Talk Title: S-unit equations and curves with few primes over cyclotomic Z_l extensions

Abstract: Let K be a number field, and S a finite set of non-archimedean places of K. A famous theorem of Siegel asserts that the S-unit equation u+v=1 has only finitely many solutions in S-units of K. A famous theorem of Shafarevich asserts that there are only finitely many isomorphism classes of elliptic curves over K with good reduction outside S. Now let l be a prime, and instead of a number field, let K be the Z l-cyclotomic extension of the rationals. We show that the S-unit equation u+v=1 has infinitely many solutions for l=2,3,5 or 7, where S consists only of the totally ramified prime above l. Moreover, for every prime l, we construct infinitely many elliptic or hyperelliptic curves defined over K with good reduction away from 2 and l. For certain primes l we show that the Jacobians of these curves in fact belong to infinitely many distinct isogeny classes. This talk is based on joint work with Robin Visser.