GENERATIVE FOURIER NEURAL OPERATORS FOR NON LINEAR INVERSE PROBLEMS

ANUPAM GUMBER

In scientific machine learning, neural operators have emerged as a new frontier in recent years. These neural operators are a type of deep architecture that learns the nonlinear solution operator of partial differential equations. One notable example is the Fourier neural operator (FNO), which has recently shown excellent performance by leveraging the fast Fourier transform to handle infinite-dimensional operators with a finite number of parameters. In this presentation, we introduce a novel concept called Generative Fourier Neural Operators (GFNOs), which are generative models with the output belonging to an infinite-dimensional function space. To provide a generic framework for constructing GFNOs, we propose a new neural operator architecture inspired by a pseudo-differential integral operator (PDIO), which generalizes the Fourier integral operator in FNO. Unlike the FNO, the PDIO architecture demonstrates reduced overfitting and improved performance. By combining the PDIO with the Fourier neural operator, we develop Generative Fourier Neural Operators (GFNOs). To analyze the performance of these models, we seek a mathematical framework that establishes a continuous-discrete equivalence for this architecture. Additionally, in the realm of inverse problems, generative models can be used to model prior information on the unknown with a higher level of accuracy than classical regularization methods. We aim to utilize this data-driven approach based on GFNOs and apply it to derive stability estimates for some potentially nonlinear infinite-dimensional inverse problems. This is ongoing work jointly with G. S. Alberti and M. Santacesaria (University of Genoa).