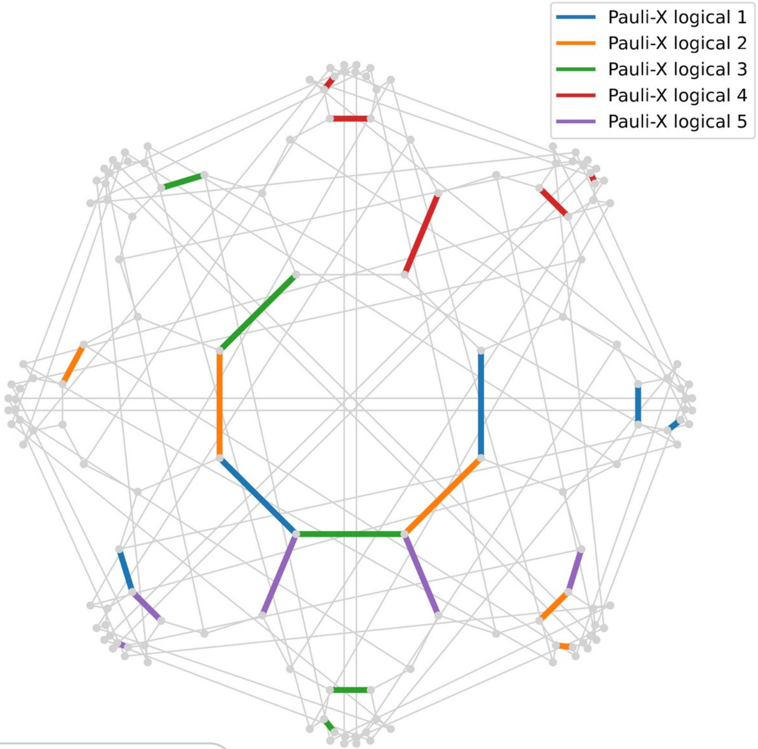
Mahmoud, Ali, and Rayan, Phys. Rev. A 113 (2026), Fig. 5 and Appendix A.

Cycles and cocycles form a canonical logical basis

Z-TYPE · PRIMAL CYCLES



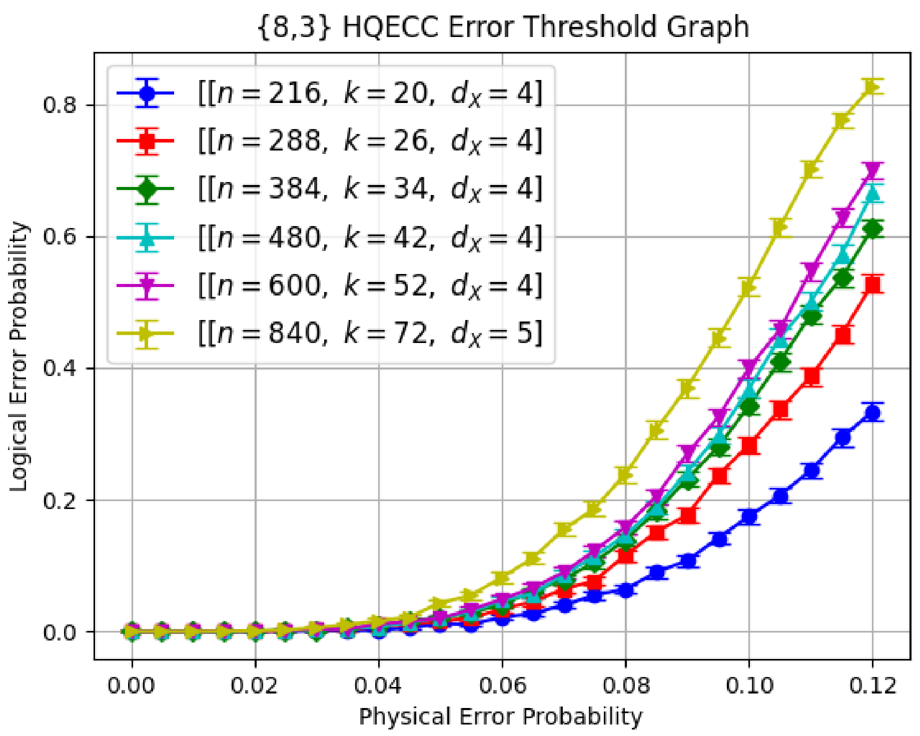
X-TYPE · DUAL COCYCLES



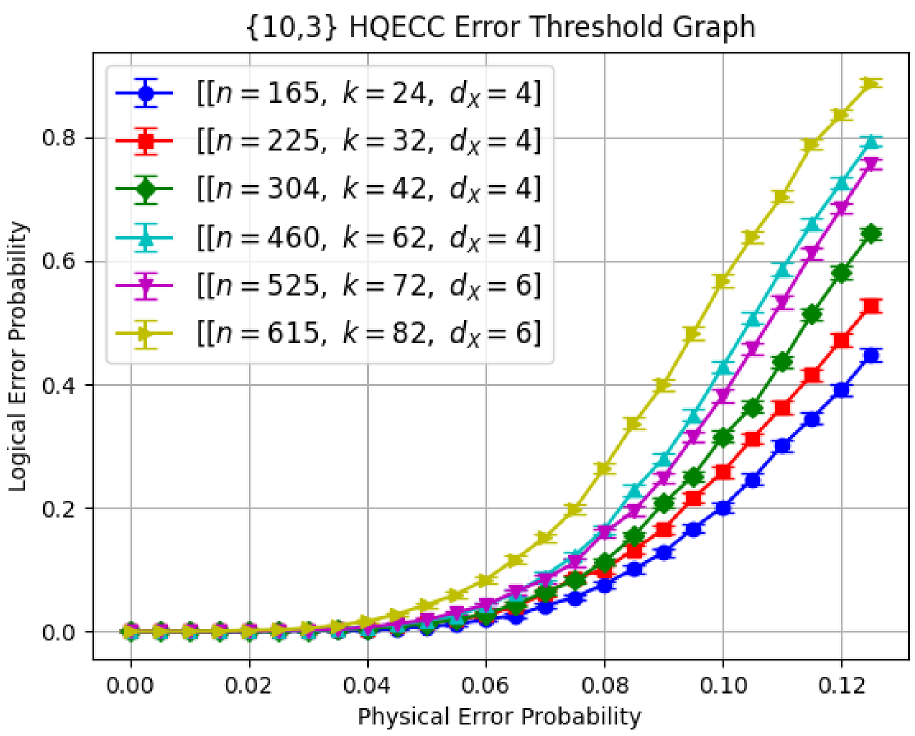
$$X_i Z_i = -Z_i X_i; \quad X_i Z_j = Z_j X_i \quad \text{for } i \neq j$$

Code-capacity thresholds for {8,3} and {10,3}

{8,3} · ≈ 3%

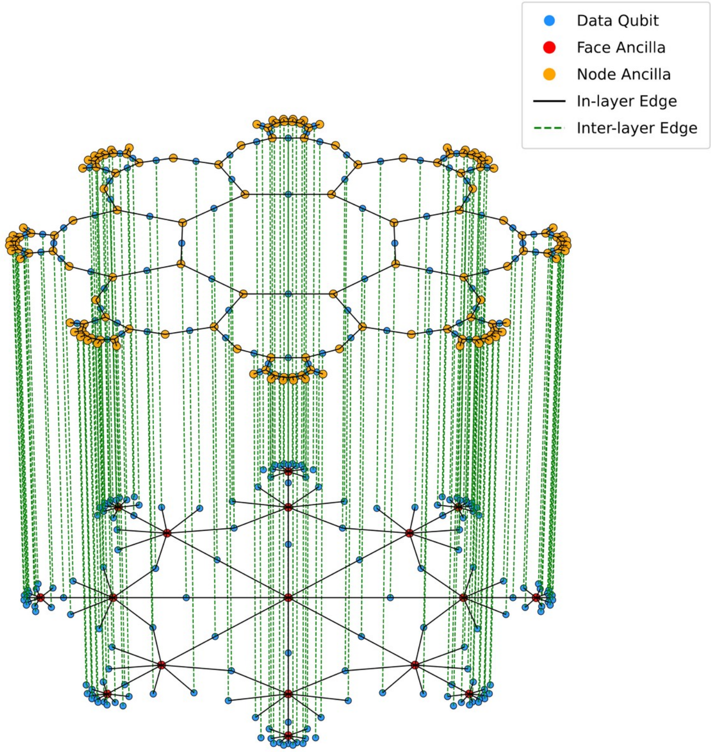


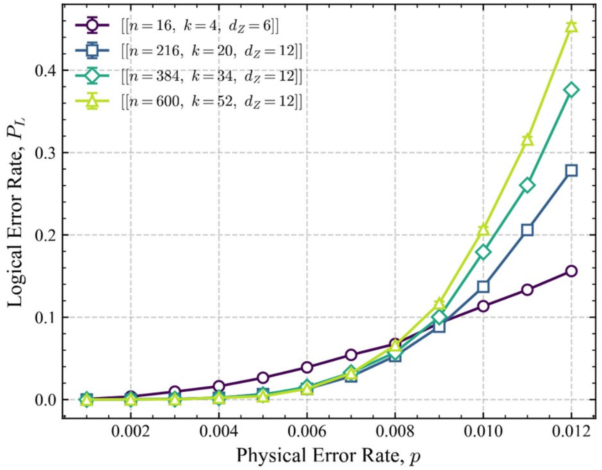
{10,3} · ≈ 2%

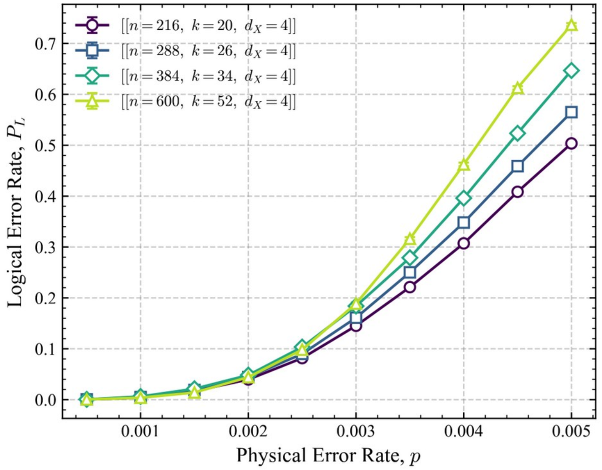


independent Z noise · perfect syndromes · MWPM · 10^4 trials per point

Foliation extends the code to a 3D cluster state







≈ 0.8%
LOGICAL Z
circuit-level depolarizing noise

≈ 0.25%
LOGICAL X
asymmetry from a non-self-dual tiling

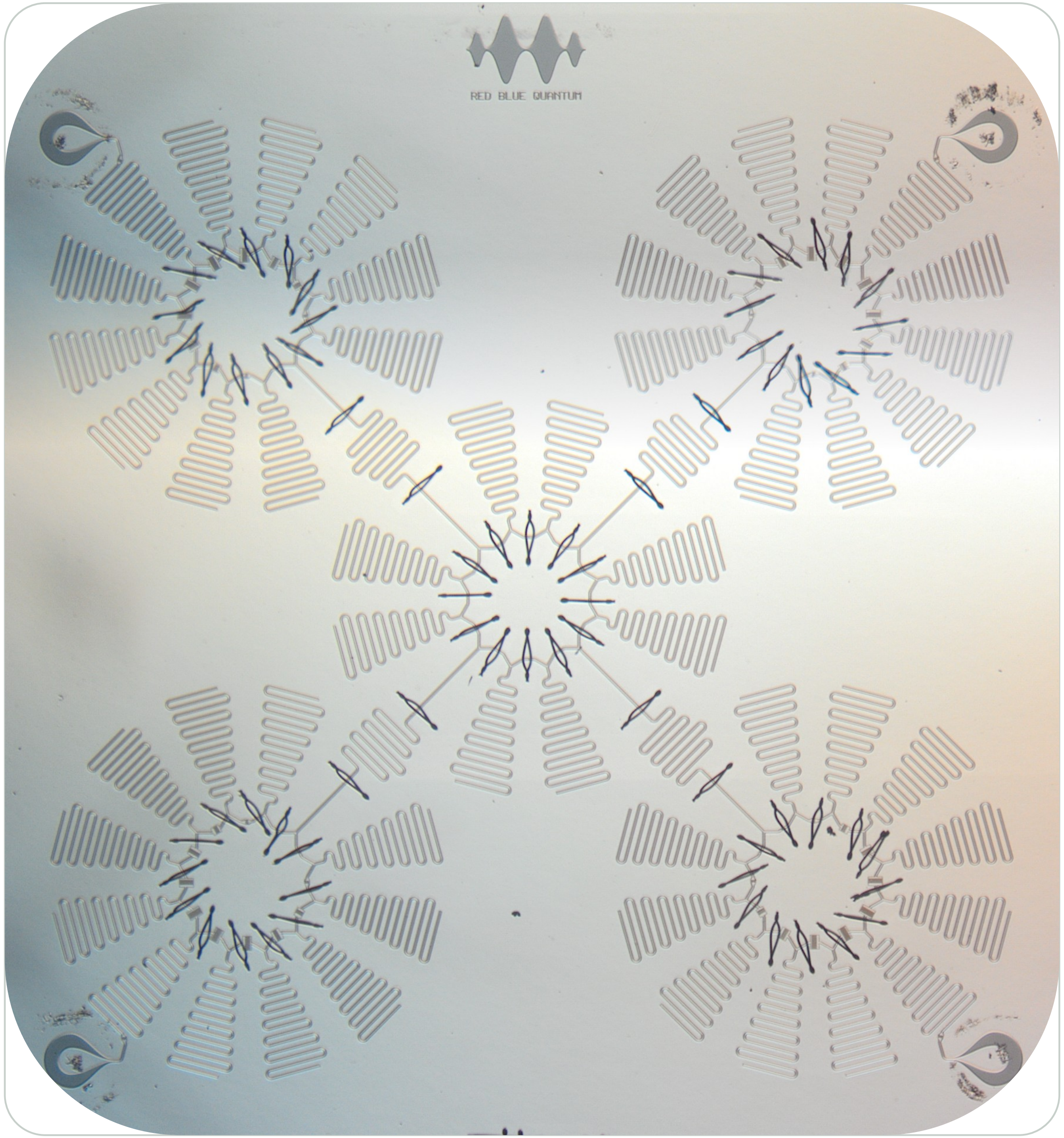
7,000
TOTAL QUBITS
largest simulated 3D resource

Circuit-level depolarizing noise: these values are not directly comparable with the previous slide.

Mahmoud, Tournaire, Bachmann, and Rayan, arXiv:2603.27004 (2026).

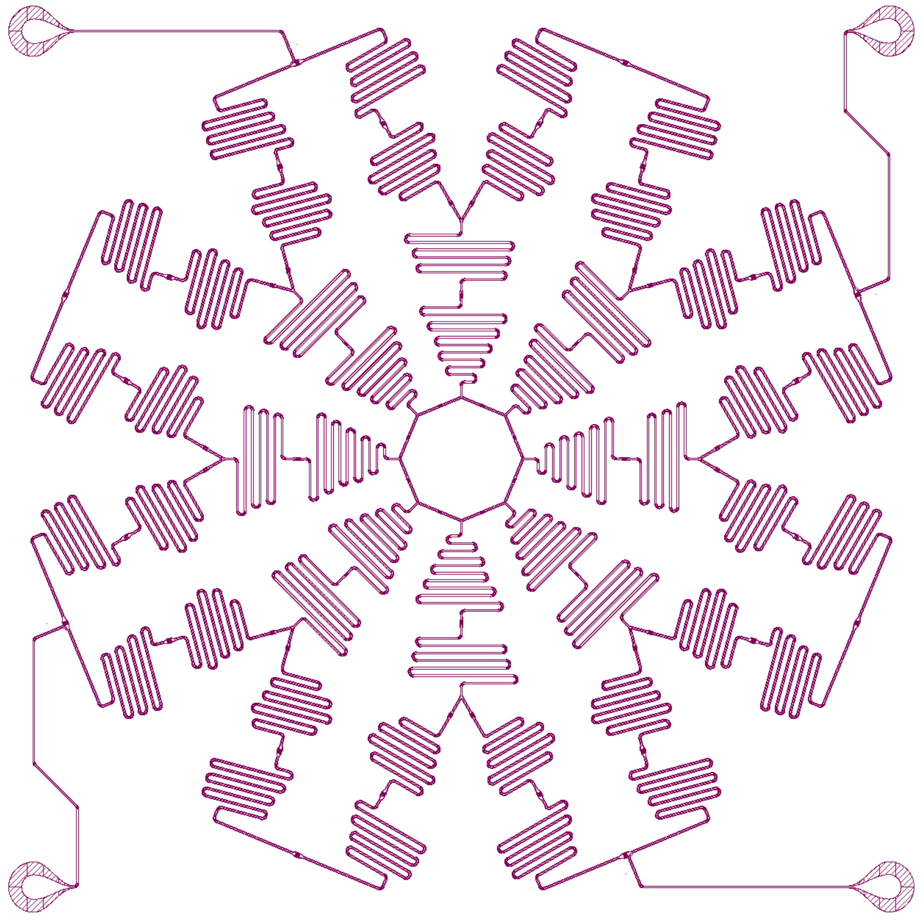
FABRICATION

Superconducting circuit simulations of hyperbolic matter



arXiv:2510.23827

A weighted open graph is the experimental Hamiltonian



$$H = -t \sum_{ij} A_{ij} a_i^\dagger a_j$$

$A_{ij} \neq 0 \;\Leftrightarrow\;$ capacitive link i–j

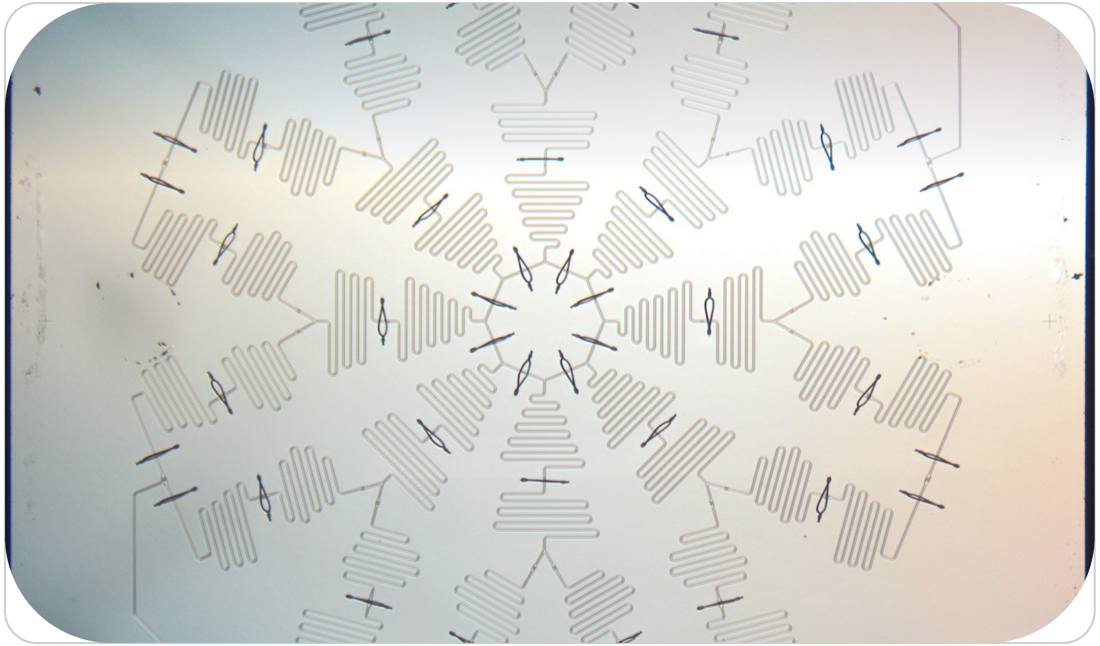
$C_{ij} \propto 1 / d_{ij}$

projected metric
sets relative weights

Global response: spectrum and resolvent of A_open
local pairwise links · physical boundary

Two open graphs probe genus-2 and genus-3 designs

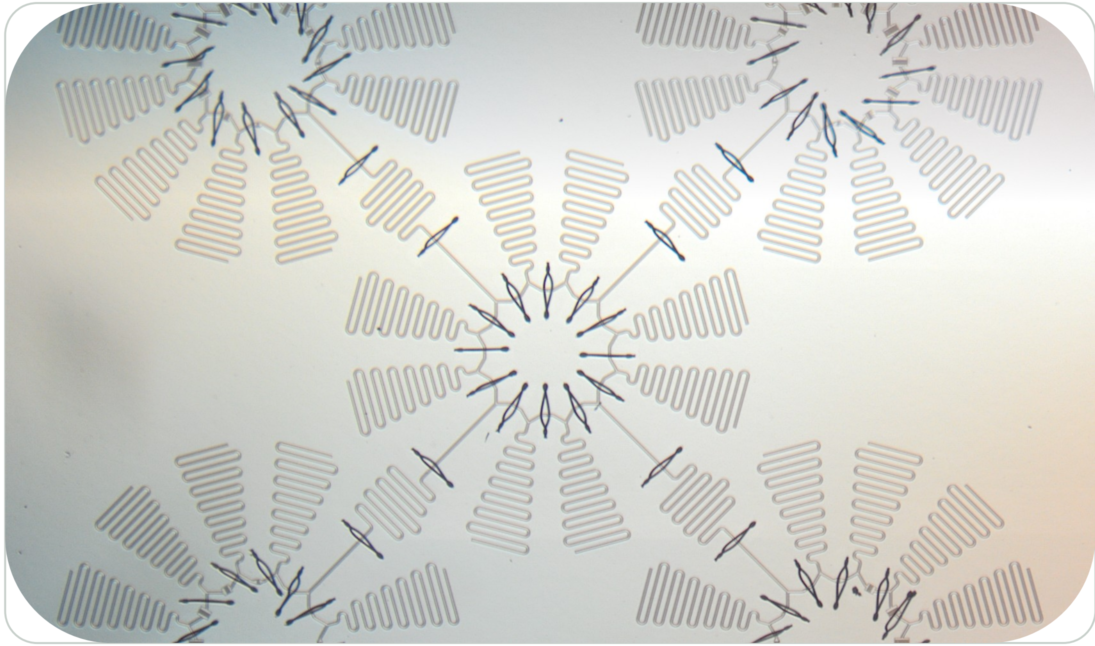
{8,3} DESIGN · ASSOCIATED g = 2



9 · 48 · 56

FACES · RESONATORS · COUPLINGS

{12,4} DESIGN · ASSOCIATED g = 3



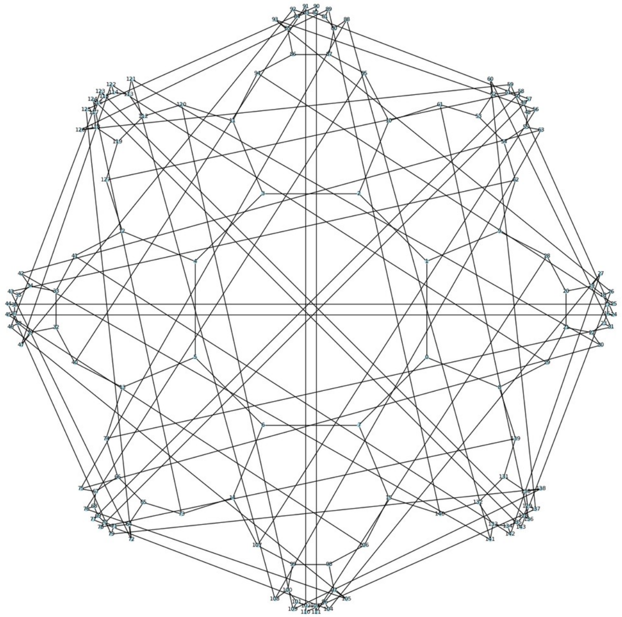
5 · 56 · 60

FACES · RESONATORS · COUPLINGS

Xu et al., arXiv:2510.23827 (2025), Figs. 2–3.

The devices are open graphs, not compact quotients

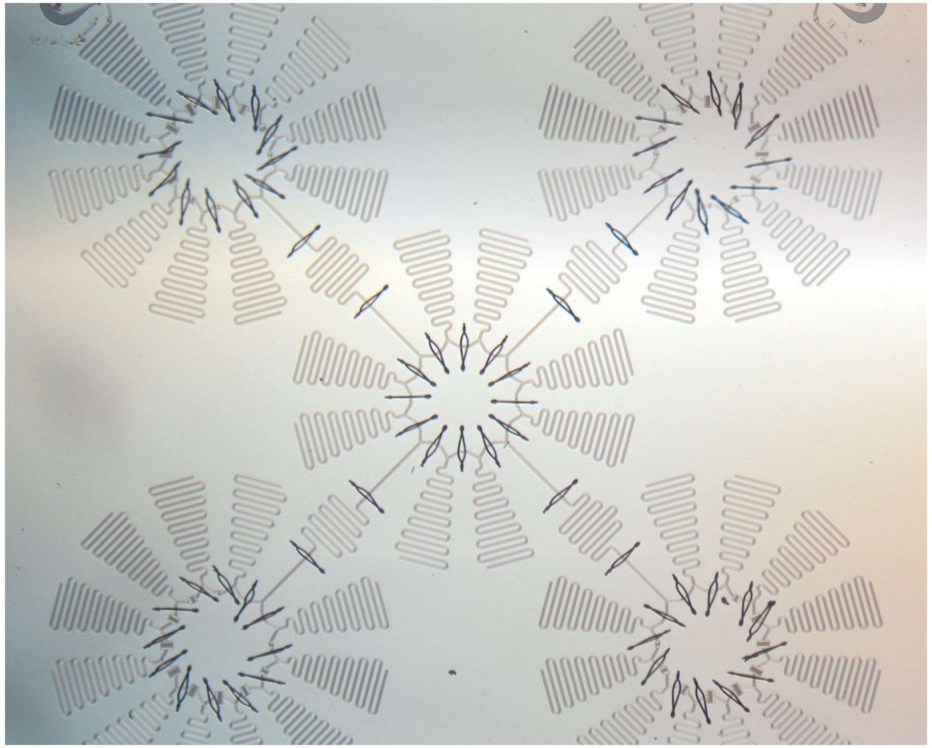
COMPACT / PERIODIC



$\Gamma_0 \lhd \Gamma$

$\neq$

FINITE / OPEN

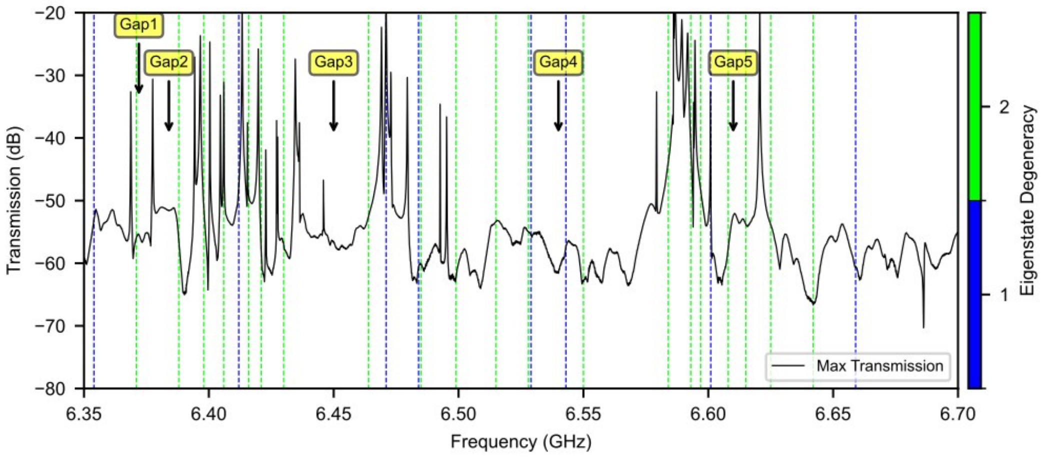


unpaired boundary edges

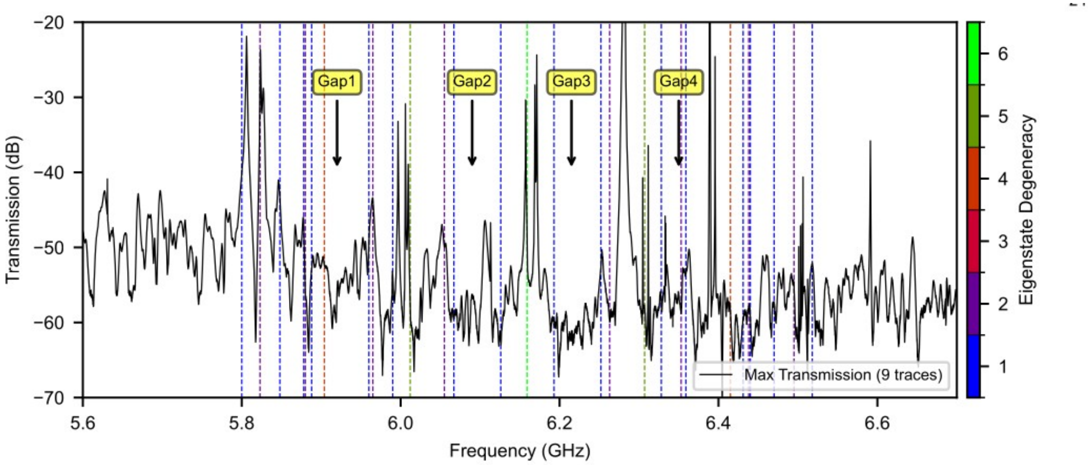
Comparison based on Maciejko–Rayan (2022), Mahmoud–Ali–Rayan (2026), and Xu et al. (2025).

Measured spectra recover graph-level clusters and gaps

GENUS-2 DESIGN



GENUS-3 DESIGN



3 principal clusters
dominant gaps retained

5 principal clusters · 4 gaps
high-degeneracy states align best

measured resonances ↔ eigenvalues of A_open

Xu et al., arXiv:2510.23827 (2025), Figs. 2d and 3d.

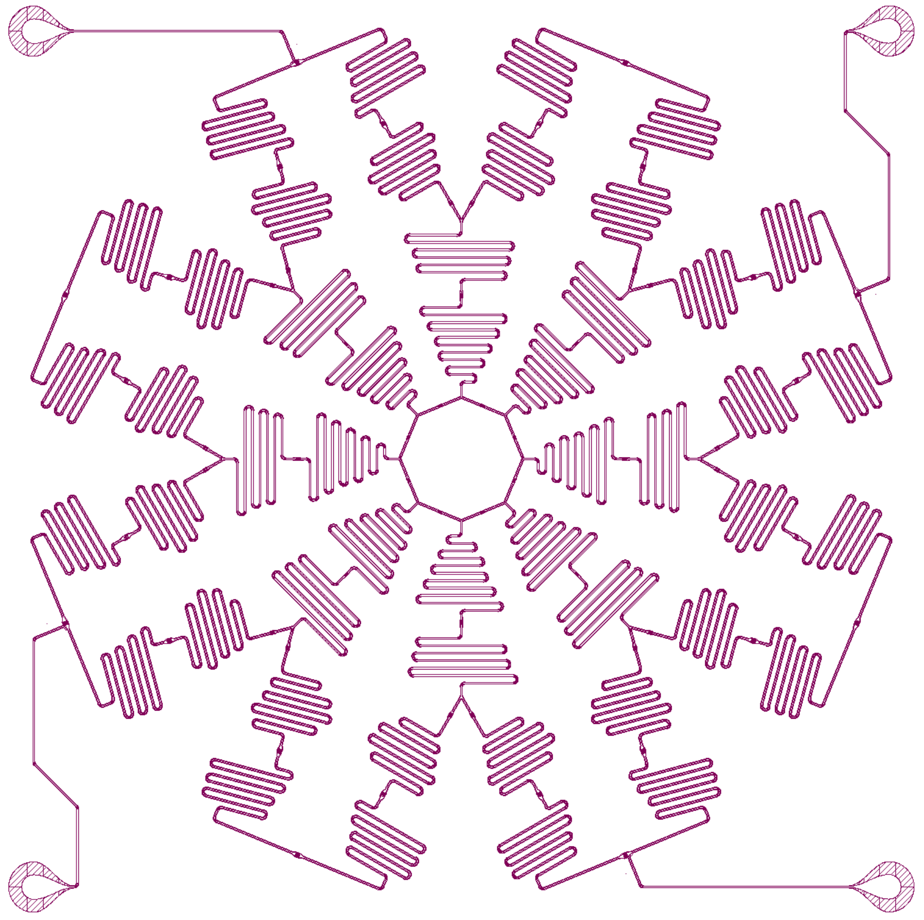
Open devices access local spectra, not global sectors

STRUCTURE / OBSERVABLE	OPEN DEVICE	COMPACT QUOTIENT
local adjacency and coupling ratios	✓	✓
tight-binding peak clusters and gaps	✓	✓
localization / participation ratios	✓	✓
noncontractible homology cycles	—	✓
flat-holonomy / flux sectors	—	✓
nonabelian Bloch sectors	—	✓
periodic surface-code logicals	—	✓

✓ demonstrated or directly represented — absent without engineered side identifications

Synthesis of the band-theory, QEC, and circuit results cited in the deck.

Tabletop AdS/CFT: observables and boundary conditions



Boundary response G(ω; a,b)

Calibrated ports along an open boundary ring

2-point → Δ

weak nonlinearity → 3-point data

Scope: analogue bulk–boundary tests; closed compact quotients have no spatial boundary

Device: Xu et al. (2025). Outlook: Dey et al., PRL 133 (2024); Chen et al. (2023).

Summary

Band theory identifies countably-many momenta sectors supporting band structures

Quantum codes turn compact-surface homology into logical operators

Resonator chips realize weighted open graphs and measure their spectra

Open Questions

- Band theory: Which higher-rank sectors govern spectra of large covers?
- Quantum codes: Which quotient towers and decoders optimize rate, systoles, and logical error?
- Holography: Can open boundaries and weak nonlinearity recover universal two- and three-point data?

Boundary-condition dictionary

CONCEPT	THEORETICAL REALIZATION	PRESENT EXPERIMENT
finite system	normal cover / quotient graph	terminated planar graph
boundary sites	paired by Γ_0	unpaired physical edges
momentum	flat holonomy on $H_1(\Sigma)$	no global momentum label
Bloch sector	irreps of Γ / Γ_0	open-graph eigenmodes
logical qubits	nontrivial cycles and cocycles	not implemented
observable	bands, holonomies, logical errors	S-parameter peaks and localization

Device numbers at a glance

DEVICE	UNIT-CELL GENUS	FACES	RESONATORS	EDGES / COUPLINGS	COUPLING
{8,3} hyperbolic	2	9	48	56	direct
{12,4} hyperbolic	3	5	56	60	direct
{8,3} kagome-like	open parent	9	56	80	mediated

≈ 6.5 GHz
FUNDAMENTAL MODE

< 10 mK
BASE TEMPERATURE

4 ports
THREE INPUTS + ONE OUTPUT

Noise models and thresholds at a glance

CONSTRUCTION	NOISE MODEL	SYNDROMES	DECODER	TRIALS / POINT	THRESHOLD
{8,3} HQECC	independent Pauli-Z	perfect	MWPM	10^4	≈ 3%
{10,3} HQECC	independent Pauli-Z	perfect	MWPM	10^4	≈ 2%
{8,3} cluster · logical Z	circuit depolarizing	faulty circuit	MWPM	2×10^4	≈ 0.8%
{8,3} cluster · logical X	circuit depolarizing	faulty circuit	MWPM	2×10^4	≈ 0.25%

Thresholds are meaningful only together with their noise and measurement assumptions.

Mahmoud–Ali–Rayan (2026) and Mahmoud–Tournaire–Bachmann–Rayan (2026).

Fabrication and cryogenic readout workflow

Prepare silicon	Deposit Al	Pattern	Etch + bond	Cool + probe
01	02	03	04	05
RCA-1 + HF clean	100 nm Al; e-beam evaporation	S1811; maskless aligner	wet etch; ground bonds	<10 mK; VNA transmission

Readout: three inputs, one output, and vector-network-analyzer S-parameters.

Multiport spectral readout and calibration

S_{21} →

S_{31} →

S_{41} →

HYPERBOLIC RESONATOR NETWORK

→ output

$$S_{max}(f) = \max\{S_{21}, S_{31}, S_{41}\}$$

$$f_n = f_1 + (\lambda_n - \lambda_1)(f_2 - f_1)/(\lambda_2 - \lambda_1)$$

S_{max} retains a mode even when it has a node at one input port.

Fabrication disorder lifts degeneracies into clusters of nearby resonances.

Selected papers behind this talk

BAND THEORY	J. Maciejko and S. Rayan, Hyperbolic band theory, Science Advances 7, eabe9170 (2021).	**arXiv:2008.05489**
BLOCH THEOREM	J. Maciejko and S. Rayan, Automorphic Bloch theorems for hyperbolic lattices, PNAS 119, e2116869119 (2022).	**arXiv:2108.09314**
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